String-Inspired Triplet See-Saw from Diagonal Embedding of $SU(2)_L \subset SU(2)_A \times SU(2)_B$

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Motivated by string constructions, we consider a variant on the Type II see-saw mechanism involving the exchange of triplet representations of $SU(2)_L$ in which this group arises from a diagonal embedding into $SU(2)_A \times SU(2)_B$. A natural assignment of Standard Model lepton doublets to the two underlying gauge groups results in a bimaximal pattern of neutrino mixings and an inverted hierarchy in masses. Simple perturbations around this leading-order structure can accommodate the observed pattern of neutrino masses and mixings.

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Introduction

Observations by a variety of experimental collaborations have now firmly established the hypothesis that neutrino oscillations occur and that they are the result of non-vanishing neutrino masses and mixing angles [1, 2, 3]. While our knowledge of neutrino massdifferences and mixings has continued to improve over recent years, there continues to be as yet no consensus on the correct mechanism for generating the quite small neutrino masses implied by the experimental data. In some respects this is similar to the case of quark masses and mixings: despite having access to even more of the relevant experimental data for an even longer period of time, no compelling model of the hierarchies of masses and mixings in the quark sector has emerged either. But most theoretical effort in the area of neutrinos goes beyond the simple Dirac-mass Yukawa operator by introducing new structures in the superpotential to account for neutrino masses, such as the see-saw mechanism (in one of its various forms, to be defined more precisely below).¹ Thus neutrinos are likely to be very special in the Standard Model – and its supersymmetric extensions – and may thereby provide a unique window into high-scale theories that the quark sector fails to illuminate.

It has thus far been mostly in vain that we might look to string theory for some guidance in how to approach the issue of flavor. In part this is because of the vast number of possible vacua in any particular construction, each with its own set of fields and superpotential couplings between them. On the other hand the problem of generating small neutrino masses may be one of the most powerful discriminants in finding realistic constructions. This was one of the conclusions of a recently completed survey [9] of a large class of explicit orbifold compactifications of the heterotic string for the standard (or "Type I") see-saw in its minimal form. The fact that no such viable mechanism was found may suggest that often-neglected alternatives to the standard see-saw may have more theoretical motivation than considerations of simplicity, elegance, or GUT structure would otherwise indicate.

In this work we study the properties of a new construction of see-saw mechanisms that is motivated by known string constructions. The mechanism is an example of the Higgs triplet or "Type II" seesaw [10, 11, 12, 13, 14, 15], but the stringy origin has important implications for the mixings and mass hierarchy that distinguish it from conventional "bottom-up" versions of the triplet model. After outlining the model in a general way below we will motivate its plausibility in string theory by considering a particular $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbifold of the heterotic string [16], where several of the properties needed for a fully realistic model are manifest.

I. GENERAL FEATURES OF SU(2) TRIPLET MODELS

Let us briefly review the form of the effective neutrino mass matrix to establish our notation and to allow the contrast between models involving triplets of $SU(2)_L$ and those involving singlets to be more apparent. While models of neutrino masses can certainly be considered without low-energy supersymmetry, our interest in effective Lagrangians deriving from string theories which preserve N = 1 supersymmetry leads us to couch our discussion in a supersymmetric framework. Then the effective mass operator involving only the light (left-handed) neutrinos has mass dimension five. Once the Higgs fields acquire vacuum expectation values (vevs) the effective neutrino masses are given by

$$(\mathbf{m}_{\nu})_{ij} = (\lambda_{\nu})_{ij} \frac{v_2^2}{M}, \qquad (1)$$

where v_2 is the vev of the Higgs doublet H_2 with hypercharge Y = +1/2. The 3×3 matrix of couplings λ_{ν} is necessarily symmetric in its generation indices *i* and *j*.

Such an operator can be induced through the exchange of heavy singlet (right-handed) neutrinos N_R – as in the standard or "Type I" see-saw approach [17, 18, 19] – or through the exchange of heavy triplet states T [10, 11, 12, 13, 14, 15], or both. In either case, the mass

¹ For some recent reviews of theoretical models of neutrino masses and mixings, see [4, 5, 6, 7, 8] and references therein.

scale M is given by the scale at which lepton number is broken (presumably the mass scale of the heavy state being exchanged). In the presence of both contributions to the light neutrino masses we have the general mass matrix

$$\mathcal{L} = \frac{1}{2} \begin{pmatrix} \overline{\nu}_L & \overline{N}_L^c \end{pmatrix} \begin{pmatrix} \mathbf{m}_T & \mathbf{m}_D \\ \mathbf{m}_D^T & \mathbf{m}_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{ h.c.} \quad (2)$$

Each of the four quantities in (2) are understood to be 3×3 matrices in flavor space. That is, we imagine a model with one *species* of lepton doublet L (with three generations) and, if present, one *species* of right-handed neutrino field (with three generations).

We will use the name "triplet models" to refer to any model which uses electroweak triplet states alone to generate neutrino masses. That is, such a model dispenses with right-handed neutrinos altogether, and the effective neutrino mass in (1) is then simply identified with the entry \mathbf{m}_T in (2). A supersymmetric extension of the MSSM capable of giving small effective masses to lefthanded neutrinos would involve two new sets of fields T_i and \overline{T}_i which transform as triplets under $SU(2)_L$ and have hypercharge assignments $Y_T = +1$ and $Y_{\overline{T}} = -1$, respectively [14, 15]. For the time being we will consider just one pair of such fields, which couple to the Standard Model through the superpotential

$$W_{\nu} = (\lambda_T)_{ij} L_i T L_j + \lambda_1 H_1 T H_1 + \lambda_2 H_2 \overline{T} H_2 + M_T T \overline{T} + \mu H_1 H_2, \qquad (3)$$

where the SU(2) indices on the doublets and triplet have been suppressed. Strictly speaking, the coupling λ_1 is not necessary to generate the required neutrino masses, but given the Standard Model charge assignments of the fields T and \overline{T} there is no *a priori* reason to exclude this coupling. The mass scale M in (1) is to be identified with M_T in this case, and the matrix λ_T is symmetric in its generation indices. From the Lagrangian determined by (3), it is clear that should the auxiliary fields of the chiral supermultiplets for the triplets vanish in the vacuum $\langle F^T \rangle = \langle F^{\overline{T}} \rangle = 0$, and we assume no vevs for the left-handed sneutrino fields, then there is a simple solution for the vevs of the neutral components of the triplet fields

$$\langle T \rangle = -\frac{\lambda_2 \langle H_2 \rangle^2}{M_T}; \quad \langle \overline{T} \rangle = -\frac{\lambda_1 \langle H_1 \rangle^2}{M_T}, \qquad (4)$$

implying $m_{\nu} = \lambda_T \langle T \rangle$.

Recent models of the Type II variety [4, 5, 6, 7, 8] would typically retain the right-handed neutrinos and utilize all the components in the mass matrix of (2) to explain the neutrino masses and mixings. These examples are often inspired by SO(10) GUT considerations or are couched in terms of left-right symmetry more generally. The latter commonly employ additional Higgs fields transforming as (1, 3) under $SU(2)_L \times SU(2)_R$, which

acquire vevs to break the gauge group to the Standard Model.

Instead we imagine a process by which the SU(2) of the Standard Model emerges as the result of a breaking to a *diagonal* subgroup $SU(2)_A \times SU(2)_B \rightarrow SU(2)_L$ at a very high energy scale. Furthermore, while we will employ two conjugate triplet representations which form a vector-like pair under the Standard Model gauge group with $Y = \pm 1$, we do not seek to embed this structure into a left-right symmetric model.

II. DIAGONAL EMBEDDING OF $SU(2)_L$

In attempting to embed the framework of the previous section in a model of the weakly coupled heterotic string we immediately encounter an obstacle: the simplest string constructions contain in their massless spectra only chiral superfields which transform as fundamentals or (anti-fundamentals) of the non-Abelian gauge groups of the low-energy theory. Scalars transforming as triplets of SU(2) simply do not exist for such affine level one constructions [20, 21]. Indeed, scalars transforming in the adjoint representation appear only at affine level two, while representations such as the **120** and **210** of SO(10) appear only at affine level four - and the **126** of SO(10), which contains triplets of the Standard Model $SU(2)_L$, has been shown to never appear in free-field heterotic string constructions [22].

Directly constructing four-dimensional string compactifications yielding higher affine-level gauge groups has proven to be a difficult task. But a group factor \mathcal{G} can be effectively realized at affine level k = n by simply requiring it to be the result of a breaking of $\mathcal{G}_1 \times \mathcal{G}_2 \times \cdots \times \mathcal{G}_n$ to the overall diagonal subgroup. In fact, these two ways of understanding higher affine levels – picking a particular set of GSO projections in the underlying string construction and the low energy field theory picture of breaking to a diagonal subgroup – are equivalent pictures [22]. With this as motivation, let us consider an appropriate variation on the superpotential of (3). The breaking of the gauge group $SU(2)_A \times SU(2)_B$ to the diagonal subgroup, which we identify as $SU(2)_L$, can occur through the vacuum expectation value of a field in the bifundamental representation of the underlying product group via an appropriately arranged scalar potential. For the purposes of our discussion here we will need only assume that this breaking takes place at a sufficiently high scale, say just below the string compactification scale. As such ideas for product group breaking have been considered in the past [23], we will not concern ourselves further with this step.

Any additional bifundamental representations will decompose into triplet and singlet representations under the surviving $SU(2)_L$. Gauge invariance of the underlying $SU(2)_A \times SU(2)_B$ theory then requires that the neutrino-mass generating superpotential coupling involving lepton doublets and our SU(2) triplet now be given by

$$W_{\nu} = (\lambda_T)_{ij} L_i T L'_j, \tag{5}$$

where the field T is the $SU(2)_L$ triplet representation and L and L' are two *different species* of doublets under the $SU(2)_L$ subgroup. That is, we denote by L fields which have representation (2, 1) and by L' fields which have representation (1, 2) under the original $SU(2)_A \times SU(2)_B$ gauge theory. These two sets of lepton doublets, each of which may carry a generation index as determined by the string construction, arise from different sectors of the string Hilbert space. Once the gauge group is broken to the diagonal subgroup this distinction between the species is lost except for the pattern of couplings represented by the matrix λ_T . The indices i and j carried by the lepton doublets represent internal degeneracies arising from the specific construction. It is natural to identify these indices with the flavor of the charged lepton (up to mixing effects, which we assume to be small).

In a minimal model, with only three lepton doublets charged under the Standard Model $SU(2)_L$, we are obliged to separate the generations, with two arising from one sector of the theory and one from the other. The precise form of the effective neutrino mass matrix will depend on this model-dependent identification, but one property is immediately clear: the effective neutrino mass matrix will necessarily be off-diagonal in the charged lepton flavor-basis.

We will restrict our study to the case of one triplet state with supersymmetric mass M_T as in (3). If we separate the doublet containing the electron from the other two, by defining $L_i = L_e = (\mathbf{2}, 1)$ and $L'_j = L_{\mu}, L_{\tau} = (1, \mathbf{2})$ under $SU(2)_A \times SU(2)_B$, then the matrix of couplings λ_T is (to leading order)

$$\lambda_T = \lambda_0 \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}.$$
 (6)

It is natural to assume that the overall coefficient λ_0 in (6) is of order unity. In fact if we now return to a string theory context, particularly that of the heterotic string with orbifold compactification, then the fact that the two generations of L'_j in (5) arise from the same sector of the string Hilbert space (i.e., the same fixed point location under the orbifold action) suggests that we should identify the coupling strengths: a = b.

Neutrino mass matrices based on the texture in (6) with a = b are not new to this work, but were in fact considered not long ago as a starting point for the bimaximal mixing scenario [24, 25, 26, 27]. In fact, the form of (6) can be derived from a bottom-up point of view by first postulating a new symmetry based on the modified lepton number combination $L_e - L_\mu - L_\tau$ [28, 29]. Indeed, the operator in (5) with the identification of $L = L_e$ and $L' = L_\mu, L_\tau$ does indeed conserve this quantum number. However, in the string-theory motivated (top down) approach this conserved quantity arises as an *accidental*

symmetry pertaining to the underlying geometry of the

string compactification. It reflects the different geometrical location of the fields (in terms of orbifold fixed points) of the electron doublet from the muon and tau doublets.

To make contact with data it is necessary to consider the Yukawa interactions of the charged leptons as well. To that end, our string-inspired model should have a superpotential of the form

$$W = \lambda_{T}LTL' + \lambda_{1}H_{1}TH'_{1} + \lambda_{2}H_{2}TH'_{2} + \lambda_{3}S_{3}T\overline{T} + \lambda_{4}S_{4}H_{1}H_{2} + \lambda_{5}S_{5}H'_{1}H'_{2} + \hat{\lambda}_{4}\hat{S}_{4}H_{1}H'_{2} + \hat{\lambda}_{5}\hat{S}_{5}H'_{1}H_{2} + \lambda_{6}S_{6}LH_{1} + \lambda_{7}S_{7}L'H'_{1} + \hat{\lambda}_{6}\hat{S}_{6}LH'_{1} + \hat{\lambda}_{7}\hat{S}_{7}L'H_{1},$$
(7)

where generation indices have been suppressed. The terms proportional to the couplings λ_1 and λ_2 in (3) must now be modified to reflect the fact that the Higgs doublets must also come from two different species. These are denoted in the same manner as the lepton doublets: $H_{1,2}$ for (2,1) representations and $H'_{1,2}$ for (1,2) representations. The second line in (7) are the dynamically-generated supersymmetric mass terms, with $\lambda_3 \langle S_3 \rangle \equiv M_T$. The fields S_3, S_4 and S_5 are singlets under $SU(2)_A \times SU(2)_B$ with hypercharge Y = 0, while \hat{S}_4 and \widehat{S}_5 are $SU(2)_L$ singlets with Y = 0 transforming as $(\mathbf{2},\mathbf{2})$ under $SU(2)_A \times SU(2)_B$. We anticipate a large vev for S_3 . The fields S_4, S_5, \hat{S}_4 , and \hat{S}_5 may acquire TeV scale vevs from supersymmetry breaking, leading to generalized μ terms², or some could have vevs near the string scale (or at an intermediate scale), projecting some of the Higgs states out of the low energy theory. From the point of view of both $SU(2)_L$ as well as the underlying $SU(2)_A \times SU(2)_B$ it is not necessary that S_4 and S_5 be distinct fields; there may be string selection rules forbidding their identification in an explicit construction, however. Similar statements apply to S_4 and S_5 . Of course, some of these fields could be absent.

The final line of (7) represents the Dirac mass couplings of the left-handed leptons with their right-handed counterparts. Again, the fields S_6 and S_7 , both singlets under $SU(2)_A \times SU(2)_B$, carrying only hypercharge Y = +1, may or may not be identified depending on the construction, while \hat{S}_6 and \hat{S}_7 , which may or may not be distinct, transform as (2, 2). Some of these fields may be absent. We assume that S_6 , S_7 , \hat{S}_6 , and \hat{S}_7 do not acquire vevs. Charged lepton masses are then determined by some combination of the coupling matrices λ_6 , λ_7 , $\hat{\lambda}_6$ and $\hat{\lambda}_7$ (and possibly higher-order terms that connect the two sectors) as well as appropriate choices of Higgs vevs for the neutral components of the four Higgs species.

² The μ parameters of the Higgs scalar potential could also arise as effective parameters only after SUSY breaking via the Giudice-Masiero mechanism [30].

III. MAKING CONTACT WITH EXPERIMENTAL DATA

Having laid out the framework for our string-based model, we now wish to ask how well such a structure can accommodate the measurements of neutrino mixing angles and mass differences that have been made, and what sort of predictions (if any) might this framework make in terms of future experimental observations. We use a convention in which the solar mixing data defines the mass difference between m_2 and m_1 with $m_2 > m_1$. Then the eigenvalue m_3 relevant for the atmospheric data is the "isolated" eigenvalue.

The current experimental picture is summarized by the recent three neutrino global analysis in [3]. For the solar neutrino sector we take

$$\Delta m_{12}^2 = 7.92(1 \pm 0.09) \times 10^{-5} \text{eV}^2 \tag{8}$$

$$\sin^2 \theta_{12} = 0.314(1^{+0.18}_{-0.15}), \tag{9}$$

where all measurements are $\pm 2\sigma$ (95% C.L.). The last measurement implies a value for the mixing angle θ_{12} itself of $\theta_{12} \simeq 0.595^{+0.060}_{-0.052}$, well below the maximal mixing value $\theta_{12}^{max} = \pi/4$. We take the upper bound on θ_{13} to be

$$\sin^2 \theta_{13} = 0.9^{+2.3}_{-0.9} \times 10^{-2} \quad \Rightarrow |\theta_{13}| < 0.18 \tag{10}$$

at the 2σ level. For the atmospheric oscillations

$$|\Delta m_{23}^2| = 2.4(1^{+0.21}_{-0.26}) \times 10^{-3} \text{eV}^2$$
 (11)

$$\sin^2 \theta_{23} = 0.44(1^{+0.41}_{-0.22}), \qquad (12)$$

consistent with maximal mixing $(\sin^2 \theta_{23}^{\max} = 0.5)$.

With this in mind, let us consider the general offdiagonal Majorana mass matrix

$$m_{\nu} = \begin{pmatrix} 0 & a & b \\ a & 0 & \epsilon \\ b & \epsilon & 0 \end{pmatrix} = m_{\nu}^{T}$$
(13)

with det $m_{\nu} = -2ab\epsilon$, and where we imagine the entry ϵ to be a small perturbation around the basic structure of (6), which can arise from higher-order terms in W. Without loss of generality we can redefine the phases of the lepton doublets L_i and L'_i such that the entries a, b and ϵ are real and $m_{\nu} = m_{\nu}^{\dagger}$. This implies

$$U_{\nu}^{\dagger}m_{\nu}U_{\nu} = \text{diag}(m_1, m_2, m_3) \equiv m_{\text{diag}}.$$
 (14)

We also have Tr $m_{\text{diag}} = m_1 + m_2 + m_3 = \text{Tr } m_{\nu} = 0$, where the various eigenvalues m_i are real but can be negative.

If we begin by first ignoring the solar mass difference, and take the atmospheric mass difference to be given by (11), then there is no way to accommodate the "normal" hierarchy while maintaining the requirement that $\epsilon \ll a, b$. For the inverted hierarchy (in the same approximation of vanishing solar mass difference) we would require $m_2 = -m_1 = 0.049 \text{ eV}$ with $m_3 = 0$. This could derive from (13) if $\sqrt{a^2 + b^2} = m_2$ and $\epsilon = 0$. In this case $\sum_i |m_i| = 0.098 \text{ eV}$. This is clearly in line with the form of (5) and implies a triplet mass of order

$$M_T = 2.0 \times \lambda_2 \lambda_T \left(\frac{v_2 v_2'}{(100 \text{ GeV})^2}\right) \times 10^{14} \text{ GeV}$$
(15)

where we have defined $v_2 = \langle (h_2)^0 \rangle$ and $v'_2 = \langle (h'_2)^0 \rangle$. The solar mass difference (8) can be restored in this case by taking $\epsilon \simeq \frac{1}{43}$ in the mass matrix given by

$$m_{\nu} = \sqrt{\frac{|\Delta m_{23}^2|}{2}} \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & \epsilon \\ -1 & \epsilon & 0 \end{pmatrix}.$$
 (16)

This value is particularly encouraging for theories motivated by the weakly coupled heterotic string compactified on orbifolds. Such theories generally give rise to an Abelian gauge factor with non-vanishing trace anomaly. This anomaly is cancelled by the Green-Schwarz mechanism, which involves a Fayet-Iliopoulos (FI) term $\xi_{\rm FI}$ in the 4D Lagrangian [31, 32, 33]. In general, at least one field X of the massless spectrum, charged under this anomalous U(1) factor, will receive a vev $X \simeq \sqrt{\xi_{\rm FI}}$ so as to ensure $\langle D_X \rangle = 0$ below the scale $\xi_{\rm FI}$. Explicit orbifold constructions suggest that $0.09 \leq r_{\rm FI} = \sqrt{|\xi_{\rm FI}|}/M_{\rm PL} \leq$ 0.14 for $g^2 \simeq 1/2$ [34]. Thus the perturbation ϵ could be the result of non-renormalizable operators in the superpotential of relative low-degree – perhaps involving only one or two powers of such a field vev, depending on the size of the dimensionless Yukawa couplings involved.

Considering the underlying $SU(2)_A \times SU(2)_B$ theory, fields bifundamental under both SU(2) factors will decompose into a triplet and a singlet under the breaking to the diagonal subgroup. Let us denote this singlet representation as ψ . Then terms at dimension four in the superpotential that can populate the vanishing entries in (6) include

$$\Delta W = \frac{\lambda_{11}}{M_{\rm PL}} L_1(\mathbf{2}, 1) T(\mathbf{2}, \mathbf{2}) \psi(\mathbf{2}, \mathbf{2}) L_1(\mathbf{2}, 1) + \frac{\lambda_{ij}}{M_{\rm PL}} L'_i(1, \mathbf{2}) T(\mathbf{2}, \mathbf{2}) \psi(\mathbf{2}, \mathbf{2}) L'_j(1, \mathbf{2}), \quad (17)$$

where i, j = 2, 3 and we denote the representations under $SU(2)_A \times SU(2)_B$ for convenience. The singlet field ψ must have vanishing hypercharge, so it cannot be the singlet component of the same bifundamental that led to T and \overline{T} , though it may be the singlet component of some bifundamental representation that served to generate the breaking to the diagonal subgroup in the first place, or could be identified with \hat{S}_4 or \hat{S}_5 of (7). To the extent that string models seldom give self-couplings at such a low order in the superpotential, we might expect $\lambda_{11} = \lambda_{22} = \lambda_{33} = 0$, thereby generating (16) at roughly the correct order of magnitude.

Now let us consider the leptonic (PMNS) mixing matrix defined by $U_{\text{PMNS}} = U_e^{\dagger} U_{\nu}$, where U_{ν} is the matrix

in (14) and U_e is the analogous matrix for the charged leptons. Most of the earlier studies [5, 6, 7, 8, 24, 25, 26, 27, 28, 29] of the texture in (6) assumed that this form holds in the basis for which $U_e = 1$. In that case, one has an inverted hierarchy and $U_{\rm PMNS} = U_{\nu}$ is bimaximal for a = b, i.e., $\theta_{12} = \theta_{23} = \pi/4$, while $\theta_{13} = 0$. For $|a| \neq |b|$ the solar mixing remains maximal while the atmospheric mixing angle is $|\tan \theta_{23}| = |b|/|a|$. It is now well established, however, that the solar mixing is not maximal, i.e., $\pi/4 - \theta_{12} = 0.19^{+0.05}_{-0.06}$ [3], where the quoted errors are 2σ . It is well-known that reasonable perturbations on this texture (still with $U_e = 1$) have difficulty yielding a realistic solar mixing and mass splitting. To see this, let us add a perturbation δ to the 13-entry of (16) and perturbations ϵ_{ii} to the diagonal entries. To leading order, δ only shifts the atmospheric mixing from maximal (to $\theta_{12} \sim \pi/4 - \delta/2$). ϵ_{22} and ϵ_{33} large enough to affect the Solar mixing tend to give too large contributions to $|\theta_{13}|$, so we will ignore them (their inclusion would merely lead to additional fine-tuned parameter ranges). One then finds

$$\frac{\pi}{4} - \theta_{12} \simeq 0.19 \simeq \frac{1}{4} (\epsilon - \epsilon_{11}),$$
 (18)

whereas the solar mass difference is

$$\frac{\Delta m_{12}^2}{\sqrt{2}|\Delta m_{\rm atm}^2|} \simeq \frac{1}{43} \simeq (\epsilon + \epsilon_{11}). \tag{19}$$

Satisfying these constraints would require a moderate tuning of ϵ and ϵ_{11} . Moreover, they would each have to be of order 0.4 in magnitude, somewhat large to be considered perturbations.

On the other hand, a simple and realistic pattern emerges when we instead allow for small departures from $U_e \propto \mathbf{1}$ [35, 36, 37, 38, 39, 40], and for the general superpotential in (7) there is no reason for such mixings to be absent.³ For example, starting from (16) a Cabibbo-sized 12-entry in the charged lepton mixing matrix

$$U_e^{\dagger} \sim \begin{pmatrix} 1 & -\sin\theta_{12}^e & 0\\ \sin\theta_{12}^e & 1 & 0\\ 0 & 0 & 1 \end{pmatrix},$$
(20)

leads to

$$\frac{\pi}{4} - \theta_{12} \simeq \frac{\sin \theta_{12}^e}{\sqrt{2}},\tag{21}$$

which is satisfied for $\sin \theta_{12}^e \simeq 0.27^{+0.07}_{-0.08}$. This mixing also leads to the prediction of a large

$$\sin^2 \theta_{13} \simeq \frac{\sin^2 \theta_{12}^e}{2} \simeq (0.017 - 0.059)$$
(22)

$$n_{\beta\beta} \simeq m_2 (\cos^2 \theta_{12} - \sin^2 \theta_{12}) \simeq 0.018 \text{ eV}$$
 (23)

for the effective mass relevant to neutrinoless double beta decay. This is the standard result for the inverted hierarchy, with the minus sign due to the opposite signs of m_1 and m_2 . Such a value should be observable in planned experiments [5, 6, 7, 8].

IV. REALIZATION IN HETEROTIC STRING MODELS

Having outlined in a broad manner the elementary requirements for phenomenological viability of any tripletbased model with a structure dictated by the superpotential in (7), we might now wish to ask whether such a set of fields and couplings really does arise in explicit string constructions as we have been assuming. Rather than build all possible constructions of a certain type for a dedicated scan – an undertaking that would undoubtedly produce interesting results in many areas, but which we reserve for a future study – we will here choose one particular example as a case study. The $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbifold construction of Font et al. [16] begins with a nonstandard embedding that utilizes two shift vectors and one Wilson line in the first complex plane. This Wilson line breaks the observable sector gauge group from SO(10) to $SU(3) \times SU(2)_A \times SU(2)_B$. The massless spectrum of this model contains 75 species of fields. Those from the untwisted sectors have a multiplicity of one, while twisted sectors have a multiplicity of three or nine, depending on the representation. It is natural to consider this multiplicity factor as a generation index.

There are three species of fields which are bifundamental under the observable sector $SU(2)_A \times SU(2)_B$ (one in the untwisted sector and two in various twisted sectors), five doublets under $SU(2)_A$ and eight doublets under $SU(2)_B$. There were also 17 species that were singlets under all non-Abelian groups. So the minimal set of fields needed to generate the superpotential of (7) are present, as well as an additional bifundamental representation that may be used to break the product group to the diagonal subgroup and/or generate the needed higherorder corrections in (17). We note that there are additional species that have non-trivial representations under the non-Abelian groups of *both* the observable and hidden sectors. In order to avoid potential complications should any of these hidden sector groups undergo confinement we have not considered these in what follows.

From the selection rules given in [16] it is possible to construct all possible dimension three (renormalizable) and dimension four (non-renormalizable) superpotential couplings consistent with gauge invariance. Considering only the 33 relevant fields mentioned in the previous paragraph, the selection rules and gauge invariance under the observable and hidden sector non-Abelian groups

³ The relatively large value required for $\sin \theta_{12}^e$ compared to $\sqrt{m_e/m_{\mu}} \sim 0.07$ suggests an asymmetric charged lepton mass matrix, but this would not be unexpected.

allow 32 and 135 terms at dimension three and four, respectively. Requiring in addition gauge invariance under the six U(1) factors (one of which being anomalous) reduces these numbers to a tractable 15 and 8, respectively.

To ascertain which of the terms in (7) can be identified from the above it is necessary to choose a linear combination of the five non-anomalous U(1) factors to be identified as hypercharge, and then determine the resulting hypercharges of the bifundamentals, doublets and singlets under this assignment. Our algorithm was to begin with the two bifundamentals of the twisted sector, as the untwisted bifundamental had no couplings to SU(2) doublets at dimension three or four. These two twisted sector fields had a selection-rule allowed coupling to a non-Abelian group singlet at the leading (dimension three) order, which could therefore play the role of S_3 in (7). Requiring these two fields to carry hypercharge $Y = \pm 1$ (and thus automatically ensuring that the candidate S_3 have vanishing hypercharge) placed two constraints on the allowed hypercharge embedding.

We then proceed to the coupling (5), or λ_T in (7) for the Y = +1 species. Each bifundamental had several couplings of this form to various pairs of SU(2) doublets, at both dimension three and dimension four. By considering all possible pairs and requiring that the doublets involved be assigned Y = -1/2 places two more constraints on the allowed hypercharge embedding. Finally, we proceed to the equally critical λ_2 coupling in (7) for the oppositely charged Y = -1 species. Again by considering all possible pairs of doublets with this coupling, and requiring that both have hypercharge Y = +1/2 we typically constrained the hypercharge embedding to a unique embedding. The hypercharges of all the states in the theory are then determined. Not all will have Standard Model hypercharges, and thus most will have fractional or non-standard electric charges and must be discarded as "exotics." From the set with Standard Model hypercharge assignments we can identify the surviving couplings of (7). In all, this process resulted in 35 distinct field assignment possibilities, each having as a minimum the couplings λ_T , λ_2 and λ_3 – the minimum set to generate the triplet see-saw and the mass pattern of (6). Though these couplings are not enough to generate the perturbations on the bimaximal texture, nor do they include the couplings needed to generate charged lepton masses or μ -terms to break electroweak symmetry, they still represent a complete set of needed couplings to explain the smallness of neutrino masses generally - something that a more exhaustive search of a whole class for the "standard" seesaw failed to achieve [9].

None of the 35 possibilities allowed for all of the couplings of (7), and 12 had no other couplings than the minimal set. This is yet another example of how selection rules of the underlying conformal field theory often forbid operators that would otherwise be allowed by gauge invariance in the 4D theory. Rather than present the various features of all of these assignments, we instead point out a few particular cases. One successful hypercharge assignment allows for a superpotential of the form

$$W = \lambda_T LTL' + \lambda_2 H_2 TH'_2 + \lambda_3 S_3 TT + \lambda_5 S_5 H'_1 H'_2 + \lambda_7 E_R L' H'_1, \qquad (24)$$

where in this case L, L', H_2 and H'_2 all have multiplicity three, H'_1 has multiplicity one and there is no species with the correct hypercharge to be identified as H_1 . In this case, identifying L with the doublet containing the electron leaves the electron massless after electroweak symmetry breaking up to terms of dimension five in the superpotential.

Alternatively, one can obtain candidates for all six species of doublets, such as an example in which the allowed superpotential is given by

$$W = \lambda_T LTL' + \lambda_1 H_1 TH'_1 + \lambda_2 H_2 \overline{T}H'_2 + \lambda_3 S_3 T\overline{T} + \lambda_7 E_R L'H'_1.$$
(25)

All doublets except H_2 in this case arise from twisted sectors, so have multiplicity three. It is interesting to note that in several of the 35 cases the hypercharge embedding assigned Y = 0 to the bifundamental representation of the untwisted sector, suggesting it could play the role of breaking the product group to the diagonal subgroup. Couplings of the form of (17), however, were forbidden by the string theoretic selection rules through dimension four.

Of course none of these cases are truly realistic in the sense of what is needed to explain the observed neutrino data as outlined in the previous section, and it would have been naive to have expected any to be in the first place. The above examples are instead meant to demonstrate the plausibility of this new realization of a tripletinduced seesaw from a string-theory viewpoint by means of a ready example from the literature. Having introduced the concept, defined a basic structure as in (7)and demonstrated that the structure may in fact be realized in the context of tractable string constructions, it becomes reasonable to propose a dedicated search over a wide class of constructions for precisely this model – a search that would necessarily be a separate research project in its own right but which would complement well the analysis already performed in [9].

The $\mathbb{Z}_3 \times \mathbb{Z}_3$ construction is often considered because it, like its \mathbb{Z}_3 cousin, generates a three-fold redundancy for most of the massless spectrum in a relatively straightforward way. But a minimal model would presumably prefer to break away from the three-fold degeneracy on every species, but not the requirement of three generations globally. For example, it is possible to imagine a model in which there are only three "lepton" doublets of $SU(2)_L$ once we break to the diagonal subgroup. Since species in orbifold models (and orientifold models of open strings as well) are defined by fixed point locations (i.e., geometrically) this is not unreasonable to imagine – in fact, precisely such a separation of the three "generations" occurs in the recent $\mathbb{Z}_2 \times \mathbb{Z}_3$ construction of Kobayashi et al. [41]. Nevertheless, there is no getting around the need for at least an extra pair of one, if not both, of the Higgs doublets of the MSSM. As discussed following (7), it is possible that the extra doublets are projected out near the string scale (e.g., if some of the $S_{4,5}$ and $\hat{S}_{4,5}$ are associated with the Fayet-Iliopoulos terms) or at an intermediate scale. It is also possible that one or more extra doublets survives to the TeV scale, in which case there are potential implications for FCNC [42, 43] and CP violation [44], as well as for the charged lepton mixing generated from (7). A more detailed study of such issues is beyond the scope of this paper.

Conclusions

We have presented a new construction of Type II seesaw models utilizing triplets of $SU(2)_L$ in which that group is realized as the diagonal subgroup of an $SU(2)_A \times$ $SU(2)_B$ product group. The triplets in this construction begin as bifundamentals under the two original SU(2)factors, and this identification immediately leads to a bimaximal mixing texture for the effective neutrino mass matrix provided generations of lepton doublets are assigned to the two underlying SU(2) factors in the appropriate way. The observed atmospheric mass difference can be accommodated if the triplets obtain a mass of order 10^{14} GeV, and the solar mass difference can easily be incorporated by a simple perturbation arising at dimension four or five in the superpotential. The observed deviation of the Solar mixing from maximal can be accommodated by a small (Cabibbo-like) mixing in the charged lepton sector, leading to predictions for θ_{13} and neutrinoless double beta decay.

We were led to consider this construction by imagining

the simplest possible requirements for generating a triplet of $SU(2)_L$ from string constructions – particularly the weakly coupled heterotic string, though the model can be realized in other constructions as well. Though inspired by string theory, the model is not itself inherently stringy and is interesting in its own right. Some of the properties of this model are known to phenomenologists, who have arrived at a similar mass matrix from other directions. Interestingly, however, to the best of our knowledge the particular texture has not emerged from other versions of heavy triplet models, e.g., motivated by grand unification or left-right symmetry. The simplest version of the construction requires at least one additional pair of Higgs doublets, which may or may not survive to the TeV scale.

Having laid out a concrete model as a plausible alternative to the standard Type I seesaw in string-based constructions, it is now possible to examine large classes of explicit string models to search for both types of neutrino mass patterns. Given the difficulty in finding a working example of the minimal Type I seesaw in at least one otherwise promising class of string construction, having alternatives with clear "signatures" (in this case, at least two SU(2) factors, with at least two fields bifundamental under both, capable of forming a hypercharge-neutral mass term) is welcome.

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- M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, Phys. Rev. D 68, 113010 (2003).
- [2] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, JHEP 0408, 016 (2004).
- [3] G. L. Fogli, E. Lisi, A. Marrone and A. Palazzo, "Global analysis of three-flavor neutrino masses and mixings," [arXiv:hep-ph/0506083].
- [4] S. F. King, Rept. Prog. Phys. 67, 107 (2004).
- [5] A. De Gouvea, Mod. Phys. Lett. A **19**, 2799 (2004).
- [6] P. Langacker, "Neutrino physics (theory)," [hepph/0411116].
- [7] G. Altarelli and F. Feruglio, New J. Phys. 6, 106 (2004).
- [8] R. N. Mohapatra *et al.*, "Theory of neutrinos," [arXiv:hep-ph/0412099].
- [9] J. Giedt, G. L. Kane, P. Langacker and B. D. Nelson, "Massive neutrinos and (heterotic) string theory," [hepth/0502032].
- [10] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B 181, 287 (1981).
- [11] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 23, 165 (1981).
- [12] J. Schechter and J. W. F. Valle, Phys. Rev. D 25, 774

(1982).

- [13] E. Ma and U. Sarkar, Phys. Rev. Lett. 80, 5716 (1998).
- [14] T. Hambye, E. Ma and U. Sarkar, Nucl. Phys. B 602, 23 (2001).
- [15] A. Rossi, Phys. Rev. D 66, 075003 (2002).
- [16] A. Font, L. E. Ibanez, F. Quevedo and A. Sierra, Nucl. Phys. B **331**, 421 (1990).
- [17] M. Gell-Mann, P. Ramond and R. Slansky in *Super-gravity*, ed. P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam), 1979.
- [18] T. Yanagida in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto, KEK Report KEK-97-18, 1979.
- [19] J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980).
- [20] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, Nucl. Phys. B 256, 253 (1985).
- [21] A. Font, L. E. Ibanez and F. Quevedo, Nucl. Phys. B 345, 389 (1990).
- [22] K. R. Dienes and J. March-Russell, Nucl. Phys. B 479, 113 (1996).

- [23] R. Barbieri, G. R. Dvali and A. Strumia, Phys. Lett. B 333, 79 (1994).
- [24] V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Lett. B 437, 107 (1998).
- [25] R. Barbieri, L. J. Hall, D. R. Smith, A. Strumia and N. Weiner, JHEP 9812, 017 (1998).
- [26] G. Altarelli and F. Feruglio, Phys. Lett. B 439, 112 (1998).
- [27] M. Jezabek and Y. Sumino, Phys. Lett. B 440, 327 (1998).
- [28] R. N. Mohapatra, A. Perez-Lorenzana and C. A. de Sousa Pires, Phys. Lett. B 474, 355 (2000).
- [29] K. S. Babu and R. N. Mohapatra, Phys. Lett. B 532, 77 (2002).
- [30] G. F. Giudice and A. Masiero, Phys. Lett. B 206, 480 (1988).
- [31] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289, 589 (1987).
- [32] M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B 293, 253 (1987).

- [33] J. J. Atick, L. J. Dixon and A. Sen, Nucl. Phys. B 292, 109 (1987).
- [34] J. Giedt, Annals Phys. **297**, 67 (2002).
- [35] P. H. Frampton, S. T. Petcov and W. Rodejohann, Nucl. Phys. B 687, 31 (2004).
- [36] W. Rodejohann, Phys. Rev. D 69, 033005 (2004).
- [37] S. T. Petcov and W. Rodejohann, Phys. Rev. D 71, 073002 (2005).
- [38] G. Altarelli, F. Feruglio and I. Masina, Nucl. Phys. B 689, 157 (2004).
- [39] A. Romanino, Phys. Rev. D 70, 013003 (2004).
- [40] A. Datta, L. Everett and P. Ramond, "Cabibbo haze in lepton mixing," [arXiv:hep-ph/0503222].
- [41] T. Kobayashi, S. Raby and R. J. Zhang, Phys. Lett. B 593, 262 (2004).
- [42] M. Sher and Y. Yuan, Phys. Rev. D 44, 1461 (1991).
- [43] D. Atwood, L. Reina and A. Soni, Phys. Rev. D 55, 3156 (1997).
- [44] L. J. Hall and S. Weinberg, Phys. Rev. D 48, 979 (1993).