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Dark matter clues in the muon anomalous magnetic moment

J. A. R. Cembranos*, A. Dobado†, and A. L. Maroto†
*Department of Physics and Astronomy, University of California, Irvine, CA 92697 USA
†Departamento de Física Teórica, Universidad Complutense de Madrid, 28040 Madrid, Spain
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We study the possibility to explain the non-baryonic dark matter abundance and improve the present fits on the muon anomalous magnetic moment through the same new physics. The only viable way to solve simultaneously both problems which is known to date is by using supersymmetric theories. However in this work we show that massive brane fluctuations (branons) in large extra-dimensions models can provide a more economical alternative to supersymmetry. This is so because the low-energy branon physics depends effectively on only three parameters. Next collider experiments, such as LHC or ILC, will be sensitive to branon phenomenology in the natural parameter region where the theory is able to account for the two effects.

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The existence of dark matter (DM) is a one of the long-standing problems in astrophyiscs and cosmology, dating back to the early thirties when F. Zwicky observed for the first time that the total and visible masses of rich galaxies disagree in a factor 10-100. Since then, additional evidence has been obtained from galaxy rotation curves, galaxy motions in clusters and, more recently, by precise measurements of the temperature fluctuations of the Cosmic Microwave Background (CMB) radiation [1], Type Ia supernovae [2], large scale distribution of galaxies [3] or Lyα clouds [4].

The fact that dark matter cannot be made of any of the known particles is one of the most pressing arguments for the existence of new physics, be it in the form of new particles or as a modificaction of gravity at large distances. The most favoured particle candidate to account for the DM energy density is a Weakly Interacting Massive Particle (WIMP) which can provide with the non-baryonic DM abundance $\Omega_{\rm NBDM}h^2 = 0.129 - 0.095$ measured by WMAP [1], in the form of a standard thermal relic. The particle density is in approximate thermal equilibrium until $T \sim M/20$, where M is the mass of the WIMP which we are equaling to the scale of New Physics (NP), $\Lambda_{\rm NP} \sim M$. WIMPs thin out by annihilation until their relic density freezes out when the annihilation rate equals the Hubble expansion rate, $n_{\rm Wimp}\langle\sigma_{\rm A} v\rangle \sim H$. If we assume a typical annihilation cross section $\sigma_{\rm A} \sim \alpha^2/\Lambda_{\rm NP}^2$ (where α is the electromagnetic coupling constant), the present abundance can be roughly estimated to be:

$$\Omega_{\rm \,Wimp} \sim \left(\frac{\Lambda_{\rm NP}}{100\,{\rm GeV}}\right)^2 . \tag{1}$$

The interesting feature of this result is that the NP which is able to explain the missing matter problem ($\Omega_{\rm \,Wimp} \sim 0.1$), could be related with the electroweak sector ($\Lambda_{\rm NP} \sim 100\,{\rm GeV}$) and be accessible in the next generation of collider experiments. The most popular WIMP candidate is the stable lightest supersymmetric particle which typically corresponds to a neutralino [5].

On the other hand, the success of the Standard Model (SM) of particles and interactions has been tested in many different experiments without finding very important discrepancies so far. A very remarkable example is the electron magnetic moment: $\vec{\mu}_e = g_e(e/(2m_e))\vec{s}$, whose gyromagnetic ratio deviates from the value $g_e = 2$, given by the Dirac equation, as predicted by quantum radiative corrections. This fact has been tested up to a relative precision of 0.03 parts per million (ppm) and confirms Quantum Electrodynamics (QED) as the most precise physical theory [6].

Curiously one of the most interesting deviation from the SM prediction is provided by the muon magnetic moment. Indeed $a_\mu = (g_\mu - 2)/2$ is not only more sensitive to strong and weak interactions than the electron moment, but also to NP. The 821 Collaboration at the Brookhaven Alternating Gradient Synchrotron has reached a precision of 0.5 ppm in the measurement of such a parameter [7]. Taking into account e^+e^- collisions data in order to calculate the $\pi^+\pi^-$ spectral functions, the deviation with respect to the SM prediction is at $2.6\,\sigma$ [8]: $\delta a_\mu \equiv a_\mu({\rm exp}) - a_\mu({\rm SM}) = (23.4 \pm 9.1)\times 10^{-10}$. On the other hand, the contribution of NP to this parameter can be written generically as:

$$\delta a_\mu = k \times \left(\frac{m_\mu}{\Lambda_{\rm NP}}\right)^2 , \tag{2}$$

where the order of magnitude of the constant k depends on the particular model under consideration. Notice that in order to be able to explain the current discrepancy ($\delta a_\mu \sim 10^{-9}$) with the same NP as for dark matter, i.e. $\Lambda_{NP} \sim 100$ GeV, we should have $k \sim 10^{-3}$. This is again the case for some particular supersymmetric models in which the deviation mainly comes from new loop diagrams containing neutralinos, charginos and sleptons. Thus, if there is really new physics in the Brookhaven results, supersymmetry would be so far the only known theory in which the two problems, dark matter and the muon anomaly, could be solved simultaneously [9]. In

Experiment	$\sqrt{s}$(TeV)	$\mathcal{L}$(pb^{-1})	f_0(GeV)	M_0(GeV)
HERA [1]	0.3	110	16	152
Tevatron-I [1]	1.8	78	157	822
Tevatron-I [2]	1.8	87	148	872
LEP-II [2]	0.2	600	180	103
Tevatron-II [1]	2.0	10^3	256	902
Tevatron-II [2]	2.0	10^3	240	952
ILC [2]	0.5	2×10^5	400	250
ILC [2]	1.0	10^6	760	500
LHC [1]	14	10^5	1075	6481
LHC [2]	14	10^5	797	6781
CLIC [2]	5	10^6	2640	2500

TABLE I: Limits from direct branon searches in colliders (results at the 95 % c.l.). Upper indices [1,2] denote monojet and single photon channels respectively. The results for HERA, LEP-II and Tevatron run I have been obtained from real data, whereas those for Tevatron run II, ILC, LHC and CLIC are estimations. $\sqrt{s}$ is the center of mass energy of the total process; $\mathcal{L}$ is the total integrated luminosity; f_0 is the bound on the brane tension scale for one massless branon ($N=1$) and M_0 is the limit on the branon mass for small tension $f\to 0$ (see [15] for details).

this work, however, we point out that the brane-world scenario, originally proposed as an alternative to supersymmetry in the context of the gauge hierarchy problem [10], is also a viable alternative here.

It has been recently found that massive brane fluctutations (branons) are natural candidates to dark matter in brane-world models with low tension [11, 12]. Branon physics can be described at low energies by an effective action which depends essentially on only three parameters: the branon mass M, the brane tension scale f and the cut-off Λ which sets the range of validity of the effective theory. This should be compared with the more than one hundred free parameters of the Minimal Supersymmetric Standard Model (MSSM) or the five parameters of the constrained MSSM.

In order to have a sensible effective theory the following hierarchy among the three parameters is expected $M \lesssim f \lesssim \Lambda$. From the point of view of the four dimensional effective phenomenology, the massive branons are new pseudoscalar fields which can be understood as the pseudo-Goldstone bosons corresponding to the spontaneous breaking of translational invariance in the bulk space produced by the presence of the brane [13]. They are stable due to parity invariance on the brane. The SM-branon low-energy effective Lagrangian [14, 15] can be written as:

$$\mathcal{L}_{Br} = \frac{1}{2}g^{\mu\nu}\partial_\mu\pi^\alpha\partial_\nu\pi^\alpha - \frac{1}{2}M^2\pi^\alpha\pi^\alpha + \frac{1}{8f^4}(4\partial_\mu\pi^\alpha\partial_\nu\pi^\alpha - M^2\pi^\alpha\pi^\alpha g_{\mu\nu})T^{\mu\nu}\,. \quad (3)$$

where $\alpha = 1\dots N$, with N the number of branon species. We see that branons interact by pairs with the SM energy-momentum tensor $T^{\mu\nu}$, and that the coupling is suppressed by the brane tension f^4. Limits on the model parameter from tree-level processes in colliders are briefly summarized in Table I, where one can find not only the present restrictions coming from HERA, Tevatron and LEP-II, but also the prospects for future colliders such as ILC, LHC or CLIC [15, 16]. Additional bounds from astrophysics and cosmology can be found in [17].

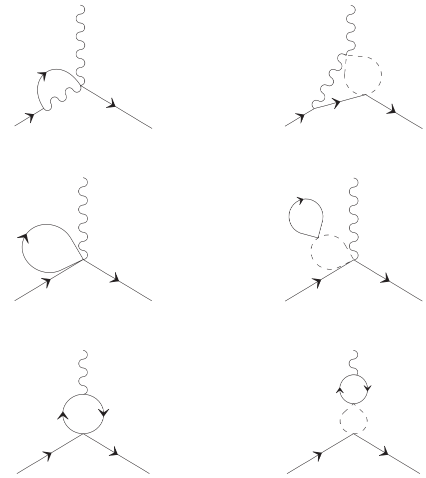

FIG. 1: The three diagrams on the left are the different types of contributions from Lagrangian (4) to the anomalous magnetic moment of the muon at one loop. The three diagrams on the right are the equivalent two loop contributions from the branon theory described by (3). The continuous, dashed and wavy lines represent the muon, branon and photon fields respectively. Notice that in the first type of contribution, the fermion loop can also be attached to the outgoing muon.

In order to obtain the first branon contribution to the μ anomalous magnetic moment, we compute the one-loop effective action for SM particles, by integrating out the branon fields with cutoff regularized integrals. At the level of two-point functions, branon loops result only in a renormalization of the SM particle masses, which is not observable. However new couplings appear at higher-point functions which can be described by an effective Lagrangian [16, 18] whose more relevant terms are:

$$\mathcal{L}^{(1)}_{SM} \simeq \frac{N\Lambda^4}{192(4\pi)^2 f^8}\left\{2T_{\mu\nu}T^{\mu\nu} + T^\mu_\mu T^\nu_\nu\right\}\,. \quad (4)$$

As we have commented above, Λ is the cutoff which limits the validity of the effective description of branon and SM dynamics. This new parameter appears when dealing with branon radiative corrections since the Lagrangian in (3) is not renormalizable. A one-loop calculation with the new effective four-fermion vertices coming from (4), whose Feynman digrams are given in Figure 1, is equivalent to a two-loop computation with the Lagrangian in (3), and allows us to obtain the contribution of branons to the anomalous magnetic moment:

$$\delta a_\mu \simeq \frac{5\,m_\mu^2}{114\,(4\pi)^4}\frac{N\Lambda^6}{f^8}. \quad (5)$$

Experiment	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (pb^{-1})	$f^2/(N^{1/4}\Lambda)$ (GeV)
HERA ^{c}	0.3	117	52
Tevatron-I ^{a,b}	1.8	127	69
LEP-II ^{a}	0.2	700	59
LEP-II ^{b}	0.2	700	75
Tevatron-II ^{a,b}	2.0	2×10^3	83
ILC ^{b}	0.5	5×10^5	261
ILC ^{b}	1.0	2×10^5	421
LHC ^{b}	14	10^5	383

TABLE II: Limits from virtual branon searches at colliders (results at the 95 % c.l.) The indices ^{a,b,c} denote the two-photon, e^+e^- and e^+p (e^-p) channels respectively. The first three analysis have been performed with real data: HERA [23], LEP-II [25] and Tevatron-I [24]; whereas the final four are estimations. The first two columns are the same as in Table I, and the third one corresponds to the lower bound on $f^2/(N^{1/4}\Lambda)$.

This result is qualitatively similar to other $g_\mu - 2$ contributions obtained in different analyses in the brane-world scenario [19, 20]. We can observe that the correction has the correct sign and that it is thus possible to improve the agreement with the observed experimental value by the E821 Collaboration. In fact, by using the commented difference between the experimental and the SM prediction [7, 8], we can estimate the *preferred* parameter region for branon physics:

$$6.0 \text{ GeV} \gtrsim \frac{f^4}{N^{1/2}\Lambda^3} \gtrsim 2.2 \text{ GeV } (\, 95 \text{ \% } c.l.\,) \qquad (6)$$

However branon loops can have additional effects which should also be compatible with SM phenomenology. The most relevant ones could be the four-fermion interactions or the fermion pair annihilation into two gauge bosons. Following [21, 22], we have used the data coming from HERA [23], Tevatron [24] and LEP [25] on this kind of processes in order to set bounds on the parameter combination $f^2/(\Lambda N^{1/4})$. The results are shown in Table II, where it is also possible to find the prospects for the future colliders mentioned above. These limits show that an important consequence of the relation (6) is that the first branon signals at colliders would be associated to radiative corrections [18] and not to the direct production studied in previous works [15].

Indeed, if there is new physics in the muon anomalous magnetic moment and it is due to branon radiative corrections, the phenomenology of these particles should be observed at the LHC and in a possible future ILC, which have larger sensitivities for virtual effects working at a center of mass energy of 1 TeV (in contrast with the direct branon production, where the LHC presents a larger sensitivity in any case, see Tables I and II, and Figures 2 and 3). In particular, the LHC should observe an important difference with respect to the SM prediction in channels like $pp \to e^+e^-$. The ILC should observe the most important effect in the Bhabha scattering.

Another limitation to the branon parameters could be obtained from electroweak precision measurements,

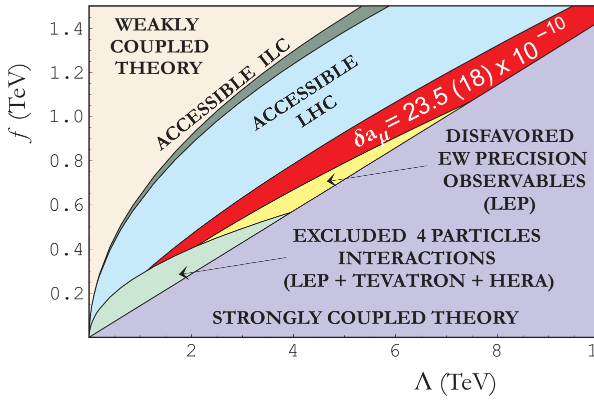

FIG. 2: Main limits from branon radiative corrections in the $f-\Lambda$ plane for a model with $N = 1$. The (red) central area shows the region in which the branons account for the muon magnetic moment deficit observed by the E821 Collaboration [7, 8], and at the same time, are consistent with present collider experiments (whose main constraint comes from the Bhabha scattering at LEP) and electroweak precision observables. Prospects for future colliders are also plotted.

which use to be very useful to constrain models of new physics. The so called oblique corrections (the ones corresponding to the W, Z and γ two-point functions) use to be described in terms of the S, T, U [26] or the ϵ_1, ϵ_2 and ϵ_3 parameters [27]. The experimental values obtained by LEP [25, 28] are consistent with the SM prediction for a light Higgs $m_H \leq 237$ GeV at 95 % c.l. In principle, it is necessary to know this parameter in order to put constraints on new physics, but one can talk about disfavored regions of parameters in order to avoid fine tunings. We can estimate this area by performing a computation of the parameter $\bar{\epsilon} \equiv \delta M_W^2/M_W^2 - \delta M_Z^2/M_Z^2$, in a similar way as it was done for the first order correction coming from the Kaluza-Klein gravitons in the ADD models for rigid branes [20]. The experimental value of $\bar{\epsilon}$ obtained from LEP [25, 28] is $\bar{\epsilon} = (1.27 \pm 0.16) \times 10^{-2}$. The theoretical uncertainties are one order of magnitude smaller [27] and therefore, we can estimate the constraints for the branon contribution at 95 % c.l. as $|\delta\bar{\epsilon}| \lesssim 3.2 \times 10^{-3}$ with the result [18]:

$$\frac{f^4}{N^{1/2}\Lambda^3} \gtrsim 3.1 \text{ GeV } (\, 95 \text{ \% } c.l.\,) \qquad (7)$$

This bound has a different dependence on Λ than the SM interactions induced by virtual branons. The constraints coming from this analysis are complementary to the previous ones. In Figure 2, we have included all the limits in the $f-\Lambda$ plane from virtual branon effects. We have also plotted the region in which the effective theory can be considered as strongly coupled, ($\Lambda \gtrsim 4\sqrt{\pi} f N^{-1/4}$) and for which the loop expansion is no longer valid [18]. We see that the region compatible with the Brookhaven results extends for $1100 \lesssim \Lambda \lesssim 15100\, N^{-1/2}$ (GeV) and $300\, N^{1/8} \lesssim f \lesssim 2130\, N^{-1/4}$ (GeV).

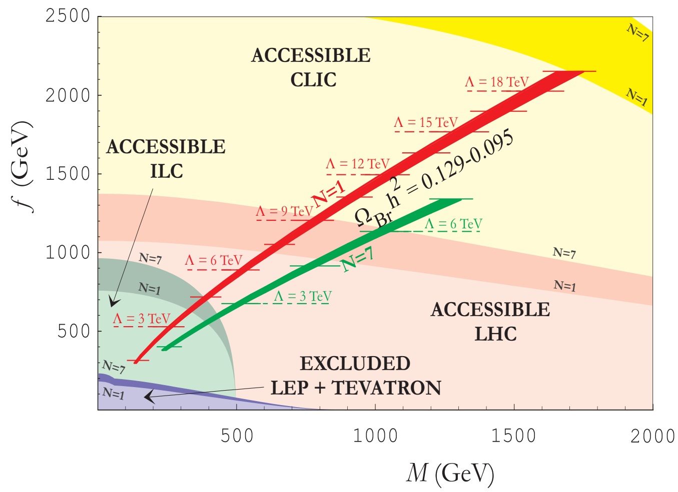

FIG. 3: Branon abundance in the range: $\Omega_{Br}h^2 = 0.129 - 0.095$, in the $f - M$ plane (see [17] for details). The regions are only plotted for the preferred values of the brane tension scale f. One can find on these lines the central values of Λ corresponding to the observed difference between the experimental and the SM prediction on the muon anomalous magnetic moment [7, 8]. The lower area is excluded by single-photon processes at LEP-II together with monojet signals at Tevatron-I [15]. Prospects for the sensitivity in collider searches of real branon production are plotted also for future experiments (See [15] and Table I). In this figure one can observe explicitly the dependence on the number of branons N, since all these regions are plotted for the extreme values $N = 1$ and $N = 7$.

It is remarkable to note that the same parameter space which explains the magnetic moment deficit of the muon, is able to explain the DM content of the Universe and, in addition, the preferred scale is related with the electroweak sector. In fact, if the branon mass is of the order of the electroweak scale, or more precisely, it is between $M \sim 100$ GeV and $M \sim 1.7$ TeV, then branons could form the total non baryonic DM abundance observed by WMAP [1, 11]. In Figure 3, we have plotted the $f - M$ regions in which branons could explain the WMAP measurements. We include also the limits from colliders and the values of Λ corresponding to the central values of the muon anomalous magnetic moment observed at Brookhaven. In these regions branons decouple at $T < M < f < \Lambda$, i.e., they are non-relativistic, behave as cold DM and the effective theory described by the Lagrangian (3) can be used to properly evaluate their thermal relic abundance [11].

To summarize, we have shown that massive branons could offer an alternative explanation for the observed dark matter abundance and the recent measurements of the muon anomalous magnetic moment. The model only contains three parameters and, in the region compatible with the experiments, their values satisfy the natural hierarchy of a weakly coupled low-energy effective theory i.e. $M \lesssim f \lesssim \Lambda$. We have also shown that in the preferred parameter region, future colliders such as LHC or ILC should be sensitive to branon phenomenology.

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