

The clash of symmetries in a Randall-Sundrum-like spacetime.

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We present a toy model that exhibits clash-of-symmetries style Higgs field kink configurations in a Randall-Sundrum-like spacetime. The model has two complex scalar fields $\Phi_{1,2}$, with a sextic potential obeying global $U(1)\otimes U(1)$ and discrete $\Phi_1 \leftrightarrow \Phi_2$ interchange symmetries. The scalar fields are coupled to 4+1 dimensional gravity endowed with a bulk cosmological constant. We show that the coupled Einstein-Higgs field equations have an interesting analytic solution provided the sextic potential adopts a particular form. The 4+1 metric is shown to be that of a smoothed-out Randall-Sundrum type of spacetime. The thin-brane Randall-Sundrum limit, whereby the Higgs field kinks become step functions, is carefully defined in terms of the fundamental parameters in the action. The “clash of symmetries” feature, defined in previous papers, is manifested here through the fact that both of the $U(1)$ symmetries are spontaneously broken at all non-asymptotic points in the extra dimension w . One of the $U(1)$'s is asymptotically restored as $w \rightarrow -\infty$, with the other $U(1)$ restored as $w \rightarrow +\infty$. The spontaneously broken discrete symmetry ensures topological stability. In the gauged version of this model we find new flat-space solutions, but in the warped metric case we have been unable to find any solutions with nonzero gauge fields.

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I. INTRODUCTION

The breaking of symmetries is one of the most important issues in particle physics. Nature has taught us the relevance of symmetry principles, especially through Noether's connection of global symmetries with conservation laws and the successful description of fundamental interactions via local gauge invariance. But to reconcile symmetric field equations with the real world, one needs that some of the solutions to those equations have a reduced symmetry compared to the equations themselves. This is, of course, what we mean by ‘spontaneous symmetry breaking (SSB)’ or ‘hidden symmetry’.

The precise mechanism by which electroweak SSB happens has yet to be experimentally determined. In the standard model (SM), the default option is to use an elementary scalar or Higgs field doublet with a quartic potential that induces a nonzero vacuum expectation value for one of the components. This will soon be tested at the Large Hadron Collider through its physical-Higgs boson search programme.

The purpose of this paper is to further develop a different method of symmetry breaking called the ‘clash of symmetries’ [1, 2, 3, 4] (see also [5]). This mechanism is relevant for brane-world extensions to the regular 3+1 dimensional standard model [6, 7]. The important development we report here is the rigorous incorporation of gravity into one of the toy Higgs models explored in the previous papers on this topic [3]. Since gravity is fundamental to brane-world models, this is a very welcome step forward.

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The toy model we shall analyse consists of two complex scalar fields $\Phi_{1,2}$ coupled to $4 + 1$ dimensional gravity endowed with a bulk cosmological constant. The action is constructed to be invariant under global $U(1) \otimes U(1)$ transformations and the discrete $\Phi_1 \leftrightarrow \Phi_2$ interchange operation. The Higgs potential we construct is sextic, and its specific form is driven by our desire to find an analytic solution to the coupled Einstein-Higgs field equations. The spacetime metric tensor we posit has the Randall-Sundrum non-factorisable form [6], but with the warp factor a completely smooth function of the extra dimension coordinate. The precise Randall-Sundrum metric emerges by taking a certain limit. The Higgs field configurations we seek are kink-like, associated with the spontaneously broken discrete symmetry.

We also study the gauged version of this model, extending the analysis of Ref.[3]. We find additional non-trivial flat-space solutions, but despite an extensive numerical search we have been unable to find any warped metric solutions with nonzero gauge fields.

Section II reviews the clash of symmetries mechanism, while Sec. III defines and analyses the ungauged version of our toy model. This is followed by an account of the gauged cases in Sec. IV. The final section is a conclusion and outlook for future work.

II. THE CLASH OF SYMMETRIES

The default SSB mechanism employed in the SM and many extensions to it utilises a simple spatially homogeneous, static and stable solution to the field equations: the Higgs fields $\phi_i(x^\mu)$ are set equal to those constant values corresponding to an absolute minimum of the Higgs potential. The locations of the nonzero constants within the Higgs multiplets then determine the stability group of the vacuum state, what we commonly call the unbroken symmetry after SSB. The nonzero Higgs fields become constant background scalar fields throughout the universe.

There are other types of solutions to Higgs field equations that can also serve as stable, static background fields: topological solitons, such as kinks, strings and monopoles. While there is much interest in such configurations, they are not usually considered as viable candidates for playing a major role in spontaneous symmetry breaking. This is simply because they are spatially inhomogeneous. The energy densities required for electroweak and higher symmetry breaking are so high as to be incompatible with the strong evidence for a universe that is spatially homogeneous at large scales. This objection, however, does not apply to brane-world models, because the non-trivial spatial dependence can be restricted to the extra dimension coordinates only.

For definiteness, consider the case of one extra dimension described by Cartesian coordinate w and topologically-stable Higgs configurations $\phi_i(w)$, some of which have kink-form with respect to w . The pattern of spontaneous breaking then effectively becomes a function of w . The $3 + 1$ -brane is located at, say, $w = 0$. If the degrees of freedom that are confined to the brane [10] are confined absolutely, then the unbroken symmetry is the stability group of $\phi_i(w = 0)$, to be denoted by $H(w = 0)$ from now on (and let the internal symmetry group of the action be G). The symmetry breaking will be communicated to the brane-world denizens through appropriate interactions. But in a quantal (and perhaps even in a classical) world, one would not expect strict confinement to the brane: there should be some leakage off it into the bulk. One therefore expects the brane-localised fields to also couple, but with reduced strength, to $\phi_i(w)$ states where w is slightly different from zero. If the stability group of $\phi_i(0 < |w| < \epsilon)$, call it $H(w \sim \epsilon)$, is different from $H(w = 0)$, then a rather rich effective symmetry breaking outcome could ensue for the brane world. For example, a hierarchical breaking pattern $G \rightarrow H(w = 0) \rightarrow H(w \sim \epsilon)$ was speculated upon in Ref.[2], where in that case $G = SO(10)$, $H(w = 0) = SU(4) \otimes SU(2) \otimes U(1)$ and $H(w \sim \epsilon) = SU(3) \otimes SU(2) \otimes U(1)^2$. The potential application to grand unified models and the gauge hierarchy puzzle is clear.

The ‘clash of symmetries’ phenomenon allows the unbroken symmetry at non-asymptotic points in the extra dimension, $|w| < \infty$, to be smaller than the symmetry holding at $|w| = \infty$ [1, 2, 3, 4]. The boundary conditions at infinity for kink configurations are set to be the Higgs vacuum expectation values (VEVs). They force the spontaneous breaking $G \rightarrow H(|w| \rightarrow \infty)$. In usual SSB, the Higgs VEV configuration is used throughout the whole of space, so the symmetry is broken to the group we are calling $H(|w| \rightarrow \infty)$ everywhere, homogeneously. In the clash of symmetries mechanism, this symmetry breaking pattern only holds asymptotically with respect to the extra dimension(s).

The clash of symmetries can occur if the isomorphic subgroups left unbroken at $w = -\infty$ and $w = +\infty$, $H(w \rightarrow -\infty) \equiv H(-\infty)$ and $H(w \rightarrow +\infty) \equiv H(+\infty)$ respectively, can be *differently embedded* within the parent group G . In the cases examined so far in the literature, the breakdown at finite values of w is to the *intersection* of the asymptotic stability groups,

$$H(|w| < \infty) = H(-\infty) \cap H(+\infty) \equiv H_{\text{clash}}. \quad (2.1)$$

Although $H(-\infty)$ and $H(+\infty)$ are isomorphic, their different embeddings within G leads their intersection to be a smaller group.¹ Some special values of w , most often $w = 0$, may correspond to stability groups larger than H_{clash} because the spatially varying Higgs configurations may instantaneously pass through special patterns as w varies. For instance, a component of a Higgs multiplet may be an odd function of w such as $\tanh(w)$, so it would vanish at $w = 0$ and thus the unbroken symmetry would be enhanced at that point. This possibility lies behind our remarks two paragraphs above about an application to the gauge hierarchy problem.

Discrete symmetries also play a vital role: their spontaneous breakdown leads the vacuum manifold to have disconnected pieces. Kinks interpolate between vacuum states from disconnected pieces. Within a given topological class of kinks, the one with lowest energy is then guaranteed to be (topologically) stable.

The easiest example of the clash of symmetries uses $G = \text{SU}(3)$ spontaneously breaking to $\text{SU}(2)$. An $\text{SU}(2)$ subgroup can be embedded in three different ways in $\text{SU}(3)$: I -spin, U -spin and V -spin. Clash of symmetries kinks interpolate between VEVs respecting (I, U) , (U, V) or (V, I) at $w = (-\infty, +\infty)$. The differently embedded asymptotic $\text{SU}(2)$ symmetries “clash” at non-asymptotic points, leading H_{clash} to be $\text{SU}(2)_I \cap \text{SU}(2)_U = \{1\}$ and so on.²

The model-building promise of the clash of symmetries lies in the greater symmetry breaking power of spatially inhomogeneous Higgs configurations over the simple homogeneous alternative of standard SSB. We hope that this will lead to models with simpler Higgs sectors and fewer associated parameters. Some of us have already speculated that the ultimate application might be to an E_6 model, because of the fact that there are three different E_6 generators that can serve as electric charge, corresponding to three different embeddings of electromagnetism within the full group [1]. Standard E_6 models can only use one at a time. Even more speculatively, a connection between that threefold structure and the three families of quarks and leptons might exist³ [1].

But many preparatory studies are needed before such an ambitious vision can be realised. In this paper, we take another important step along that road: the incorporation of gravity. To facilitate the study of gravity, we begin by simplifying the group theoretic structure as much as possible without losing the essence of the clash of symmetries mechanism: we take $G = \text{U}(1)_1 \otimes \text{U}(1)_2$, with $H(-\infty) = \text{U}(1)_1$ and $H(+\infty) = \text{U}(1)_2$ leading to $H_{\text{clash}} = \{1\}$. The discrete symmetry we need for topological stability interchanges the two Abelian sectors.

In the next section we show that the ungauged version of this model can yield a clash-of-symmetries-style kink in a warped metric spacetime. The section after that revisits the gauged extension in both flat and curved spacetime.

¹ There will also typically be kink configurations that interpolate between *identically* embedded subgroups. Such kinks do not display the clash of symmetries, and one must ensure that they have higher energy than those displaying the clash.

² To actually implement this simple $\text{SU}(3)$ scheme requires more work: see Ref.[1] for details.

³ There is some similarity between the clash of symmetries mechanism and orbifold symmetry breaking [8]. In particular, Ref. [9] constructs a model where different subgroups of $\text{SO}(10)$ are left unbroken on different branes, with the overall unbroken symmetry being the intersection of those individual subgroups, and with families also distributed amongst the branes. While there are similarities, it is clear that the mechanisms are not identical, because the clash of symmetries has a physical Higgs field while the orbifold method is Higgsless.

III. THE UNGAUGED MODEL

A. The field equations

The model has two complex Higgs fields Φ_1 and Φ_2 coupling to 4 + 1 dimensional gravity, and it respects the $\Phi_1 \leftrightarrow \Phi_2$ discrete symmetry. The action is

$$S = - \int \left[\frac{\kappa}{2} R + g^{MN} (\partial_M \bar{\Phi}_1 \partial_N \Phi_1 + \partial_M \bar{\Phi}_2 \partial_N \Phi_2) + V(\bar{\Phi}_1 \Phi_1, \bar{\Phi}_2 \Phi_2) \right] \sqrt{-g} d^5 x \quad (3.1)$$

where g_{MN} is the 4 + 1 dimensional metric tensor, with $M, N = 0, 1, 2, 3, 5$, R is the curvature scalar, and overbars denote complex conjugation. The matter action S_m is defined as S with the Einstein-Hilbert contribution absent. The Higgs potential V is as yet unspecified except that it must be invariant under $\Phi_1 \leftrightarrow \Phi_2$. It will have a constant piece that serves as the bulk cosmological constant. We employ the $(-, +, +, +, +)$ signature and our sign conventions are as in Weinberg (apart from our definition of $g \equiv \text{Det}(g)$) [11].

The Einstein equation found by requiring a stationary action under metric variations is

$$G^{MN} = -\frac{1}{\kappa} T^{MN} \quad (3.2)$$

where the Einstein tensor is, as usual, $G^{MN} = R^{MN} - (1/2)g^{MN}R$ with R^{MN} being the Ricci tensor. The stress-energy tensor,

$$T^{MN} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{MN}}, \quad (3.3)$$

evaluates to

$$T^{MN} = 2g^{MP}g^{NQ}t_{PQ} - g^{MN}(g^{PQ}t_{PQ} + V), \quad (3.4)$$

where

$$t_{MN} \equiv \partial_M \bar{\Phi}_1 \partial_N \Phi_1 + \partial_M \bar{\Phi}_2 \partial_N \Phi_2. \quad (3.5)$$

The Higgs Euler-Lagrange equations are

$$\partial_M (\sqrt{-g}g^{MN} \partial_N \Phi_i) = \sqrt{-g} \frac{\partial V}{\partial \bar{\Phi}_i}, \quad (3.6)$$

where $i = 1, 2$, and the complex conjugate equations also hold.

We now specify our solution ansatz. We seek configurations that only depend on the extra dimension $x^5 \equiv w$. The imaginary parts of the Higgs fields are set to zero and we write

$$\Phi_i = \frac{\phi_i(w)}{\sqrt{2}}, \quad (3.7)$$

where $\phi_i(w)$ is real. The metric ansatz, of Randall-Sundrum type, is defined through the 4 + 1 dimensional line element

$$ds_5^2 = dw^2 + e^{2f(w)} ds_4^2 \quad (3.8)$$

where $f(w)$ is an as yet unknown function and $ds_4^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ is the 3 + 1 dimensional Minkowski line element (our brane will be Minkowski), with $\mu, \nu = 0, 1, 2, 3$ or t, x, y, z . The field equations then reduce to

$$3f'' + 6(f')^2 = -\frac{1}{\kappa} \left[\frac{1}{2} [(\phi_1')^2 + (\phi_2')^2] + V \right], \quad (3.9)$$

$$6(f')^2 = \frac{1}{\kappa} \left[\frac{1}{2} [(\phi_1')^2 + (\phi_2')^2] - V \right], \quad (3.10)$$

$$\phi_i'' + 4f'\phi_i' = \frac{\partial V}{\partial \phi_i}, \quad (3.11)$$

where prime denotes differentiation with respect to w . Adding and subtracting Eqs. (3.9) and (3.10) gives

$$f'' + 4(f')^2 = -\frac{2}{3\kappa}V, \quad (3.12)$$

$$f'' = -\frac{1}{3\kappa}[(\phi_1')^2 + (\phi_2')^2]. \quad (3.13)$$

Note that Eq. (3.13) is independent of the potential, a feature we will use below.

We will construct a potential V that has degenerate global minima at

$$\text{vacuum 1 : } \phi_1 = u, \phi_2 = 0 \quad \text{and} \quad \text{vacuum 2 : } \phi_1 = 0, \phi_2 = u, \quad (3.14)$$

where u would be the VEV in the standard homogeneous SSB mechanism. The vacua are related by the now spontaneously broken discrete interchange symmetry. The Higgs field boundary conditions will be

$$\phi_i(-\infty) = \text{vacuum 2} \quad \text{and} \quad \phi_i(+\infty) = \text{vacuum 1}. \quad (3.15)$$

B. Constructing the solution

In $3+1$ dimensions without gravity, renormalisability limits us to quartic potentials. With extra dimensions, and with gravity included, this is no longer a well-motivated requirement. In fact, the only general constraint on the form of V we will impose is that it be bounded from below. A very important topic for the future of the clash of symmetries mechanism will indeed be the rules by which potentials should be specified.

Our more modest goal here is to provide an existence proof that kinks featuring the clash of symmetries form can be consistently coupled to Randall-Sundrum-like gravity. We will search for an analytically tractable model for reasons of both simplicity and elegance. To do this, it makes sense to invert the problem by first specifying the solutions $\phi_i(w)$ and $f(w)$ and then constructing a potential that gives those solutions. This, in itself, is non-trivial and interesting.

It turns out that a good choice, satisfying the boundary conditions of Eq. (3.15) and also the Higgs-potential-independent Eq. (3.13), is

$$\phi_1^s(w) = \frac{u}{\sqrt{2}}\sqrt{1 + \tanh \beta w}, \quad (3.16)$$

$$\phi_2^s(w) = \frac{u}{\sqrt{2}}\sqrt{1 - \tanh \beta w}, \quad (3.17)$$

$$f^s(w) = -\frac{u^2}{12\kappa} \ln(\cosh \beta w), \quad (3.18)$$

where β is a free parameter that specifies the brane thickness, and the superscript s denotes ‘‘solution’’. The Higgs configurations are the square roots of the most naïve choices, and $f^s(w)$ is essentially a regularisation of $|w|$. The latter is, of course, the corresponding function in the Randall-Sundrum scenario. The smoothing out of the Randall-Sundrum cusp goes hand-in-hand with the existence of Higgs field kinks coupled to gravity, and the brane is dynamically generated rather than put in by hand. Note also that

$$\phi_1^s(w)^2 + \phi_2^s(w)^2 = u^2, \quad (3.19)$$

a feature that will turn out to be very useful.

The first inkling for what V should be comes from Eq. (3.12). It suggests that

$$V = -\frac{\beta^2 u^4}{24\kappa} + \frac{\beta^2}{2u^2} \left(1 + \frac{u^2}{3\kappa}\right) \phi_1^2 \phi_2^2 + U(\phi_1^2, \phi_2^2), \quad (3.20)$$

where U is a function that vanishes at the level of the solution, that is $U(\phi_1^s(w)^2, \phi_2^s(w)^2) = 0$. Notice that a negative bulk cosmological constant,

$$\Lambda_5 = -\frac{\beta^2 u^4}{24\kappa}, \quad (3.21)$$

follows from the ansatz: the bulk has anti-de Sitter qualities, as usual in Randall-Sundrum-like models, though in our case it is of course not precisely anti-de Sitter.

More information about U is supplied by the Higgs field equation (3.11). To solve it, one must specify that

$$U = -\frac{\beta^2}{u^4} \left(\frac{3}{2} + \frac{u^2}{3\kappa} \right) \phi_1^2 \phi_2^2 (\phi_1^2 + \phi_2^2 - u^2) + W(\phi_1^2 + \phi_2^2 - u^2), \quad (3.22)$$

where the new function W obeys

$$W(0) = 0, \quad (3.23)$$

$$\left. \frac{\partial W}{\partial \phi_{1,2}} \right|_{\phi_1^s, \phi_2^s} = 0, \quad (3.24)$$

so as not to affect the satisfaction of the field equations, Eq. (3.12) and Eq. (3.13) respectively. As far as they are concerned, W can vanish. The resulting sextic potential, however, would not be bounded from below because the coefficient of the first term in U is negative.

We therefore construct W so that, as well as obeying Eqs. (3.23) and (3.24), it leads to a potential that is both bounded from below and has global minima at Eq. (3.14). The simplest suitable function is

$$W = \zeta \frac{\beta^2}{4u^2} \left(\frac{3}{2} + \frac{u^2}{3\kappa} \right) (\phi_1^2 + \phi_2^2 - u^2)^2 \left(\eta + \frac{\phi_1^2 + \phi_2^2 - u^2}{u^2} \right) \quad (3.25)$$

where η and ζ are dimensionless parameters.

The full Higgs potential for the model is specified by Eqs. (3.20), (3.22) and (3.25). We will present a global minimisation analysis of it in the next subsection. (To reinstate the complex nature of the Higgs fields, the substitution $\phi_i^2 \rightarrow 2\bar{\Phi}_i \Phi_i$ should be made, though for the forthcoming discussion we need only work with the ϕ_i .)

Before doing so, let us identify the Randall-Sundrum limit of the model. We need to identify a limit in which the warp factor, e^{2f^s} , becomes $e^{-a|w|}$ where a is some positive constant. Following Ref.[12], we note that $(\cosh \beta w)^{-(1/\beta)} \rightarrow e^{-|w|}$ as $\beta \rightarrow \infty$. Writing

$$e^{2f^s(w)} = \left[(\cosh \beta w)^{-\frac{1}{\beta}} \right]^{\frac{u^2 \beta}{6\kappa}}, \quad (3.26)$$

we see that the Randall-Sundrum warp factor, $e^{-\frac{u^2 \beta}{6\kappa}|w|}$, is obtained in the limit

$$\beta \rightarrow \infty, \quad u \rightarrow 0 \quad \text{such that } u^2 \beta \rightarrow \text{finite constant}. \quad (3.27)$$

We also require η and ζ to be unchanged when taking the limit. Note that the Higgs field configurations, Eqs. (3.16) and (3.17), become step functions as $\beta \rightarrow \infty$, with their magnitude tending to zero because $u \rightarrow 0$. The Randall-Sundrum fine-tuning condition between the brane tension and the bulk cosmological constant is a feature of our solution. Defining the brane tension via

$$\Lambda_b \equiv \lim_{RS} \int_{-\infty}^{+\infty} \left[\frac{1}{2} [(\phi_1^{s'})^2 + (\phi_2^{s'})^2] + V(\phi_1^s, \phi_2^s) \right] \sqrt{-g^s} dw, \quad (3.28)$$

it is easy to show that

$$\Lambda_b^2 = -\frac{3}{2} \kappa \Lambda_5, \quad (3.29)$$

which is the Randall-Sundrum condition.

C. Minimisation analysis

We now identify the parameter space region where our potential has global minima specified by Eq. (3.14).

To establish the parameter region where the potential is bounded from below, we look at the sixth-order terms in isolation. They are

$$V_6 = \lambda (\phi_1^2 + \phi_2^2) [\zeta (\phi_1^2 + \phi_2^2)^2 - 4\phi_1^2 \phi_2^2], \quad (3.30)$$

where $\lambda \equiv (3\beta^2/8u^4)[1 + 2u^2/(9\kappa)] > 0$. The substitutions $\phi_1 = \chi \cos \theta$ and $\phi_2 = \chi \sin \theta$ reveal that $V_6 = \lambda \chi^6 (\zeta - 4 \sin^2 \theta \cos^2 \theta)$, from which the condition

$$\zeta > 1 \quad (3.31)$$

is trivially seen to be the only requirement to produce boundedness from below.

The minimisation analysis is a four-parameter problem. For some of the local minima, both their (ϕ_1, ϕ_2) locations and the values of the potential at those points are complicated functions of the parameters. But there is a smart way to evade these unenlightening complications while yielding physically meaningful results: approach the Randall-Sundrum limit defined by Eq. (3.27). Our task then becomes a two parameter problem in (η, ζ) that is readily visualised.

To see this, note that in Eqs. (3.20), (3.22) and (3.25), the combinations

$$1 + \frac{u^2}{3\kappa} \quad \text{and} \quad \frac{3}{2} + \frac{u^2}{3\kappa} \quad (3.32)$$

frequently appear. In the Randall-Sundrum limit, which includes $u \rightarrow 0$, these combinations simplify to 1 and 3/2 respectively. In the exact Randall-Sundrum limit, the meaning to be ascribed to the potential is obscure, so we cannot work exactly in that limit. Instead, we will approach the limit without reaching it, and write an approximation to the potential justified by the above remarks. It is

$$\begin{aligned} V \simeq V_{\text{approx}} &= -\frac{\beta^2 u^4}{24\kappa} + \frac{\beta^2}{2u^2} \phi_1^2 \phi_2^2 - \frac{3\beta^2}{2u^4} \phi_1^2 \phi_2^2 (\phi_1^2 + \phi_2^2 - u^2) + \zeta \frac{3\beta^2}{8u^2} (\phi_1^2 + \phi_2^2 - u^2)^2 \left(\eta + \frac{\phi_1^2 + \phi_2^2 - u^2}{u^2} \right), \\ &= \frac{3\beta^2 u^2}{8} \left[-\frac{u^2}{9\kappa} + \frac{4}{3} p_1^2 p_2^2 - 4 p_1^2 p_2^2 (p_1^2 + p_2^2 - 1) + \zeta (p_1^2 + p_2^2 - 1)^2 (\eta + p_1^2 + p_2^2 - 1) \right], \end{aligned} \quad (3.33)$$

where

$$p_i \equiv \frac{\phi_i}{u} \quad (3.34)$$

is a dimensionless Higgs field. The fact that β contributes only to an overall scale, and that κ appears solely in the constant term, reduces the analysis to a two-parameter problem in (η, ζ) . While κ contributes to the potential values, it does so equally for all local minima in this approximation, which means that only η and ζ need be considered when comparing the local minima to see which are also the global minima.

The potential has three varieties, denoted (I, II, III) , of possible local minima (or saddle points),

$$(I) \quad p_1^2 = 1, p_2 = 0 \quad \text{degenerate with} \quad p_1 = 0, p_2^2 = 1 \quad (\text{discrete symmetry breaking}), \quad (3.35)$$

$$(II) \quad p_1^2 = p_2^2 \neq 0 \quad (\text{discrete symmetry preserving}), \quad (3.36)$$

$$(III) \quad p_1 = p_2 = 0 \quad (\text{no SSB}). \quad (3.37)$$

Which of these are actual local minima depends on (η, ζ) . We want I to describe the global minima.

Of the three, it is II that has the complicated parameter dependence. In fact it is quite easy to show, even for the full potential of Eqs. (3.20), (3.22) and (3.25), that $V(I) < V(III)$ requires that

$$\eta > 1 \quad (3.38)$$

which is thus a necessary but not sufficient condition to ensure that I gives the global minima. Sufficiency requires an examination of the parameter dependence of $V(II)$ also. The results are shown in Fig. 1, where the shaded area is the Higgs parameter region where I is the global minimum.

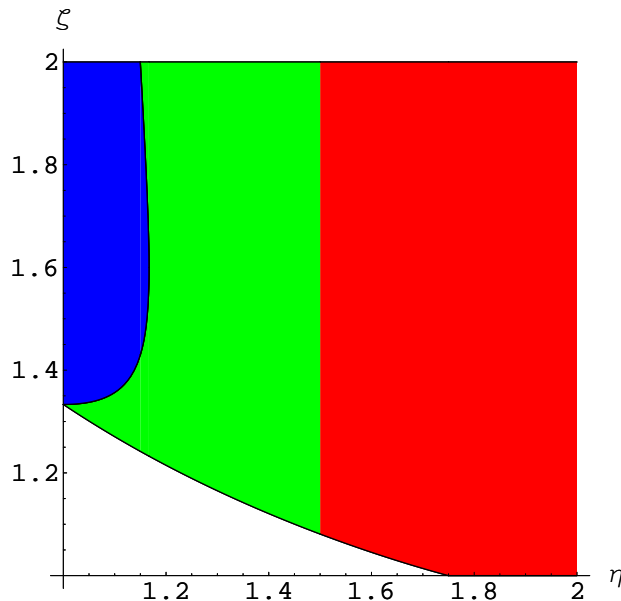


FIG. 1: (Colour online.) The regions in the (η, ζ) plane where the approximate potential has the required discrete symmetry breaking global minimum. The potential is bounded from below for $\zeta > 1$. In the red (or medium-shaded) region, $\phi_1 = \phi_2 = 0$ is a local maximum, $\phi_1^2 = \phi_2^2 \neq 0$ is a saddle point, and $\phi_1^2 = u^2$, $\phi_2 = 0$ and $\phi_1 = 0$, $\phi_2^2 = u^2$ are degenerate global minima. The green (or light-shaded) region differs from the red (medium-shaded) only in that $\phi_1 = \phi_2 = 0$ becomes a local minimum. As η decreases further, the $\phi_1 = \phi_2 = 0$ minimum keeps falling and in the blue (or dark-shaded) region it lies below the $\phi_1^2 = \phi_2^2 \neq 0$ saddle point. For $\eta < 1$, $\phi_1 = \phi_2 = 0$ becomes the global minimum. In the unshaded region in the $\eta, \zeta > 1$ quadrant, $\phi_1 = \phi_2 \neq 0$ is the global minimum. The border between the shaded and unshaded regions is a smooth but complicated function $\zeta(\eta)$ given in the main text.

In the red (or medium-shaded if viewing in black-and-white) region, I is a local minimum and II a saddle point with $V(I) < V(II)$, while III is a local maximum. As η decreases below $3/2$, III develops into a local minimum also; the ordering is $V(I) < V(II) < V(III)$ in the green (light-shaded) region and $V(I) < V(III) < V(II)$ in the blue (dark-shaded). For $\eta < 1$, III is the global minimum so there is no SSB. The border line between the shaded and unshaded areas is given by

$$\zeta = \frac{-27 - 18\eta + 61\eta^2 + \sqrt{729 + 972\eta - 2970\eta^2 + 1900\eta^3 - 375\eta^4}}{24\eta^3}. \quad (3.39)$$

Lowering ζ has the effect of lowering $V(II)$. Below this border line, $V(II) < V(I) < V(III)$, so the symmetry preserving vacuum is the global minimum.

The border between the blue (dark-shaded) and green (light-shaded) regions is given by

$$\eta = \frac{8\zeta - 3\zeta^2 + 2\sqrt{3}\sqrt{4\zeta^2 - 7\zeta^3 + 3\zeta^4}}{3\zeta^2}. \quad (3.40)$$

Figures 2-4 show contour plots of the approximate potential for representative points in the red (medium-shaded), green (light-shaded) and blue (dark-shaded) areas of Fig. 1, respectively. The colour- or shade-coding is explained in the captions. The plots illustrate the pattern of extrema described above.

IV. THE GAUGED MODEL

It is natural to extend our toy model by gauging $U(1) \otimes U(1)$, and to study the dependence of the gauge fields on w by self-consistently solving all of the coupled Euler-Lagrange equations. In the flat space case, Rozowsky, Volkas

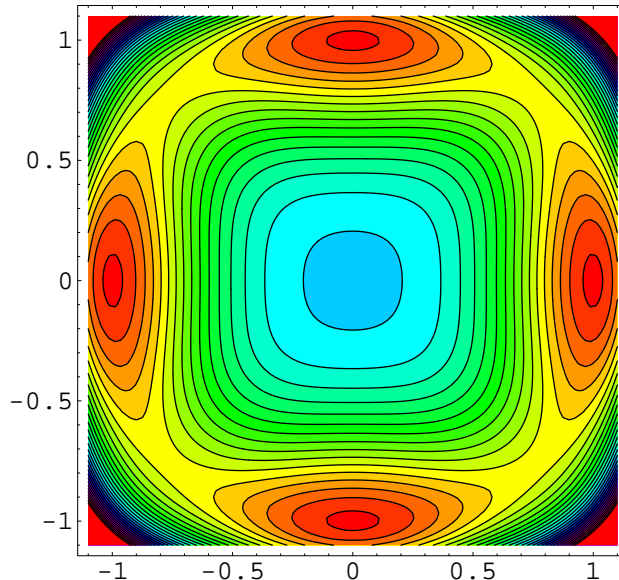


FIG. 2: (Colour online.) Contour plot of the approximate potential for $\eta = \zeta = 1.6$ to illustrate the qualitative features of the extrema for a representative point in the red (medium-shaded) region of Fig. 1. Within this plot, red colour codes for minima and blue for maxima. (These look similar in black-and-white, but the description below clarifies which is which.) The origin, $p_1 = p_2 = 0$, is a local maximum. The degenerate global minima are at $(p_1, p_2) = (0, \pm 1)$ and $(\pm 1, 0)$, while extrema *II*, where $p_1^2 = p_2^2$, are saddle points (light green or very-lightly shaded).

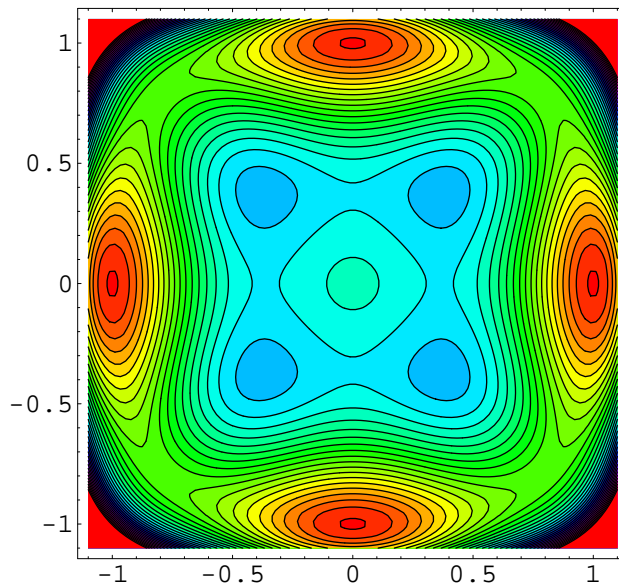


FIG. 3: (Colour online.) Contour plot of the approximate potential for $\eta = 1.3$ and $\zeta = 1.6$ to illustrate the qualitative features of the extrema for a representative point in the green (light-shaded) region of Fig. 1. Within this plot, red colour codes for minima and blue for maxima. (In black-and-white, the red region appears darker than the blue in this plot; the description below clarifies which is which.) The origin, $p_1 = p_2 = 0$, is now a local minimum, but the degenerate global minima are still at $(p_1, p_2) = (0, \pm 1)$ and $(\pm 1, 0)$. The relevant $p_1^2 = p_2^2$ extrema are again saddle points.

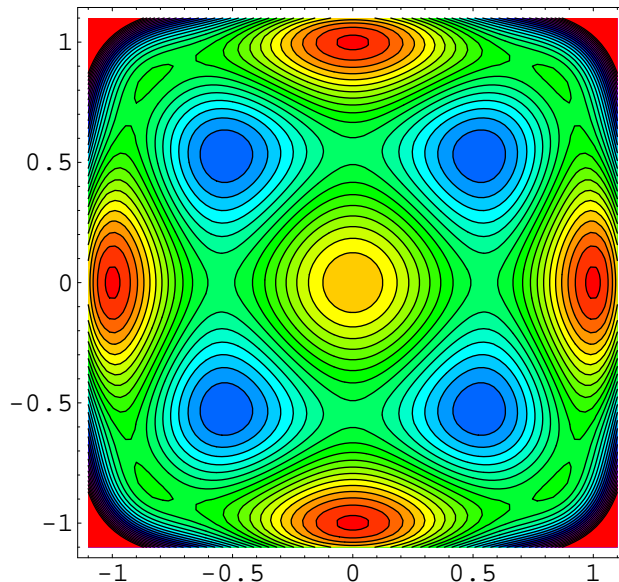


FIG. 4: (Colour online.) Contour plot of the approximate potential for $\eta = 1.05$ and $\zeta = 1.6$ to illustrate the qualitative features of the extrema for a representative point in the blue (dark-shaded) region of Fig. 1. Within this plot, red colour codes for minima and blue for maxima. (In black-and-white they appear similar, but the description below clarifies which is which.) The origin, $p_1 = p_2 = 0$, is now a deeper local minimum (yellow, or medium-shaded), having dropped below the $p_1^2 = p_2^2$ saddle points (green, or medium-shaded valleys). The local maxima are at approximately $(|p_1|, |p_2|) = (0.5, 0.5)$. The degenerate global minima are at $(p_1, p_2) = (0, \pm 1)$ and $(\pm 1, 0)$.

and Wali [3] found some interesting solutions where the gauge field coupled to Φ_i tended towards a linear function of w on the side of the wall where $\Phi_i \rightarrow 0$, and was exponentially suppressed on the other side (Meissner effect). The linearly rising gauge field implies an asymptotically constant magnetic field, so the physical picture was of two infinite, uniform sheets (one for each sector) of supercurrent density flowing along the domain wall. In this section we will first generalise the Rozowsky et al. flat-space analysis and then consider the curved space problem.

A. Flat space

The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}F_1^{MN}F_{1MN} - \frac{1}{4}F_2^{MN}F_{2MN} - (D^M\Phi_1)^*(D_M\Phi_1) - (D^M\Phi_2)^*(D_M\Phi_2) - V, \quad (4.1)$$

where

$$\begin{aligned} D_M\Phi_1 &= (\partial_M - ieA_{1M} - i\tilde{e}A_{2M})\Phi_1, \\ D_M\Phi_2 &= (\partial_M - i\tilde{e}A_{1M} - ieA_{2M})\Phi_2, \end{aligned} \quad (4.2)$$

and we take V to be quartic

$$V = \lambda_1(\overline{\Phi}_1\Phi_1 + \overline{\Phi}_2\Phi_2 - u^2)^2 + \lambda_2\overline{\Phi}_1\Phi_1\overline{\Phi}_2\Phi_2. \quad (4.3)$$

In the $\lambda_{1,2} > 0$ parameter space region, the global minima of V are

$$\text{Vacuum 1: } \overline{\Phi}_1\Phi_1 = u^2, \Phi_2 = 0 \quad \text{and} \quad \text{Vacuum 2: } \Phi_1 = 0, \overline{\Phi}_2\Phi_2 = u^2. \quad (4.4)$$

The Lagrangian is invariant under the discrete symmetry

$$\Phi_1 \leftrightarrow \Phi_2, \quad A_1 \leftrightarrow A_2. \quad (4.5)$$

This model generalises that considered in Ref.[3] by having $\tilde{e} \neq 0$.

The Euler-Lagrange equations are

$$\begin{aligned} D^M D_M \Phi_i &= 2 \lambda_1 \Phi_i (\bar{\Phi}_i \Phi_i + \bar{\Phi}_j \Phi_j - u^2) + \lambda_2 \Phi_i \bar{\Phi}_j \Phi_j, \\ \partial_M F_i^{MN} &= -2 \text{Im} (e \bar{\Phi}_i D^N \Phi_i + \tilde{e} \bar{\Phi}_j D^N \Phi_j), \end{aligned} \quad (4.6)$$

where $(i, j) = (1, 2)$ or $(2, 1)$. We now specialise to configurations that depend only on w , impose Lorentz gauge $\partial_M A_i^M = 0$, and write $\Phi_i = R_i e^{i\theta_i}$. The equations become,

$$R_i'' = R_i [(e A_i^\mu + \tilde{e} A_j^\mu) (e A_{i\mu} + \tilde{e} A_{j\mu})] + 2 \lambda_1 R_i (R_i^2 + R_j^2 - u^2) + \lambda_2 R_i R_j^2, \quad (4.7)$$

$$A_i^{\mu''} = 2 e R_i^2 (e A_i^\mu + \tilde{e} A_j^\mu) + 2 \tilde{e} R_j^2 (e A_j^\mu + \tilde{e} A_i^\mu), \quad (4.8)$$

$$\theta_i' = -(e A_i^5 + \tilde{e} A_j^5). \quad (4.9)$$

Observe from the last of these equations that the $A = 5$ gauge field components are pure gauge, so they and the phase fields θ_i decouple from the problem.

We choose to look for solutions that do not break rotational invariance in the x, y and z directions. This means that we can always rotate the x, y, z coordinate system so that the only nonzero component of (A_i^x, A_i^y, A_i^z) is in, say, the x -direction (this is tantamount to the requirement that the ratios of these components are w -independent). We also choose that the same component is nonzero for both $i = 1$ and $i = 2$. Equations (4.7) and (4.8) now simplify to

$$R_i'' = R_i [-(e A_i^t + \tilde{e} A_j^t)^2 + (e A_i^x + \tilde{e} A_j^x)^2] + 2 \lambda_1 R_i (R_i^2 + R_j^2 - u^2) + \lambda_2 R_i R_j^2, \quad (4.10)$$

$$A_i^{t,x''} = 2 e R_i^2 (e A_i^{t,x} + \tilde{e} A_j^{t,x}) + 2 \tilde{e} R_j^2 (e A_j^{t,x} + \tilde{e} A_i^{t,x}) \quad (4.11)$$

The final choice is whether to choose the gauge fields to be timelike ($A_i^t \neq 0, A_i^x = 0$), lightlike ($A_i^t = A_i^x \neq 0$), or spacelike ($A_i^t = 0, A_i^x \neq 0$). As is obvious from Eq. (4.10), the timelike ansatz gives rise to asymptotic oscillatory behaviour for the R 's and is thus unacceptable. We first discuss the spacelike case.

As a boundary condition, we require that $R_{1,2}$ tend to vacua 1 and 2 on opposite sides of the wall:

$$R_1(+\infty) = u, \quad R_2(+\infty) = 0 \quad \text{and} \quad R_1(-\infty) = 0, \quad R_2(-\infty) = u. \quad (4.12)$$

The appropriate boundary conditions for the gauge fields are then determined by examining Eqs. (4.11) at the asymptotic points. As $w \rightarrow +\infty$, we find that

$$\begin{pmatrix} A_1'' \\ A_2'' \end{pmatrix} = 2 u^2 \begin{pmatrix} e^2 & e \tilde{e} \\ e \tilde{e} & \tilde{e}^2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad (4.13)$$

where $A_i \equiv A_i^x$. Inputting trial asymptotic solutions of the form

$$A_{1,2} = a_{1,2} e^{-\kappa w}, \quad (4.14)$$

we obtain an eigenvalue equation yielding

$$\kappa = \sqrt{2} u \sqrt{e^2 + \tilde{e}^2} \quad \text{and} \quad \kappa = 0. \quad (4.15)$$

The zero eigenvalue means that one linear combination of A_1 and A_2 is always unsuppressed, and the vanishing second derivative tells us it grows linearly with w (provided its amplitude is nonzero). By the symmetry of the problem, a similar situation obtains as $w \rightarrow -\infty$. The exponentially suppressed linear combination embodies the Meissner effect. Numerical solutions are depicted in Fig. 5.

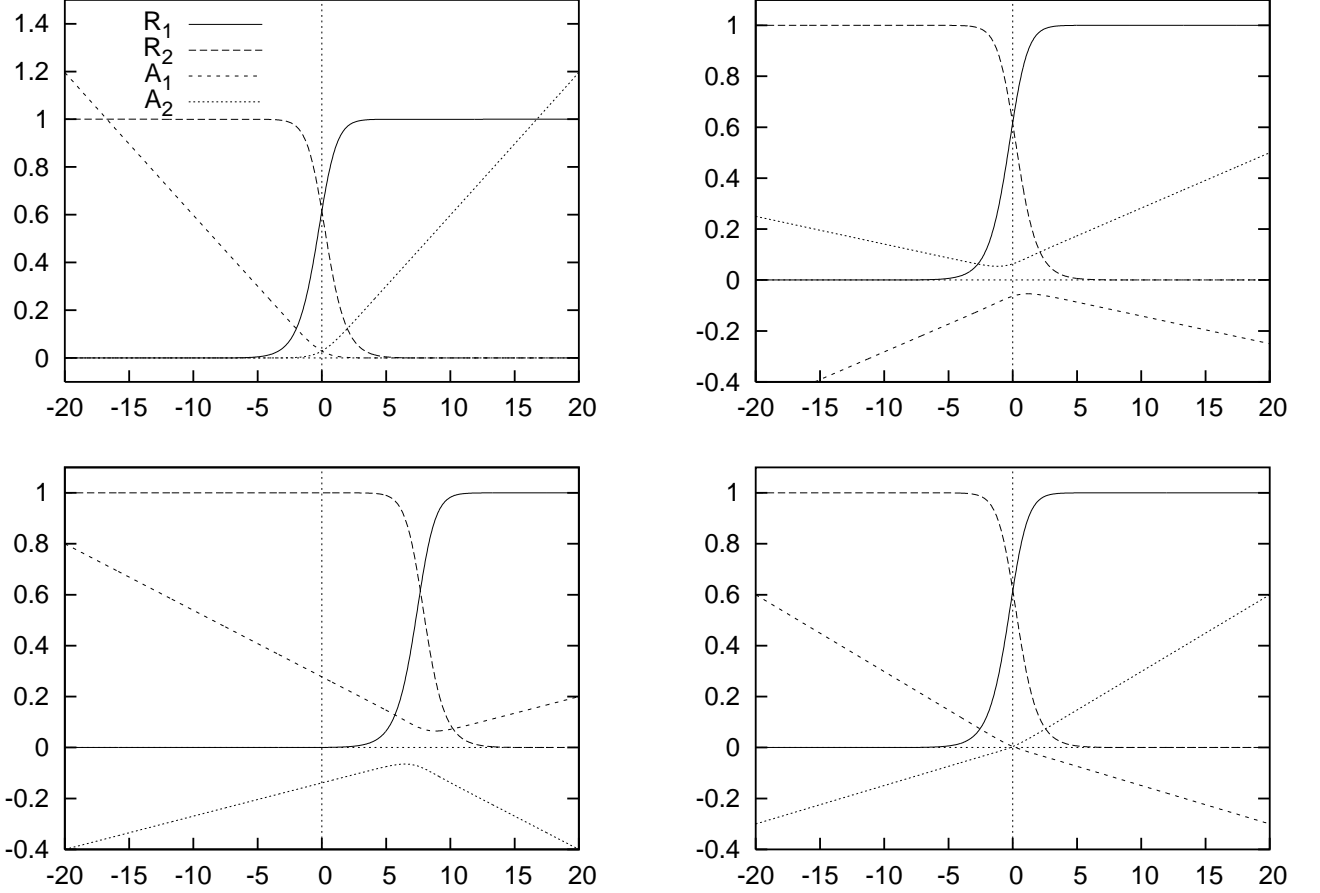


FIG. 5: Static solutions to the two scalar and two gauge fields in the $U(1) \otimes U(1)$ model. In all cases $\lambda_1 = \lambda_2 = e = 1$, and the fields are all plotted in units of u . In the top left plot, $\tilde{e} = 0$ as in the Rozowsky et al. model and the gauge fields show Meissner-like suppression under their respective scalar field. In the other three plots, $\tilde{e} = \frac{1}{2}$ and we show cases with various values for the free gauge field boundary conditions. In the two right plots, reflection symmetric boundary conditions are imposed for the pair of linear combinations $eA_i^x + \tilde{e}A_j^x$. In the bottom left plot, the gauge boundary conditions are asymmetric and their coupling to the kink shifts it to the right.

The lightlike case is qualitatively similar to the spacelike case. But it is amusing to point out that for the parameter choice $\lambda_2 = 4\lambda_1$ an analytic solution exists. For the lightlike ansatz, the gauge fields cancel out of Eq. 4.10, and with the stated parameter choice and $e = 1$, $\tilde{e} = 0$, the resulting equations yield the solutions

$$R_{1,2}(w) = \frac{u}{2} \left[1 \pm \tanh(\sqrt{\lambda_1} u w) \right]. \quad (4.16)$$

Inputting these results into Eq. 4.11, the analytic solutions obeying the correct boundary conditions for the gauge field functions $A_i^t = A_i^x \equiv A_i$ are

$$A_{1,2}(w) = \text{const.} \times {}_2F_1 \left(\frac{1+\sqrt{2}-\sqrt{3}}{2}, \frac{1+\sqrt{2}+\sqrt{3}}{2}; 1+\sqrt{2}; (1+e^{\pm 2\sqrt{\lambda_1} u w})^{-1} \right) \left(1 \mp \tanh(\sqrt{\lambda_1} u w) \right)^{\frac{1}{\sqrt{2}}}, \quad (4.17)$$

where ${}_2F_1$ is a hypergeometric function.

B. Warped spacetime

We now generalise the above model through the inclusion of gravity [13]. The action is

$$S = - \int \left[\frac{\kappa}{2} R + g^{MN} t_{MN} + \frac{1}{4} g^{MP} g^{NQ} f_{MNPQ} + V \right] \sqrt{-g} d^5 x, \quad (4.18)$$

with

$$t_{MN} \equiv \sum_i (D_M \Phi_i)^* D_N \Phi_i, \quad (4.19)$$

$$D_M \equiv \partial_M - \sum_i i Q_i A_{iM}, \quad (4.20)$$

$$f_{MNPQ} \equiv \sum_i F_{iMN} F_{iPQ}, \quad (4.21)$$

and the charges (Q_1, Q_2) of the scalar fields are (e, \tilde{e}) for Φ_1 and (\tilde{e}, e) for Φ_2 .

The Higgs and gauge field equations of motion are

$$D_M (\sqrt{-g} g^{MN} D_N \Phi_i) - \sqrt{-g} \frac{\partial V}{\partial \Phi_i^*} = 0 \quad (4.22)$$

$$\partial_M (\sqrt{-g} g^{MP} g^{NQ} F_{iPQ}) + \sqrt{-g} g^{NP} 2 (e \operatorname{Im} (\Phi_i^* D_P \Phi_i) + \tilde{e} \operatorname{Im} (\Phi_j^* D_P \Phi_j)) = 0, \quad (4.23)$$

while the Einstein equations, $\kappa G_{MN} = -T_{MN}$, feature

$$T_{MN} = 2t_{MN} + g^{PQ} f_{MPNQ} + g_{MN} \mathcal{L}_M, \quad (4.24)$$

$$\mathcal{L}_M = -g^{MN} t_{MN} - \frac{1}{4} g^{MP} g^{NQ} f_{MNPQ} - V. \quad (4.25)$$

The metric ansatz is generalised to

$$ds^2 = -e^{f(w)} dt^2 + e^{h(w)} dx^2 + e^{j(w)} (dy^2 + dz^2) + dw^2, \quad (4.26)$$

in order to have enough degrees of freedom to be able to look at sufficiently general gauge field configurations. We look for solutions of the form

$$A_{iM} = (A_i(w), B_i(w), 0, 0, Z_i(w)), \quad (4.27)$$

$$\Phi_i = R_i(w) e^{i\alpha_i(w)}, \quad (4.28)$$

To simplify the algebra it is useful to define the following:

$$\mathcal{F} = \frac{1}{2} (f + h + 2j), \quad (4.29)$$

$$\overline{\mathcal{F}} = \frac{1}{2} (-f + h + 2j), \quad (4.30)$$

$$\overline{\overline{\mathcal{F}}} = \frac{1}{2} (f - h + 2j), \quad (4.31)$$

$$\mathcal{A}_i = e A_i + \tilde{e} A_j, \quad (4.32)$$

$$\mathcal{B}_i = e B_i + \tilde{e} B_j, \quad (4.33)$$

$$\mathcal{Z}_i = e Z_i + \tilde{e} Z_j - \alpha'_i. \quad (4.34)$$

In terms of these, the equations of motion for the gauge field components are

$$A_i'' + \overline{\mathcal{F}}' A_i' - 2e R_i^2 \mathcal{A}_i - 2\tilde{e} R_j^2 \mathcal{A}_j = 0, \quad (4.35)$$

$$B_i'' + \overline{\overline{\mathcal{F}}}' B_i' - 2e R_i^2 \mathcal{B}_i - 2\tilde{e} R_j^2 \mathcal{B}_j = 0, \quad (4.36)$$

$$(e R_i^2 + \tilde{e} R_j^2) \mathcal{Z}_i = 0. \quad (4.37)$$

Equation (4.37) implies $\mathcal{Z}_i = 0$ which we use to simplify the rest of the equations of motion. For the scalar fields we have

$$R_i'' + \mathcal{F}' R_i' + e^{-f} \mathcal{A}_i^2 R_i - e^{-h} \mathcal{B}_i^2 R_i - e^{-i\alpha_i} \frac{\partial V}{\partial \Phi_i^*} = 0, \quad (4.38)$$

and for the three metric functions we have

$$f'' + \frac{1}{2} f'^2 + \frac{1}{3} f' h' + \frac{2}{3} f' j' - \frac{1}{3} h' j' - \frac{1}{6} j'^2 - \frac{10}{3\kappa} e^{-f} \Phi - \frac{2}{3\kappa} e^{-h} \Psi + \frac{2}{3\kappa} \Omega = 0, \quad (4.39)$$

$$h'' + \frac{1}{3} f' h' - \frac{1}{3} f' j' + \frac{1}{2} h'^2 + \frac{2}{3} h' j' - \frac{1}{6} j'^2 + \frac{2}{3\kappa} e^{-f} \Phi + \frac{10}{3\kappa} e^{-h} \Psi + \frac{2}{3\kappa} \Omega = 0, \quad (4.40)$$

$$j'' - \frac{1}{6} f' h' + \frac{1}{6} f' j' + \frac{1}{6} h' j' + \frac{5}{6} j'^2 + \frac{2}{3\kappa} e^{-f} \Phi - \frac{2}{3\kappa} e^{-h} \Psi + \frac{2}{3\kappa} \Omega = 0, \quad (4.41)$$

where

$$\Phi = \mathcal{A}_1^2 R_1^2 + \mathcal{A}_2^2 R_2^2 + \frac{1}{2} A_1'^2 + \frac{1}{2} A_2'^2, \quad (4.42)$$

$$\Psi = \mathcal{B}_1^2 R_1^2 + \mathcal{B}_2^2 R_2^2 + \frac{1}{2} B_1'^2 + \frac{1}{2} B_2'^2, \quad (4.43)$$

$$\Omega = R_1'^2 + R_2'^2 + V. \quad (4.44)$$

We performed an extensive numerical search for solutions to Eqs. (4.35, 4.36, 4.38-4.41), subject to the boundary conditions that the Higgs fields tend to VEVs, and that the metric functions tend to Randall-Sundrum form. For definiteness, we used the same Higgs potential as in the non-gauged model, namely as given by Eqs. (3.20), (3.22) and (3.25). The result of this investigation was the perhaps disappointing one that all solutions found required the gauge fields to vanish. Since the investigation was numerical, we cannot be certain that no non-trivial solutions exist, although we strongly suspect this to be the case.

V. CONCLUSION

We have constructed a $U(1) \otimes U(1)$ model with two complex scalar fields interchanged by a discrete symmetry coupled to 4 + 1 dimensional gravity. It has a solution featuring clash-of-symmetries-style Higgs kink configurations in a Randall-Sundrum-like spacetime. Gravity is localised to the dynamically generated (smooth) brane, and the Randall-Sundrum limit of the solution can be defined. For the chosen Higgs kink and metric configurations, the Higgs potential had to be of a certain sextic form whose properties we studied in some depth. The symmetry breaking pattern varies as a function of the extra dimension coordinate w , and displays the clash of symmetries phenomenon: at all points $|w| < \infty$ both $U(1)$'s are broken, with alternate $U(1)$'s restored as $w \rightarrow +\infty$ and $w \rightarrow -\infty$. The spontaneous breaking of the discrete interchange symmetry guarantees topological stability for the Higgs-gravity-induced brane. We also examined the gauged version of the theory, but we found that to have nonzero gauge fields the spacetime has to be flat.

This work sets the stage for incorporating gravity into more complicated models displaying the clash of symmetries. The eventual aim is to construct a realistic brane-world model that uses the clash of symmetries to understand spontaneous symmetry breaking in a more satisfactory fashion than obtained with the default mechanism using homogeneous Higgs vacuum expectation values.

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