Chiral symmetry breaking via constituent quarks for $\bar{q}q$ pseudoscalar mesons

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We base our chiral symmetry approach on the quark–level linear sigma model Lagrangian. Then we review the Nambu–Goldstone theorem with vanishing π, K, η_8 masses. Next we dynamically generate the π, K, η_8 masses away from the chiral limit. Then we study pion and kaon Goldberger–Treiman relations. Finally we extend this $q\bar{q}$ scheme to scalar and vector mesons. We also show the above $q\bar{q}$ meson scheme fits the higher mass octet baryon qqq pattern as well.

I. INTRODUCTION

Chiral symmetry breaking for pseudoscalar mesons(π, K, η_8) requires that although these masses vanish in the chiral limit [then satisfying Goldberger–Treiman relations (GTRs)], m_π, m_K, m_{η_8} are non-vanishing away from the chiral limit – hopefully near their observed values. In a quark–level linear sigma model (LσM) $q\bar{q}$ scheme (with constituent quarks) of Sec. II, the massless Nambu–Goldstone (NG) limits are reviewed in Sec. III and the chiral–broken π, K, η_8 $q\bar{q}$ meson masses are extracted in Secs. IV and V. Then GTRs are studied in Sec. VI. The analogue ground–state vector and scalar masses are obtained in Sec. VII. Finally this LσM $q\bar{q}$ scheme is summarized in Sec. VIII.

II. QUARK–LEVEL LINEAR σ MODEL

The strong interaction quark–level LσM Lagrangian density is

$$\mathcal{L}_{\text{int}} = g\bar{\psi}(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi + g'\sigma(\sigma^2 + \boldsymbol{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2)^2 - m_q\bar{\psi}\psi \ , \tag{1}$$

where

$$m_q = f_\pi g \ , \quad g' = \frac{m_\sigma^2}{2f_\pi} = \lambda f_\pi \tag{2}$$

in the chiral limit (CL) for f_π being approximately 93 MeV with $g = 2\pi/\sqrt{3}$ [1]. See, *e.g.*, Ref. [2] for the original nucleon–level version. The latter LσM also manifests a) the chiral current algebra b) the fermion and meson Goldberger–Treiman relations in Eq. (2), c) the partially conserved axial current (PCAC) scheme as well.

III. REVIEW OF NAMBU–GOLDSTONE THEOREM IN CL

Tree level $\partial\cdot A^{\text{CL}} = 0 \Rightarrow m_\pi^{\text{CL}} = 0$ [3] due to the chiral symmetry extended via quark and pion loops as Quark loops (ql):

$$(m_\pi^2)_{\text{ql}} = i8\,N_c\,g\left(-g + \frac{2g'm_q}{m_\sigma^2}\right)\int\frac{\hat{d}^4p}{p^2 - m_q^2} \to 0 \ \ \text{in CL} \ , \tag{3}$$

where $\hat{d}^4p = \frac{d^4p}{(2\pi)^4}$. Meson loops (ml):

$$(m_\pi^2)_{\text{ml}} = (-2\lambda + 5\lambda - 3\lambda)\,i\int\frac{\hat{d}^4p}{p^2 - m_\pi^2} + (2\lambda + \lambda - 3\lambda)i\int\frac{\hat{d}^4p}{p^2 - m_\sigma^2} = 0 + 0 = 0 \ . \tag{4}$$

Then the Nambu–Goldstone theorem in the CL is

$$m_\pi^2 = (m_\pi^2)_{\rm ql} + (m_\pi^2)_{\rm ml} = 0 + 0 = 0 . \tag{5}$$

The coefficients multiplying the three (formally divergent) integrals in Eqs. (3) and (4) are identically zero *before* cutoffs keep these integrals finite.

Extending this to SU(3) LσM we get the GTRs: $f_\pi\, g = \hat{m}$, $f_K\, g = \frac{1}{2}(m_s + \hat{m})$ where $\hat{m} = (m_u + m_d)/2$ along with the GTR ratio 1.22 from data [4]

$$\frac{f_K}{f_\pi} = \frac{1}{2}\left(\frac{m_s}{\hat{m}} + 1\right) \approx 1.22 \;\; \Rightarrow \;\; \frac{m_s}{\hat{m}} \approx 1.44 , \tag{6}$$

$$
\begin{aligned}
(m_K^2)_{\rm ql} &= i\,4\,N_c\,g \int \hat{d}^4 p \left(-2g\frac{p^2 - m_s\hat{m}}{(p^2 - m_s^2)(p^2 - \hat{m}^2)} + \frac{g'_{ns}}{m_{\sigma ns}^2}\frac{2\hat{m}}{p^2 - \hat{m}^2} + \sqrt{2}\frac{g'_s}{m_{\sigma s}^2}\frac{m_s}{p^2 - m_s^2}\right) \\
&= 0 \;\; {\rm in\ CL}
\end{aligned}
\tag{7}
$$

(see Ref. [3], Eqs. (23) and (24), or [1]) leading via quark loops to

$$m_K^2 = M_{\rm VP}^K + M_{{\rm qktad},ns}^K + M_{{\rm qktad},s}^K = 0 \tag{8}$$

in the CL (see Ref. [5], Eq. (14a)).

IV.   PION AND KAON MASSES AWAY FROM CL

The average nonstrange constituent quark mass is approximately

$$\hat{m} = \frac{1}{2}(m_u + m_d) \approx \frac{M_N}{3} \approx 313\ {\rm MeV} . \tag{9}$$

Equivalently, the low energy QCD 1 GeV scale $\langle -\bar{q}q\rangle \approx (245\ {\rm MeV})^3$ with the usual coupling $\alpha_s \approx 0.50$ suggests a dynamical mass [6, 7]

$$m_{\rm dyn} = \left(\frac{4\pi}{3}\alpha_s\langle -\bar{q}q\rangle\right)^{1/3} \approx 313\ {\rm MeV} . \tag{10}$$

Either the latter scale or Eq. (9) then predict charge radii via the vector meson dominance (VMD) and LσM schemes

$$r_\pi^{\rm VMD} = \frac{\hbar c\sqrt{6}}{m_\rho} \approx 0.623\ {\rm fm} , \tag{11}$$

$$r_\pi^{\rm L\sigma M} = \frac{\hbar c}{\hat{m}} \approx 0.63\ {\rm fm} , \tag{12}$$

for $\hbar c = 197.3\ {\rm MeV} \cdot {\rm fm}$ and with $(\hat{m} + m_s)/2 = (337.5 + 486)/2 \approx 411.75\ {\rm MeV}$ [see Eqs. (23) and (24)],

$$r_K^{\rm VMD} = \frac{\hbar c\sqrt{6}}{m_K^\star} \approx 0.54\ {\rm fm} , \tag{13}$$

$$r_K^{\rm L\sigma M} = \frac{2\hbar c}{\hat{m} + m_s} \approx 0.49\ {\rm fm} , \tag{14}$$

near present data [4]

$$r_{\pi^+} = (0.672 \pm 0.008) \text{ fm} , \quad r_{K^+} = (0.560 \pm 0.031) \text{ fm} . \tag{15}$$

Also the u and d constituent quark masses are [8]

$$\hat{m}(\text{mag. dipole moment}) = \frac{m_p}{2.792847} \left[1 + \frac{14 \text{ MeV}}{9\hat{m}} \right] \approx 337.5 \text{ MeV} , \tag{16}$$

$$m_u = 335.5 \text{ MeV} , \quad m_d = 339.5 \text{ MeV} , \tag{17}$$

due to

$$m_d - m_u \simeq m_{K^0} - m_{K^+} = 3.97 \text{ MeV} , \tag{18}$$

away from the CL and isospin limit [9]. See Ref. [10] for a global $\bar{q}q$ picture of mesons.
    The quark-level LσM predicts

$$N_c = 3, \quad m_\sigma = 2m_q, \quad g = \frac{2\pi}{\sqrt{3}} \approx 3.6276 \tag{19}$$

via either the LσM [1], QCD in infrared limit [11], $Z = 0$ compositeness condition (Z=0 c.c.) [12, 13]. This implies via the GTR

$$\hat{m} = f_\pi \, g \approx 93 \text{ MeV} \cdot \frac{2\pi}{\sqrt{3}} \approx 337.4 \text{ MeV} , \tag{20}$$

very near (16). Given the above constituent quark masses away from the CL, the chiral–breaking pion and kaon masses are found via [14] Eq. (4.4).
    The difference between the constituent and dynamical quark mass defines an effective current quark mass which vanishes in the CL [14]:

$$\delta\hat{m} = \hat{m}_{\text{con}} - \frac{\hat{m}_{\text{CL}}^3}{\hat{m}_{\text{con}}^2} = 337.5 \text{ MeV} - 269.2 \text{ MeV} = 68.3 \text{ MeV} , \tag{21}$$

where $\delta\hat{m} \to 0$ when $\hat{m}_{\text{con}} \to \hat{m}_{\text{CL}} \approx m_N/3 \approx 313$ MeV.
Then because mesons are taken as $q\bar{q}$ states,

$$m_\pi = \delta\hat{m} + \delta\hat{m} \approx 136.6 \text{ MeV} \tag{22}$$

midway between $m_{\pi^+} = 139.57$ MeV and $m_{\pi^0} \approx 134.98$ MeV experimental masses [4]. Also from Eq. (6) above,

$$m_s = 1.44 \, \hat{m} \approx 486.0 \text{ MeV} \tag{23}$$

away from the CL or with $g = 2\pi/\sqrt{3}$ and $f_K = 1.22 f_\pi \approx 113.46$ MeV,

$$m_s = 2 f_K \, g - \hat{m} = (823.2 - 337.5) \text{ MeV} \approx 485.7 \text{ MeV} \tag{24}$$

close to (23). However with $m_s \approx 510$ MeV $\approx m_\phi(1020)/2$ or via magnetic dipole moments, one finds the average constituent quark masses extending Eq. (23) to [14]

$$m_s^{\text{avg}} = (486 + 510) \text{ MeV}/2 = 498 \text{ MeV} \quad \Rightarrow \tag{25}$$

$$\bar{m} = (498 + 337.5) \text{ MeV}/2 = 417.75 \text{ MeV} \quad \Rightarrow \tag{26}$$

$$\delta'\hat{m} = \hat{m}_{\text{con}} - \frac{\hat{m}_{\text{CL}}^3}{\bar{m}^2} \approx (337.5 - 175.7) \text{ MeV} = 161.8 \text{ MeV} \tag{27}$$

$$\delta'm_s = m_{s,\text{con}}^{\text{avg}} - \frac{\hat{m}_{\text{CL}}^3}{\bar{m}^2} \approx (498 - 175.7) \text{ MeV} \approx 322.3 \text{ MeV} \quad \Rightarrow \tag{28}$$

$$m_K = \delta'\hat{m} + \delta'm_s = (161.8 + 322.3) \text{ MeV} = 484.1 \text{ MeV} \tag{29}$$

not far away from average [4] K mass $m_{K^0} \approx 497.7$ MeV and $m_{K^+} \approx 493.7$ MeV.

V. η_8 MASS

In the CL $m_{\eta_8} = 0$, consistent with the *squared* Gell-Mann–Okubo (GMO) mass

$$m_{\eta_8}^2 = \frac{4m_K^2 - m_\pi^2}{3} \to 0 \quad \text{in CL limit ,} \tag{30}$$

$$\frac{f_8}{f_\pi} = \frac{3}{5 - 2\hat{m}/m_s} \approx 0.831 \tag{31}$$

for $m_s/\hat{m} \approx 1.44$. ($f_8/f_\pi \to 1$ in the U(3) symmetry limit.) Quark–level GTR for η_8 [4] gives $(f_{\eta_8}/f_\pi) \approx 1.2$. Then

$$\sqrt{3}\frac{f_8}{f_\pi}\left(\frac{f_{\eta_8}}{f_\pi}\right) f_\pi \, g = \sqrt{3} \cdot 0.831 \cdot 1.2 \cdot 92.42 \text{ MeV} \cdot \frac{2\pi}{\sqrt{3}} = m_{\eta_8} \approx 579.1 \text{ MeV} , \tag{32}$$

a dynamical estimate reasonably close to the GMO value $m_{\eta_8} \approx 566.6$ MeV from Eq. (30). Also η_{ns} and η_s $\bar{q}q$ mixing masses are [15] $m_{\eta_{ns}} \approx 758.1$ MeV and $m_{\eta_s} \approx 801.2$ MeV, respectively, so that

$$m_\eta^2 + m_{\eta'}^2 = m_{\eta_1}^2 + m_{\eta_8}^2 = m_{\eta_{ns}}^2 + m_{\eta_s}^2 \approx 1.217 \text{ GeV}^2 . \tag{33}$$

Reference [15] suggests using $m_{\eta_8} = 575.56$ MeV as in Eqs. (34) and (35) below

$$|\theta_P| = \arctan\sqrt{\frac{m_{\eta_8}^2 - m_\eta^2}{m_{\eta'}^2 - m_{\eta_8}^2}} \approx 13° , \tag{34}$$

$$m_{\eta_8}^2 = \cos^2\theta_P \, m_\eta^2 + \sin^2\theta_P \, m_{\eta'}^2 \approx (575.56 \text{ MeV})^2 , \tag{35}$$

$$\phi_P = \arctan\sqrt{\frac{m_{\eta_{ns}}^2 - m_\eta^2}{m_{\eta'}^2 - m_{\eta_{ns}}^2}} \approx 41.84° , \tag{36}$$

$$m_{\eta_{ns}}^2 = \cos^2\phi_P \, m_\eta^2 + \sin^2\phi_P \, m_{\eta'}^2 = (758.1)^2 \text{ MeV} , \tag{37}$$

$$\theta_P = \phi_P - \arctan\sqrt{2} \approx 41.84° - 54.74° = -12.9° , \tag{38}$$

close to $-13°$ in Eq. (34) and near the resulting GMO mass $m_{\eta_8} \approx 566.6$ MeV in Eq. (30) away from the CL with average

$$m_{\eta,\eta'}^{\text{avg}} = \frac{547.75 + 957.78}{2} \text{ MeV} = 752.77 \text{ MeV} , \tag{39}$$

near $m_{\eta_{ns}} = 758.1$ MeV above. With hindsight, the above tightly bound $q\bar{q}$ meson masses are near data [4] in spite of their NG vanishing values.

VI. CHIRAL GOLDBERGER–TREIMAN RELATIONS

Given the massless NG pseudoscalars $m_\pi = m_K = m_{\eta 8} = 0$ and their massive version in Secs. IV and V, the massive chiral symmetry breaking poles combined with axial current conservation then lead to the quark–level GTRs for pions and for kaons:

$$f_\pi g \;=\; \hat{m} = \frac{1}{2}(m_u + m_d) \approx 337.5 \text{ MeV} , \tag{40}$$

$$f_K g \;=\; \frac{1}{2}(m_s + \hat{m}) \approx 411.8 \text{ MeV} , \tag{41}$$

where we have invoked the constituent quark masses, Eqs. (16,17,23). Also evaluating the lhs of Eqs. (40) and (41) for $f_\pi \approx 93$ MeV, $f_K \approx 1.22 f_\pi \approx 113.5$ MeV and the meson–quark coupling $g \approx 2\pi/\sqrt{3} \approx 3.6276$ for $N_c = 3$ [1, 6, 7], the lhs of Eqs. (40,41) becomes 337.4 MeV, 411.6 MeV, in very close agreement with the rhs of Eqs. (40,41), respectively.

For finite UV cutoff Λ, the pion coupled to the axial current via the quark loop with $g = 2\pi/\sqrt{N_c}$ leads to

$$\int \frac{d^4 p}{(p^2 - m_q^2)^2} = i\pi^2 , \tag{42}$$

or equivalently to the log–divergent gap equation [1, 16]

$$-i 4 N_c g^2 \int \frac{\hat{d}^4 p}{(p^2 - m_q^2)^2} = 1 . \tag{43}$$

Explicitly accounting for Λ, the lhs of Eq. (43) can be written as [1]

$$\ln\left(1 + \frac{\Lambda^2}{m_q^2}\right) - \frac{1}{1 + \frac{m_q^2}{\Lambda^2}} = 1 , \tag{44}$$

with the numerical solution

$$\frac{\Lambda^2}{m_q^2} \approx (2.3)^2 , \tag{45}$$

or

$$\Lambda^{\text{CL}} \approx 2.3 \, \hat{m}_q^{\text{CL}} \approx 750 \text{ MeV} , \tag{46}$$

for CL quark mass 325.7 MeV and $\Lambda \approx 2.3 \hat{m}_q \approx 776$ MeV for chiral–broken mass 337.5 MeV.

It is significant that the above quark mass scales from Eq. (46) correspond to the Z=0 c.c. [12, 13] with $\Lambda^{\text{CL}} < \Lambda \approx 776$ MeV near the $\rho(775)$ and $\omega(782)$ slightly bound $\bar{q}q$ masses, but slightly heavier then $\Lambda^{\text{CL}} \approx 750$ MeV. In a similar fashion, the Λ' cutoff for the vector $K^\star$ $\bar{q}q$ mass is (for $m_s \approx 469$ MeV and $\hat{m} \approx 325.7$ MeV see Eq. (54) below) in the CL:

$$\Lambda' \approx 2.3\sqrt{m_s \, \hat{m}} \approx 899 \text{ MeV} , \tag{47}$$

reasonably near the observed $K^\star(894)$ mass. Lastly, the Z=0 c.c. also requires the meson–quark coupling to be [12] $g \approx 2\pi/\sqrt{3}$, analogous to the infrared QCD limit [6, 7] and also found via the quark–level LσM [1].

VII. EXTENSION TO $\bar{q}q$ SCALAR AND VECTOR MASSES

To complete the ground state $\bar{q}q$ meson scheme, we summarize and update the results of Ref. [10], first obtaining the SU(3) ground state octet vector meson $\bar{q}q$ masses from the bare plus symmetry–breaking terms as $m_V = \sqrt{2/3}\, m_V^0 - d_{i8i}\, \delta m_V$:

$$m_{\rho,\omega} = \sqrt{\frac{2}{3}} m_V^0 - \frac{1}{\sqrt{3}} \delta m_V \approx 779 \text{ MeV} , \tag{48}$$

$$m_{K^\star} = \sqrt{\frac{2}{3}} m_V^0 + \frac{1}{2\sqrt{3}} \delta m_V \approx 894 \text{ MeV} , \tag{49}$$

$$m_\phi = \sqrt{\frac{2}{3}} m_V^0 + \frac{2}{\sqrt{3}} \delta m_V \approx 1019 \text{ MeV} , \tag{50}$$

leading to the average scales

$$m_V^0 \approx 961 \text{ MeV} , \quad \delta m_V \approx 139 \text{ MeV} , \quad \frac{\delta m_V}{m_V^0} \approx 14\% . \tag{51}$$

Also for scalar meson masses, the model–independent nonstrange scalar sigma mass is [17]

$$m_{\sigma ns} \approx 665 \text{ MeV} . \tag{52}$$

This can be verified by first working in the CL with NJL–LσM mass

$$m_\sigma^{\mathrm{CL}} = 2\hat{m}^{\mathrm{CL}} \approx 651.4 \text{ MeV} , \tag{53}$$

due to the GTR. In the CL

$$\hat{m}^{\mathrm{CL}} = f_\pi^{\mathrm{CL}}\, g = (89.775 \text{ MeV}) \frac{2\pi}{\sqrt{3}} \approx 325.7 \text{ MeV} , \tag{54}$$

leading to (53). Also f_π^{CL} above follows from the once–subtracted dispersion relation [18]

$$\frac{f_\pi}{f_\pi^{\mathrm{CL}}} - 1 = \frac{m_\pi^2}{8\pi^2 f_\pi^2} \left(1 + \frac{m_\pi^2}{10\, \hat{m}^2} \right) \approx 2.946\% , \tag{55}$$

giving the observed pion decay constant [4] – extended in the CL as

$$f_\pi \approx 92.42 \text{ MeV} , \qquad f_\pi^{\mathrm{CL}} = \frac{f_\pi}{1.02946} \approx 89.775 \text{ MeV} . \tag{56}$$

Then the nonstrange scalar mass satisfies

$$m_{\sigma ns}^2 - m_\pi^2 = (m_\sigma^{\mathrm{CL}})^2 \quad \text{or} \quad m_{\sigma ns} \approx 665.82 \text{ MeV} , \tag{57}$$

compatible with (52). Also the scalar kappa mass satisfies

$$m_\kappa = 2\sqrt{\hat{m} m_s} \approx 810 \text{ MeV} \tag{58}$$

for $\hat{m} \approx 337.5$ MeV and from (6), $m_s \approx 1.44\, \hat{m} \approx 486$ MeV, compatible with E791 data [19] $m_\kappa = 797 \pm 19$ MeV. Finally, the pure strange scalar mass satisfies

$$m_{\sigma_{\mathrm{S}}} \simeq 2m_s \approx 972 \text{ MeV} , \tag{59}$$

	data Ref. [4]	quark sum
σ_{ns}	665	$2\hat{m} \approx 675$
κ	(797 ± 19)	$2\sqrt{\hat{m}m_s} \approx 810$
σ_s	(980 ± 10)	$2m_s \approx 972$

TABLE I: Touching–quark scalar meson masses (in MeV) for $\hat{m} \approx 337.5$ MeV and $m_s \approx 486$ MeV.

near the almost pure $\bar{s}s$ vector mass $m_\phi \approx 1019$ MeV.

Then the scalar analog of the SU(3) vector masses (48)–(50) are

$$m_{\sigma ns} = \sqrt{\frac{2}{3}}m_s^0 - \frac{1}{\sqrt{3}}\delta m_s \approx 665 \text{ MeV} , \tag{60}$$

$$m_\kappa = \sqrt{\frac{2}{3}}m_s^0 + \frac{1}{2\sqrt{3}}\delta m_s \approx 810 \text{ MeV} , \tag{61}$$

$$m_{\sigma s} = \sqrt{\frac{2}{3}}m_s^0 + \frac{2}{\sqrt{3}}\delta m_s \approx 972 \text{ MeV} , \tag{62}$$

giving the average SU(3) scalar $\bar{q}q$ masses,

$$m_s^0 \approx 933 \text{ MeV} , \quad \delta m_s \approx 177 \text{ MeV} , \quad \frac{\delta m_s}{m_s^0} \approx 19\% , \tag{63}$$

reasonably near the average SU(3) vector $\bar{q}q$ masses in (51).

Also given the closeness of these scalar masses in (60)–(62) to the approximate quark mass sums in Tab. I, all scalar meson masses have essentially "touching quarks". However, the vector masses in (48)–(50) are "loosely bound quarks" as they are 115 to 50 MeV heavier then the above "touching quark" scalar meson masses. In Ref. [14] a nonrelativistic quark model $\boldsymbol{L} \cdot \boldsymbol{S}$ coupling roughly accounts for the difference between vector and scalar meson masses.

VIII. CONCLUSION

In this paper we have primarily focused on the ground state $\bar{q}q$ pseudoscalar mesons, which we model via the quark–level LσM, briefly described in Sec. II.

The vanishing chiral NG pion and kaon masses are discussed in Sec. III. Their non–vanishing mass values away from the CL are discussed in Sec. IV, characterized by the tightly–bound $\bar{q}q$ charge radii $\hbar c/\hat{m}$, $2\hbar c/(\hat{m} + m_s)$ for charged pions and kaons, respectively. The latter are also close to the VMD values. In Sec. V we have extended the vanishing NG η_8 mass to its non–vanishing GMO and its tightly bound $\bar{q}q$ meson–mixing value. Sec. VI deals with chiral–symmetric pion and kaon GTRs. In Sec. VII we extended the LσM scheme to $\bar{q}q$ vector and scalar masses. Note that the loosely bound SU(3) masses $m_S^0 \approx 933$ MeV and $m_V^0 \approx 961$ MeV are close to the tightly bound m_P mass characterized by the observed [4] η' mass $m_{\eta'} = (957.78 \pm 0.14)$ MeV.

The above $\bar{q}q$ scheme appears somewhat counter to the PDG p.848 "non–$\bar{q}q$ candidates" [20]. However, in Ref. [10] we remind the reader of the standard ground state SU(3) qqq octet and decuplet baryon states with [10] $m_O^0 \sim$ 1150 MeV, $m_D^0 \sim 1230$ MeV being about 250 MeV greater then the above $\bar{q}q$ ground states $m_V^0 \sim m_S^0 \sim m_{\eta'} \sim$ 950 MeV. Taking $\phi \sim \bar{s}s$ or $m_s \sim 500$ MeV and $J/\psi(3100) \sim \bar{c}c$ or $m_c \sim 1550$ MeV, one can reasonably model the (higher mass) ground state qqq baryon states vs. data [4] as in Tab. II.

With hindsight, the recent paper [21] complements the present LσM $q\bar{q}$ picture quite well. In particular, Ref. [21] shows in detail why the quark triangle LσM predictions (involving no arbitrary parameters for at least 15 decays) match data [4] to within 5%. This is for $V \to PV$ or $P \to VV$ strong or electromagnetic decays. Also, the dynamic Schwinger–Dyson approach describes π, K, and η decays in conformity with empirical constraints.

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constituent quark predictions		data from [4]	
qqq	m[MeV]		m[MeV]
sdd	1180	Σ^-	1197.45 ± 0.30
ssd	1340	Ξ^-	1321.31 ± 0.13
sss	1500	Ω^-	1672.45 ± 0.29
cud	2225	Λ_c^+	2284.9 ± 0.6
csu	2390	Ξ_c^+	2466.3 ± 1.4
css	2550	Ω_c^0	2697.5 ± 2.6
ccd	3440	Ξ_{cc}^+	3519 ± 1

TABLE II: Loosely bound qqq baryons (in MeV) for $m_u \approx 335.5$ MeV and $m_d \approx 339.5$ MeV, $m_s \approx 500$ MeV, and $m_c \approx 1550$ MeV.

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