Dissipation, hydrodynamics and the fireball

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We investigate the hydrodynamics of the QCD plasma using dimensionless numbers built from the thermodynamics and transport theory of the plasma and characteristic dimensions of the fireball produced in heavy-ion collisions. We find that by the usual measures, dissipation is strong, and the fireball is on the borderline of equilibrium. As a result, the system is richer in phenomena than ideal hydrodynamics would predict. One general implication is that it may be possible to get a direct view of the QCD plasma phase rather than having to infer its existence indirectly from signals that come from the freezeout isotherm after the fireball has cooled into the hadronic phase.

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There is an emerging concordance of the first results from AdS/CFT estimates [1], lattice gauge theory [2, 3], and analysis of RHIC data [4] that the QCD plasma has small transport coefficients, *i. e.*, small mean free times. One of the transport coefficients that has been measured on the lattice is the electrical conductivity [2]. This is much smaller than weak coupling estimates would predict, implying that the resistivity is larger. Large resistivity means that if an external potential difference is established across the plasma, it would not drive large currents, but instead generate large amounts of entropy.

In a similiar vein, if the viscosity of the fluid is small, then the implication is that Reynolds numbers are large (all other conditions being equal). Large Reynolds numbers are typically associated with turbulence and other sources of entropy production when the fluid is subjected to external forces. On the face of it, this seems to be in contradiction to statements that the quark-gluon fluid is an "ideal liquid", so it pays to examine this matter more closely. We do so, and find that the situation is complex and interesting— turbulence is unlikely to occur, but the fireball is on the borderline of dis-equilibrium. This conclusion is driven in equal measure by analysis of the data that the RHIC has produced [4], the understanding of QCD gleaned through lattice computations [2, 3], and limits from AdS/CFT [1]. More detailed conclusions are enumerated at the end of the article.

One distinction between a liquid and a gas that seems to be easy to generalize is that in a gas the mean free path is much longer than the interparticle spacing, whereas in a liquid the two are of the same order. The number density should be replaced by the entropy density, s, in a relativistic system, so one should examine the dimensionless quantity called the liquidity in [5]—

$$\ell = \tau s^{1/3} \tag{1}$$

where τ is the mean free time. A little above the QCD cross-over temperature, T_c , one may use lattice measurements of the two quantities on the right to estimate this dimensionless number. Using the inferred value of $\tau = 0.3$ fm from lattice computations at $2T_c$ [2], and $s/T^3 = 5.7 \pm 0.2$ [6], one finds that $\ell = 0.8$, [5, 7]. At much higher temperatures, where weak coupling estimates [8] become reliable, one has $s/T^3 = 6\pi^2 + \mathcal{O}(g^2)$, and $\tau = \mathcal{O}(1/g^4 \log 1/g)$, so that $\ell \propto 1/g^4 \log(1/g)$ when T becomes sufficiently large. There is no phase transition between the strong coupling regime near T_c (liquid) and weak coupling (gas), so the nature of this "liquid" bears further investigation.

If one takes the first order dissipative hydrodynamics of [9] and writes it in the Eckart frame [10], one gets the Navier-Stokes' equation with a relativistic correction term. Analysis of dissipation through the Reynolds number then is a reasonable starting point. The Reynolds number is

$$R = \frac{\epsilon vr}{\eta} = \frac{(\epsilon/sT)v(rT)}{\eta/s} \tag{2}$$

where ϵ is the energy density of the QCD matter, s its entropy density, T its temperature, v the speed of the flow, and r the typical size of the fireball. Since $\epsilon/T^4 = 4.4 \pm 0.1$ for the QCD plasma at $2T_c$, $s/T^3 = 5.7 \pm 0.2$ and $v = c_s \approx 1/\sqrt{3}$ [6], using the AdS/CFT estimate [1] $\eta/s \ge 1/4\pi$, one finds $R \le 5.6(rT)$. In fact, on the lattice [2], one finds η/s which is twice this value, giving

$$R = 2.8(rT).\tag{3}$$

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This estimate is made at $2T_c \approx 350$ MeV. However, direct data on r comes from HBT analysis. At chemical freezeout temperature, $T \approx 175$ MeV, this implies r = 7 fm. By using the freezeout values in the estimate of R, one overestimates it, because η is espected to be larger at freezeout (otherwise matter would not freeze out) and s is expected to drop, pushing up the value of η/s [11]. On the other hand, if we take the temperature to be $2T_c$, then r would be smaller. So an upper bound on R is obtained by using freezeout values of r and T in eq. (3). This is R < 18.

If the Bjorken flow approximation is not destroyed by viscosity, then one could use the estimate $T \propto \tau^{-1/c_s^2} \simeq \tau^{-3}$ and $r \simeq \tau$ to write $rt \propto \tau^{-2}$. If one neglects any special kinetics that develops at the QCD cross over, and uses Bjorken flow to push the estimate above back to a time when the temperature was $2T_c$, then one obtains $R \simeq 14$. The main correction to this comes from the observation on the lattice of the slowing of sound as one approaches T_c [6]. We will later investigate the stability of Bjorken flow against dissipation.

In non-relativistic dynamics of a liquid, one also has incompressibility. Low R and incompressibility together cause the motion to be reversible. This doesn't happen in heavy-ion collisions. Write $\eta = \epsilon c_s \lambda$ where c_s is the speed of sound and λ is a typical length characteristic of momentum transport (which is $c_s \tau/3$ in a non-relativistic gas). Then

$$R = \left(\frac{v}{c_s}\right) \left(\frac{r}{\lambda}\right) = M\left(\frac{r}{\lambda}\right) \tag{4}$$

where M is the Mach number. For heavy-ion collisions, $1 \le M \le \sqrt{3}$. The lower bound comes from the fact that matter is expanding into a vacuum so the transverse flow essentially moves at the speed of sound. The upper bound is for fast particles or jets travelling through the medium. Therefore we have flow at small Reynolds number and large Mach number.

Typically a large Mach number means that compression cannot be neglected and shocks are possible. A low Reynolds number means that energy can be dissipated during a shock. So if one has viscous shock waves in heavy-ion collisions, then the energy of the expansion of the fireball can be pumped back into reheating, thereby prolonging the life of the plasma droplet. This scenario has been studied in [12, 13, 14].

But instead of stopping here to follow up on this, we proceed with dimensional analysis. We write the Knudsen number

$$K = \frac{\lambda}{r} = \frac{R}{M} \,. \tag{5}$$

Using the values of the Reynolds and Mach numbers presented before, we find that— $\lambda/r \approx 0.032-0.055$. Another estimate, taking $r \approx 7$ fm and $\lambda \approx 0.3$ fm [2], is $K \approx 0.04$, which is consistent with this. Hydrodynamics is the limit of very small Knudsen number. Is the number we obtained here small in this sense? To understand this, we note that a finite K means that the isotherms of ideal fluid flow cannot be regarded as mathematically perfect surfaces of zero thickness, but are smeared out over 3–6% of the radius of the fireball. In particular, this means that the outer skin, from which particles can leave the fireball, is 10–20% of the volume if the fireball shape is a sphere. If the fireball is elongated, then the evaporative skin can be a larger fraction of the volume. Thus, in the time a low temperature isotherm moves half the way towards the center, almost a quarter of the material from the surface has evaporated. So the values of K here are not really small enough for ideal hydro, since the hot system is already pretty close to freezeout.

A very enlightening way in which to consider the strength of dissipative effects is to compute the damping of sound waves. In the presence of viscosity, sound instensity is exponentially damped: $\exp(-r/\Gamma_s)$. The sound attenuation length is given by

$$\Gamma_s T = \frac{4\eta}{3s} \ge \frac{1}{3\pi},\tag{6}$$

where the last estimate is derived from the bound conjectured in [1]. At $2T_c$, lattice results would predict Γ_s to be twice this limiting value [2]. With this estimate, one e-fold decrease in intensity would occur over a distance of less than 0.4 fm at $2T_c$. This has strong implications on the hydrodynamic history of the fireball, since both the transverse flow and elliptic flow are acoustic in nature, driven by pressure gradients. Such a strong dissipation of sound implies that the hydrodynamic prediction of the elliptic flow must be smaller.

So, next we ask how the dissipation affects the longitudinal flow— this may be different. Using first order dissipative hydrodynamics [9], and doing a simple order of magnitude estimate of the relative importance of the entropy current, s', and the entropy s, one finds

$$\frac{s'}{s} = \left(\frac{\eta}{s}\right) \left(\frac{u^2}{Tr^2}\right) \tag{7}$$

Now u = 1, T = 150 MeV and r = 7 fm at freezeout. If we take the AdS/CFT lower bound, $\eta/s = 1/4\pi$, then $s'/s \approx 0.0014$ near freezeout. This small number is an underestimate because η/S has to be larger at freezeout, and the bound is more accurate at early times, when one has larger T and smaller r. If the entropy production were small at all times, one could just estimate it for Bjorken flow. Then, as before, $T \propto \tau^{-3}$ and $r \propto \tau$, one would have $s'/s \propto \tau$. This allows us to push the estimate above back to a time appropriate to the input value of η/S , and shows that the entropy production at earlier times would have been negligible. This analysis indicates that the Bjorken solution is stable against first order dissipative terms even if η/s is significantly larger than the limit conjectured in [1]. This, in turn, lends stability to our earlier estimate of the Reynolds number.

While we are discussing dissipation, I make a detour to ask how big is the bulk viscosity, ζ . This is more or less unknown. In a perturbative computation it would be very small. General conditions are discussed for the vanishing of ζ in [15], where a result from [16] is quoted which says that if the trace of the stress tensor, $\epsilon - 3P = f(\epsilon, n)$ (*P* is the pressure and *n*, a conserved particle number), then ζ vanishes. This condition fails to hold in many cases, most notably when the temperature is comparable to masses of the particles or when matter contains composite particles and energy can be transferred from translational modes to internal excitations [17]. In any case, a stress tensor with non-vanishing trace is a breakdown of conformal symmetry, and may exist in any field theory with a non-vanishing β -function. In particular, in QCD near T_c both these conditions may hold.

The bulk viscosity is known for a two component model of matter where one component has very small mean free time compared to the other [18], and gives

$$\frac{\zeta}{\eta} = \frac{5}{3} \left(1 - 3c_s^2 \right)^2,\tag{8}$$

where c_s is the speed of sound in the medium. Several attempts to understand the behaviour of the equation of state of QCD near T_c have effectively involved two component models which are similiar. Moreover, at least one of the components in such a model is usually understood to be composite [19], and thereby satisfy one of the conditions for a large ζ . Since the speed of sound is now known directly from lattice computations [6], one can form an estimate of this ratio. Of course, this would be correct only if QCD matter were approximately of this kind. On the other hand, if it is not, having some estimate available may allow us to design observables that would test or rule out this scenario. Using the data of [6] in conjunction with eq. (8), we find that $\zeta/\eta = 0.45$ at $T = 0.9T_c$. At $1.1T_c$, immediately above the transition, $\zeta/\eta = 0.35$. Even at $1.5T_c$ the ratio is as large as 0.1, but it decreases rapidly above that, being totally negligible at $2T_c$ and above. We end our detour on the bulk viscosity with this estimate and return to a summary of the evolution of the fireball.

The viscous corrections to Bjorken flow can be appproximately absorbed into a modified stress tensor [4]. An interesting dimensionless number to consider in this case is the ratio between the transverse and longitudinal "pressures",

$$A = \frac{P + 2\eta/3\tau}{P - 4\eta/3\tau} = \frac{(P/sT) + (2/3)(\eta/s)/\tau T}{(P/sT) - (4/3)(\eta/s)/\tau T}.$$
(9)

In dissipationless Bjorken hydrodynamics, A = 1. In this form one can use the estimate $P/T^4 = 1.26 \pm 0.04$ along with the previous estimates at $2T_c$. Then one finds A = 1.37 if one uses the Bjorken solution to propagate the freezeout value of τT backwards to a time when $T = 2T_c$. If instead one uses the freezeout value of this combination one finds A = 1.28. The small departure from A = 1 is another indication of the stability of the Bjorken solution. This is, of course, one of the bases of the analysis in [4].

The salient qualitative results of our analysis are—

- 1. The use of ideal hydrodynamics is incorrect but may not be terribly wrong in some ways. The isotherms are smeared out to 3–6% of the system size. However, it is bad in other ways; for example, the use of the Bjorken formula to extract the initial energy density is probably too optimistic. It seems possible that the initial energy density is significantly smaller— perhaps by 15–25%. A loss of another 15–25% by surface emission means that perhaps only about half of the Bjorken estimate of the energy density is available to the fireball.
- 2. The transverse flow stalls. The reason is that this is a sound wave, driven by the pressure gradient between the core and the surface of the fireball. However, the sound attenuation length is short, since R is large. Therefore the pressure gradient is unable to drive a rarefaction wave efficiently. This increases the lifetime of the plasma over that expected from ideal hydrodynamics.
- 3. This reasoning also applies to differential pressure gradients which cause elliptic flow, v_2 . Thus, the true elliptic flow must be at least 20% smaller than the ideal hydro result [22]. In that case, the increase of v_2 with s [23] must be re-examined to check whether it can be used to extract η/s .

- 4. Bjorken flow is fairly stable under the amount of dissipation that seems to exist in the QCD plasma. The amount of entropy generated by purely logitudinal flow is small. However, pressure isotropy is violated by 20–30% during Bjorken flow. Note however, that this will not drive acoustic phenomena.
- 5. Any pre-equilibrium signal that one gets at the detector comes from the outer region of the fireball, and therefore from matter at fairly low densities. These signals should be computable in a hadronic model.

We believe that all this is good news. If ideal hydrodynamics were a quantitatively good theory of the fireball's evolution, then the freezeout isotherm would have to be opaque in hadronic channels. One could not then look past it into the fireball. All observable hadronic quantities— single particle spectra, HBT correlations, fluctuations of conserved quantities, chemical abundances, would give direct information only on freezeout conditions, which lie in the hadronic phase. In this case, the quark-gluon plasma would be invisible; its existence would have to be deduced by fitting the equation of state to hydrodynamics— a notoriously ill-conditioned problem. So it is good news that the quark-gluon plasma is not totally invisible. One can look into the history of the plasma. However, this means that the tool to extract physics becomes a little complicated— one needs transport equations rather than hydrodynamics.

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