

QCD – NLC

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Abstract. We give a status report of our current theoretical work on QCD near the light-cone.

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1. INTRODUCTION

A simple extension of the quantum mechanics of bound states to a relativistic field theory of massless quanta bound into hadrons is not possible. The light-cone Hamiltonian approach attacks this problem from a quite different point of view. Take a cube of length 2 fm filled with quarks and gluons and boost it in the 3– direction with a Lorentz factor of $\gamma = 1000$. This gedanken experiment is well suited to imagine a proton moving with fast speed in the laboratory. The box will contract on one side, valence quark momenta will be high, and valence states will have very high energies. Naively vacuum properties of QCD are not important because of the high energies. By some suitable kinematic choices of coordinates one can construct invariants. Commonly, the light-cone energy $P^- = \frac{E-P_z}{\sqrt{2}}$ and the light-cone momentum $P^+ = \frac{E+P_z}{\sqrt{2}}$ are chosen and $M^2 = 2P_+P_- - P_\perp^2$ is invariant.

With these variables all light-cone energies are positive and increase as $P^- = \frac{P_\perp^2 + m^2}{\sqrt{2}P^+}$ for small light-cone momenta. Only fluctuations with small P^+ momenta may pose a problem. Their light-cone energies are very high. In light-cone physics the ultraviolet problem gets mixed up with the infrared problem. Formally, the problem reappears in the context of constraint equations for x^- independent fields [1]. These constraint equations arise in the light-cone Hamiltonian framework, since the Lagrangian contains the velocities in linear form $L = \partial_- \phi \partial_+ \phi$. The momenta related to these velocities obey constraint equations including $\partial_- \phi$. Therefore, integrals of the equations of motion over the spatial light-cone distance x^- become operator equations of reduced dimensionality (two transverse spatial dimensions and one time dimension). These equations are called zero-mode equations. For example, in equal time theory zero-mode equations determine the condensate of a scalar field. The x^- independent zero-mode field couples to the transverse fluctuations of all other fields, consequently these equations depend on the cutoff and are

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involved in the whole renormalization procedure. This feature is often overlooked in naïve pictures assuming either superrenormalizable models or models with a simple cutoff. In Nambu-Jona-Lasinio models one has been able to solve [2, 3] these zero-mode equations e.g. in large N_c approximation giving a view of chiral symmetry breaking on the light-cone, which is quite special. These zero-mode equations have not been solved in QCD.

A common argument goes as follows: Zero modes decouple from the rest of the theory, because their energies lie beyond the cutoff. Naïvely, the light-cone momentum $P^+ = 0$ means that the light-cone energy $P^- = \infty$. If, however, the mass m of the zero mode is zero, the mode does not disappear into infinity for very small transverse momenta. How is the situation in QCD? Can we just ignore this problem, buy a big computer, use some suitable Fock truncation, put all transverse gluon modes into a Hamiltonian matrix and diagonalize it? Pauli and Brodsky [4] and many others have solved successfully 1+1 dimensional theories. QCD on the light-cone is a tremendously seductive field theory, since the Euler-Lagrange equation for time-like light-cone potential can be solved directly in a gauge, where the potential along the spatial light-cone direction vanishes. The resulting Hamiltonian contains the light-cone Coulomb energy plus the kinetic energies of the transverse gluons and nothing else. The light-cone Coulomb energy is already in a form which linearly confines sources separated along the spatial light-cone directions. This is a simple consequence of the massless gluon propagator in one spatial dimension.

The massless gluon interaction has to be implemented also correctly for colored line sources smeared over the spatial light-cone direction. Otherwise, we violate the equal treatment of all spatial directions. This necessity can be demonstrated rather easily in perturbation theory, where the rotational invariance of the gluon exchange is reconstituted via the exchange of one transverse gluon. I think, one can be easily misled by the experience that QCD will always favor a finite correlation mass for color sources moving along time-like directions. At finite energies one sees this phenomenon in the hadronic cross sections which are given by the geometrical sizes of the hadrons, the low light-cone momentum partons do not matter at finite (small) energies. There is a natural transverse scale of the moving proton. The energy dependence of the high Q^2 -structure functions indicate, however, an abnormal increase of “size” in transverse direction. The proton first gets blacker, but then its transverse radius has to increase. Purely theoretical arguments point towards conformal invariance at high energies, a conjecture, which supports the view, that partons with small light-cone momenta sampling large spatial light-cone distances correlate over large transverse distances compared to normal hadronic scales.

We have analyzed QCD approaching the light-cone with a tilted near-light-cone coordinate reference system [5, 6, 7] containing a parameter $\eta \neq 0$ giving the distance away from the light-cone. The constraint equations appear in the near-light-cone Hamiltonian as terms proportional to $1/\eta^2$. We then multiply the light-cone energy with η , considering $\tilde{P}_+ = \eta T_+^+$ and divide the light-cone momentum by η , defining $\tilde{P}_- = \frac{1}{\eta} T_-^+$. The invariant masses remain unchanged up to terms higher order in η . By the trick with near-light-like coordinates we can derive a full quantum Hamiltonian for the zero modes which now depends on the QCD coupling

g , the extension L_{\parallel} of the spatial light-cone distance compared to some lattice size a (or ultraviolet cutoff $\Lambda = 1/a$) and the parameter η which gives the nearness to the light-cone. Having fixed the QCD coupling g which determines the lattice size a , we would like to study in this Hamiltonian the physics at large longitudinal distances $L_{\parallel}/a \rightarrow \infty$ close to the light-cone $\eta \rightarrow 0$. Because of dimensional reduction the product

$$s = \frac{\eta L_{\parallel}}{a} \tag{1}$$

appears as a coupling in the Hamiltonian. Its limit is not defined. The order of the limiting process is important as one knows from simple superrenormalizable models. One first has to let $L_{\parallel}/a \rightarrow \infty$ and then $\eta \rightarrow 0$ in order not to lose the nonperturbative properties of the vacuum. For QCD an analytical limiting process is impossible. Therefore, the only way out is to start for large s , corresponding to fixed η and large L_{\parallel} and then approach smaller values of s .

This procedure ends, when we have found a fixed point $s^* = \frac{\eta L_{\parallel}}{a}$ where the mass gap of the zero mode theory vanishes. Approaching this fixed point from the correct side which corresponds to a large longitudinal extension of the lattice, we include the nonperturbative dynamics of the zero modes. The trivial, wrong other side where s is arbitrarily small would be disconnected from the large L_{\parallel} limit. When the (2+1)-dimensional system has an infinite correlation length, both the infrared limit of large longitudinal distances and of nearness to the light-cone is realized. For a simplified zero mode theory in SU(2) we have demonstrated such a possibility on the lattice [5]. In principle the full (3+1)-dimensional theory can be solved for any η as long $g, L_{\parallel}/a$ are chosen in such a way that we have asymptotic scaling. But in order to synchronize the infrared behavior encoded in the zero mode system correctly with the ultraviolet behavior of small lattice size, the choice of η is no longer free for a given length of the longitudinal direction, one must choose η in agreement with the fixed point found in the zero mode calculation, i.e. in the (3+1)-dimensional calculation the number of slices L_{\parallel}/a in spatial light-cone direction determines η

$$\eta = \frac{s^*}{L_{\parallel}/a}. \tag{2}$$

It has to be demonstrated numerically that with decreasing QCD-coupling g the value s^* becomes smaller in such a way that we approach the light-cone $\eta \rightarrow 0$ having a reasonable number of slices L_{\parallel}/a in spatial light-cone direction. The reduced calculation in SU(2) [5] was done without the inclusion of transverse gluons, so we still have to prove that this procedure works. Phenomenologically [7] we have conjectured that the increase of the high-energy electron-proton cross section is due to this critical point s^* . At infinite energies when this point is approached, the correlation length of near-light-like Wilson lines of the partons increases with a critical index from Z(3) symmetry. The photon density remains power behaved beyond the short distance scale given by the resolution of the photon. According to our conjecture this critical opalescence phenomenon is the cause of the increase of the virtual photon cross section with high energies.

2. THE QCD HAMILTONIAN AND MOMENTUM

We use the near light-cone coordinates defined in ref. [6]

$$\begin{aligned} x^+ &= \frac{1}{\sqrt{2}}\left(\left(1 + \frac{\eta^2}{2}\right)x^0 + \left(1 - \frac{\eta^2}{2}\right)x^3\right) \\ x^- &= \frac{1}{\sqrt{2}}(x^0 - x^3) \end{aligned} \quad (3)$$

with the metric

$$g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -\eta^2 \end{pmatrix}, g^{\mu\nu} = \begin{pmatrix} \eta^2 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad (4)$$

with $\mu, \nu = +, 1, 2, -$, $\det g = -1$, which defines the scalar product

$$\begin{aligned} x_\mu y^\mu &= x^- y^+ + x^+ y^- - \eta^2 x^- y^- - \vec{x}_\perp \vec{y}_\perp \\ &= x_- y_+ + x_+ y_- + \eta^2 x_+ y_+ - \vec{x}_\perp \vec{y}_\perp. \end{aligned} \quad (5)$$

We refer to p_+ as ‘‘light-cone’’ energy and to p_- as ‘‘light-cone’’ momentum. We restrict ourselves to the color gauge group $SU(2)$ and to gluons. The Lagrangian density in the near light-cone coordinates reads:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} F_{+-}^a F_{+-}^a + \sum_{i=1,2} (F_{+i}^a F_{-i}^a + \frac{\eta^2}{2} F_{+i}^a F_{+i}^a) \\ &\quad - \frac{1}{2} F_{12}^a F_{12}^a. \end{aligned} \quad (6)$$

The energy momentum tensor has the general form:

$$T^{\mu\nu} = \sum_a (g^{\mu\alpha} g^{\rho\beta} F_{\alpha\beta}^a F_{\rho\nu}^a - \delta_n^\mu u g^{\rho\alpha} g^{\sigma\beta} F_{\alpha\beta}^a F_{\sigma\rho}^a) \quad (7)$$

We introduce dimensionless gauge fields and coordinates

$$\begin{aligned} \tilde{A}_\pm^a &= g a_\parallel A_\pm^a, & \tilde{x}_\pm &= x_\pm / a_\parallel; \\ \tilde{A}_i^a &= g a_\perp A_i^a, & \tilde{x}_\perp &= x_\perp / a_\perp. \end{aligned} \quad (8)$$

The dimensionless Lagrange density \mathcal{L} has the form

$$\int \mathcal{L} d^4 x = \int \tilde{\mathcal{L}} d^4 \tilde{x} \quad (9)$$

with

$$\begin{aligned}
\tilde{\mathcal{L}} &= \frac{1}{2} \frac{1}{g^2} \left(\frac{a_\perp}{a_\parallel} \right)^2 \tilde{F}_{+-}^a \tilde{F}_{+-}^a \\
&\quad + \frac{1}{g^2} \sum_i (\tilde{F}_{+i} \tilde{F}_{-i} + \frac{\eta^2}{2} \tilde{F}_{+i} \tilde{F}_{+i}) \\
&\quad - \frac{1}{2} \frac{1}{g^2} \left(\frac{a_\parallel}{a_\perp} \right)^2 \tilde{F}_{12}^a \tilde{F}_{12}^a.
\end{aligned} \tag{10}$$

The field momenta conjugate to \tilde{A}_i and \tilde{A}_- are

$$\begin{aligned}
\tilde{\Pi}_i^a &= \frac{\partial \tilde{\mathcal{L}}}{\partial \tilde{F}_{+i}} = \frac{1}{g^2} (\tilde{F}_{-i}^a + \eta^2 \tilde{F}_{+i}^a) \\
\tilde{\Pi}_-^a &= \frac{\partial \tilde{\mathcal{L}}}{\partial \tilde{F}_{+-}} = \frac{1}{g^2} \left(\frac{a_\perp}{a_\parallel} \right)^2 \tilde{F}_{+-}^a.
\end{aligned} \tag{11}$$

From now on we drop the tilde symbol from all coordinates, fields and momenta to facilitate the writing and reading of all formulas. The commutation relations between fields and momenta are standard.

$$\begin{aligned}
[\Pi_i^a(\tilde{x}_-, x_\perp, x_+), A_j^b(y_-, y_\perp, x_+)] &= \\
i\delta^{ab} \delta_{ij} \delta(x_- - y_-) \delta(x_\perp - y_\perp) & \\
[\Pi_-^a(x_-, x_\perp, x_+), A_-^b(y_-, y_\perp, y_+)] &= \\
i\delta^{ab} \delta(x_- - y_-) \delta(x_\perp - y_\perp). &
\end{aligned} \tag{12}$$

The dimensionless light-cone energy density T_+^+ and light-cone momentum density T_-^+ are obtained from the energy momentum tensor $T^{\mu\nu}$ and the skewed metric

$$P_+ = \int T_+^+ dx^- dx_\perp \tag{13}$$

$$P_- = \int T_-^+ dx^- dx_\perp \tag{14}$$

with

$$\begin{aligned}
T_+^+ &= \frac{1}{2} g^2 \left(\frac{a_\parallel}{a_\perp} \right)^2 \Pi_-^2 \\
&\quad + \frac{1}{2} \frac{1}{g^2} \left(\frac{a_\parallel}{a_\perp} \right)^2 F_{12} F_{12} \\
&\quad + \frac{1}{2} g^2 \frac{1}{\eta^2} \left(\Pi_i - \frac{1}{g^2} F_{-i} \right)^2 \\
T_-^+ &= \frac{1}{2} (\Pi_i F_{-i} + F_{-i} \Pi_i).
\end{aligned} \tag{15}$$

The light cone energy and momentum have an obvious symmetry. They have electric-magnetic duality of the transverse fields which any solution to the problem must respect.

$$\begin{aligned}\Pi_i &\rightarrow \frac{1}{g^2}F_{-i} \\ \frac{1}{g^2}F_{-i} &\rightarrow \Pi_i.\end{aligned}\tag{16}$$

Furthermore, since in the Lagrangian \mathcal{L} there are no terms with time derivatives of A_+ , the field A_+ acts like a Lagrange multiplier for G^a , the Gauss law. For any wavefunction $|\Phi\rangle$ of the system the following identity must hold.

$$\begin{aligned}G^a|\Phi\rangle &= \left(\frac{1}{g^2}\left(\frac{a_\perp}{a_\parallel}\right)^2 D_-^a F_{+-}^a\right. \\ &\quad \left. + \frac{1}{g^2}\sum_i D_i(F_{-i}^a + \eta^2 F_{+i}^a)\right)|\Phi\rangle \\ &= (D_-^a \Pi_-^a + \sum_i D_i \Pi_i^a)|\Phi\rangle = 0.\end{aligned}\tag{17}$$

This Gauss law is fulfilled as long as only closed loops exist in the wave function, or in the case of excited links the electric flux must be conserved at each site, i.e. there are also multiple connected flux loops possible.

If one chooses $a_\parallel \ll a_\perp$ and uses the same number of sites in x_- and x_\perp directions, one ends up with a real system, which is contracted in the longitudinal directions. Verlinde and Verlinde [8], and Arefeva [9], have advocated such a set up to describe high energy scattering. A contracted lattice means the minimal momenta become high in longitudinal direction and this looks a promising starting point for high energy scattering.

One sees from the Lagrangian \mathcal{L} in eq. (10) that the limit $a_\parallel/a_\perp \rightarrow 0$ enhances the terms with $F_{+-}F_{+-}$ and suppresses transverse $F_{12}F_{12}$.

Because of the enhanced couplings Verlinde and Verlinde conclude that the curvature in longitudinal directions is zero. One ends up with only one term which in the Hamiltonian is the term $\propto \frac{1}{\eta^2}$ and fixes the dual symmetry of the electric and magnetic fields.

Our current project [10] is to introduce lattice variables into this framework and solve the $1/\eta^2$ part of the Hamiltonian exactly.

3. A VALENCE-QUARK LIGHT-CONE HAMILTONIAN

In this section, I would like to present a derivation where the near-light-cone method and the field strength correlators work nicely together. This example demonstrates their practicality as a calculational and heuristic tool. Firstly, one

can analytically do the calculation in the stochastic vacuum model and secondly, the result is so close to reality that one can see the model-independent result. In our application of the stochastic vacuum model to high-energy scattering we always use Wilson loops which are on the light-cone. The expectation values of a Wilson loop along the light-cone is unity, because the area of a light-like Wilson loop is zero. I was always disturbed by this fact, because I thought that a color dipole moving with the speed of light should feel confining forces. The wavefunction renormalization due to single loops cancels out in the S -matrix, but the puzzle remained to me. So recently, Nurpeissov and myself [11] have looked into this problem again using a tilted Wilson loop corresponding to a fast moving dipole in Euclidean and in Minkowski space, i.e. we applied the near light-cone trick.

In Euclidean space the Wegner-Wilson loop can be represented with the help of the Casimir operator in the fundamental representation $C_2(3) = t^2 = 4/3$

$$\langle W[C] \rangle_G = \exp \left[-\frac{C_2(3)}{2} \chi_{ss} \right]. \quad (18)$$

We calculate χ_{ss} as the double area integral of the correlation function over the surface of the loop. Let us consider the χ_{ss} function for large separations R_0 of the quark and antiquark, where the confinement term plays the main role. For the nonperturbative (NP) confining (c) component χ_{ss}^{NPc} we get the following expression for large distances $R_0\alpha \gg 2a$

$$\chi_{ss}^{NPc} = \lim_{T \rightarrow \infty} \left[\frac{2\pi^3 a^2 G_2 \kappa T}{3(N_c^2 - 1)} \cdot R_0\alpha \right]. \quad (19)$$

Here G_2 denotes the gluon condensate, κ the weight of the confining correlator compared to the nonconfining correlator, a gives the correlation length and T the extension of the loop in Euclidean time.

The geometry of the arrangement enters into the factor α . The angle θ gives the tilting of the loop in the X_3, X_4 plane. The angle ϕ defines the angle of the $q\bar{q}$ connection in the X_1, X_3 plane.

$$\alpha = \sqrt{1 - \cos^2 \phi \sin^2 \theta}. \quad (20)$$

One recognizes that the confining interaction leads to a VEV of the tilted Wilson loop which is consistent with the area law for large distances R_0

$$\langle W[C] \rangle = e^{-\sigma R_0\alpha T} \quad (21)$$

$$\sigma = \frac{\pi^3 G_2 a^2 \kappa}{18}, \quad (22)$$

where σ is the string tension and the area is obtained from

$$Area = TR_0 \int_{-1/2}^{1/2} du \int_0^1 dv \sqrt{\left(\frac{dX_\mu}{du}\right)^2 \left(\frac{dX_\mu}{dv}\right)^2 - \left(\frac{dX_\mu}{du} \frac{dX_\mu}{dv}\right)^2} \quad (23)$$

$$= TR_0\alpha. \quad (24)$$

For the Wegner-Wilson loop in Minkowski space-time we define χ_{ss} in the following way

$$\langle W[C] \rangle_G = \exp \left[-i \frac{C_2(3)}{2} \chi_{ss} \right]. \quad (25)$$

Minkowskian geometry enters via the factor

$$\alpha_M = \sqrt{1 + \cos^2 \phi \sinh^2 \psi}, \quad (26)$$

which is consistent with the analytical continuation of the Euclidean expression $\alpha = 1 - \cos^2 \phi \sin^2 \theta$ into Minkowski space by transforming the angle $\theta \rightarrow i\psi$. This analytical continuation is similar to the analytical continuation used in high-energy scattering [12, 13, 14], where the angle between two Wilson loops transforms in the same way.

The confining contribution to χ_{ss} reads in Minkowski space:

$$\chi_{ss}^{Np_c} = \lim_{T \rightarrow \infty} \left[\frac{2\pi^3 a^2 G_2 \kappa T}{3(N_c^2 - 1)} \cdot R_0 \alpha_M \right]. \quad (27)$$

In order to interpret this result, one must define the four velocities of the particles described by the tilted loop

$$u_\mu = (\gamma, 0_\perp, \gamma\beta). \quad (28)$$

The exponent giving the expectation value of the Wilson loop acquires a new meaning now, since $-ig \int d\tau A^\mu u_\mu = -ig \int d\tau (\gamma A^0 - \gamma\beta A^3)$, which leads in the VEV to a value for $\beta \approx 1$

$$\langle W_r[C] \rangle = e^{-i\gamma(P^0 - P^3)T}. \quad (29)$$

The light-cone energy arising from the confining part of the correlation function has the form

$$P^- = \frac{1}{\sqrt{2}} \left(\sigma R_0 \sqrt{\cos^2(\phi) + \sin^2(\phi)/\gamma^2} \right). \quad (30)$$

One sees that the Wilson loop for boosts with large γ indicates that the light-cone energy does not depend on the transverse distance $R_0 \sin\phi$ between the quarks. We introduce the relative + momentum k^+ and transverse momentum k_\perp for the quarks with mass μ . By adding the above ‘‘potential’’ term to the kinetic term of relative motion of the two particles we complete the Hamiltonian P^-

$$P^- = \frac{(\mu^2 + k_\perp^2)P}{2(1/4P^2 - k^+)} + \frac{1}{\sqrt{2}} \sigma \sqrt{x_3^2 + x_\perp^2/\gamma^2}. \quad (31)$$

Next, we multiply P^- with the plus component of the momentum P^+ and use that $P^+/M = \sqrt{2}\gamma M$ to eliminate the boost variable from the Hamiltonian. Further, we introduce the fraction $\xi = k^+/P^+$ with $|\xi| < 1/2$ and its conjugate the scaled longitudinal space coordinate $\sqrt{2}\rho = P^+ x_3$ as dynamical variables. For our configuration the relative time of the quark and antiquark is zero. Then we get

the light-cone Hamiltonian in a Lorentz invariant manner, because the variables ξ, ρ, k_\perp and x_\perp are invariant under boosts

$$M^2 = 2P^+P^- = \frac{(\mu^2 + k_\perp^2)}{1/4 - \xi^2} + 2\sigma\sqrt{\rho^2 + M^2x_\perp^2}. \quad (32)$$

To solve the M^2 operator one has to replace the square root operator by introducing an auxiliary parameter s of dimension mass squared and minimize M^2 with respect to variations of s . Final self consistency must be reached with a guessed mass eigenvalue M_0

$$M^2 = \frac{(\mu^2 + k_\perp^2)}{1/4 - \xi^2} + \frac{1}{2} \left(4\sigma^2 \frac{\rho^2 + M_0^2 x_\perp^2}{s} + s \right). \quad (33)$$

In addition, one has to put the self-energy correction calculated by Simonov [15] which is $\Delta\mu^2 = -4\sigma * f(m_q)/\pi$ and get

$$M^2 = \frac{(\mu^2 - \frac{4\sigma f}{\pi} + k_\perp^2)}{1/4 - \xi^2} + \frac{1}{2} \left(4\sigma^2 \frac{\rho^2 + M_0^2 x_\perp^2}{s} + s \right). \quad (34)$$

For light quarks the function $f(m_q)$ is close to unity. We have used the above equation with a simple trial function:

$$\psi(\xi, x_\perp) = N \cos(\xi\pi) e^{-\frac{x_\perp^2}{2x_0^2}}. \quad (35)$$

We obtain two solutions [16] with positive masses due to the s -minimization. One solution is very low in mass and the other rather high. By tuning $f(m_q) = 0.8615$ away from unity the lower solution is pion-like with a really low mass, whereas the other solution lies at a typical hadronic scale

$$M_{low} = 0.138 GeV \quad (36)$$

$$M_{high} = 1.1 GeV. \quad (37)$$

Since on the light-cone the mechanism of chiral symmetry breaking is of particular interest, we would like to understand this result better. In the approach given here confinement plays an important role in contrast to Nambu-Jona-Lasinio effective models, which give an adequate description of spontaneous chiral symmetry breaking but do not include confinement.

The confining interaction in the light Hamiltonian was derived in the specific model of the stochastic vacuum. But it also can be inferred from the simple Lorentz transformation properties of the phase in the Wilson loop and a lattice determination of the tilted Wilson expectation values. In this respect the final Hamiltonian is model independent.

The inclusion of confining forces in the initial and final state wave functions can put all scattering cross sections calculated with the stochastic vacuum model on a much safer base, when wave functions and cross sections are derived consistently. For low Q^2 photon wave functions the long-distance part of the wave function

matters strongly and confinement is important cf. [17]. Especially the diffractive cross section has a large contribution from large dipole sizes and a correct behavior can only be expected when the problem of the large dipole wave function is treated adequately. Another extension of the above calculation is the coupling of the initial $q\bar{q}$ state to higher Fock states $q\bar{q}g$ with gluons which can be calculated with Wilson loops near the light-cone in Minkowski space.

4. DISCUSSION AND CONCLUSIONS

I have tried to give some impression how QCD appears near the light-cone. I think we have now a calculational framework to approach the light-cone in a systematic way. It does not look much easier than equal time lattice gauge theory. One may hope that some simplifications arise in the process of studying it. The work on a Wilson loop near the light-cone looked very complicated and intransparent at the beginning, but it reduced to some simple form. I like this example because it shows how the vacuum acts near the light-cone.

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