The relations between Bjorken polarized, Bjorken unpolarized, and Gross–Llewellyn Smith sum rules

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New relations between Bjorken polarized, Bjorken unpolarized and Gross-Llewellyn Smith sum rules are described. These relations are valid in the region, where both perturbative series and the series in power-suppressed $1/Q^2$-terms do not yet manifest the feauture of asymtotic expansions. The experimentally based consideartions support these relations, which may surve as the guide for possible in future measurements of the Bjorken unpolarized sum rule at Neutrino Factories.

The dominating theoretical research in the current studies of Neutrino Factories programs are related to the analysis of the possibility to detect parameters of neutrino oscillations. Since high-statistics measurments of cross-sections of νN-scattering with more intensive low-energu ν-beams allow one to get more reliable estimates of background effects to future oscillations experiments, theoretical analysis of these processes started to attrackt more interest. Moreover, these studies may allow one to get new information about the behaviour of cross-sections and extracted formfactors and structure functions (SFs) in elastic, quasi-elastic and deep-inelastic scattering (DIS) regimes. In view of this even at low energies the option for front-end non-oscillation physics should be investigated in more detail. In particular, the analysis of the data in DIS region may provide high-presision information about polarized parton distributions [1],[2] in the x, Q^2-regions, complementary to the ones, available at JLAB. Thus, using these parton distributions and extrapolating the poissible Neutrino Factories data for $g_1(x, Q^2)$ SF of polarized DIS one may extract the value for the isospin polarized Bjorken sum rule

$$\mathrm{Bp} = \int_0^1 \left[\mathrm{g_1^p(x,Q^2) - g_1^n(x,Q^2)}\right]\mathrm{dx} \tag{1}$$

for the low momentum transfered of about $Q^2 \leq$ 3.5 GeV^2. In general QCD expression for the Bjp sum rule can be expressed as $\mathrm{Bp} = \frac{g_A}{6}\mathrm{C_{Bp}}$) with g_A=1.26 being the neutron beta-decay coupling constant and

$$\mathrm{C_{Bp} = 1 - 4a_s - O(a_s^2) - \frac{A}{Q^2} - O\left(\frac{1}{Q^4}\right)} \tag{2}$$

where a_s=$\alpha_s(Q^2)/(4\pi)$, α_s is the QCD coupling constant and A is related to the non-perturbative $1/Q^2$-correction, calculated numerically in different models (for the details see Ref. [3]). Note, that $1/Q^2$-corrections to g_1^N were also extracted by the model-independent way from the current data for polarized DIS [4]). On another hand the kinenatical conditions of Neutrino Factories may allow one to extract all SFs, which enter into the cross-section of the unpolarized νN DIS process (i.e. $\mathrm{F_1}$, $\mathrm{F_2}$, $\mathrm{xF_3}$ SFs). In view of this it may be possible to use data of Neutrino Factories for the **first** extraction of unpolarized Bjorken sum rule, defined as

$$\mathrm{Bup} = \int_0^1 \left[\mathrm{F_1^{\nu p}(x,Q^2) - F_1^{\nu n}(x,Q^2)}\right]\mathrm{dx} \tag{3}$$

The QCD expression of $\mathrm{Bup = C_{Bup}}$, namely

$$\mathrm{C_{Bup} = 1 - \frac{8}{3}a_s - O(a_s^2) - \frac{B}{Q^2} - O\left(\frac{1}{Q^4}\right)} \tag{4}$$

is "measuring" the **violation** of the Callan-Gross relation from its parton model prediction

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$F_2/2xF_1 = 1$. The possibility to test experimentally this property of QCD through the extraction of Eq.(4) at Neutrino Factories was proposed in Ref. [5] (see also [2]).

The third sum rule we are interested in is the Gross-Llewellyn Smith sum rule

$$\mathrm{GLS} = \frac{1}{2}\int_O^1 \left[F_3^{\nu p}(x, Q^2) + F_3^{\nu n}(x, Q^2)\right] dx \quad . \quad (5)$$

It is proportional to the number of valence quarks, contained in the nucleon, namely $\mathrm{GLS} = 3C_{\mathrm{GLS}}(Q^2)$, where

$$C_{\mathrm{GLS}} = 1 - 4s_s - O(a_s^2) - \frac{C}{Q^2} - O\left(\frac{1}{Q^4}\right) \quad . \quad (6)$$

The key observation, which was made in Ref. [6], is that due to the fact that the first infrared renormalon poles are entering into the Borel integrals for these sum rules with the same residues [7] [2] not only properly normalized perturbative contributions to all three sum rules have similar value [7], but the non-perturbative $1/Q^2$ corrections as well. Thus, in the energy region where asymptotic nature of the $1/Q^2$-expansion did not yet start to manifest itself (namely in the region $Q^2 \geq 2$ GeV^2), the following **new relation** between DIS sum rules

$$\mathrm{Bp}(Q^2) \approx \frac{g_A}{6}\mathrm{Bup}(Q^2) \approx \frac{g_A}{18}\mathrm{GLS}(Q^2) \qquad (7)$$

is valid [6]. It was checked that the existing experimental data for the GLS sum rule [8] and the existing data for the Bp sum rule, extracted both by experimentalists [9] and theoreticians [10], [11] are respecting the new relation of Eq.(7) (see Ref. [6]). Indeed, using this relation one gets from the experimentally-based value $\mathrm{GLS}(Q^2 = 3.16\ \mathrm{GeV}^2) \approx 2.55$ is transformed into the value $\mathrm{Bp}(Q^2 = 3.15\ \mathrm{GeV}^2) \approx 0.178$. Within existing error bars this result agrees with the value $\mathrm{Bp}(Q^2 = 3\ \mathrm{GeV}^2) = 0.164 \pm 0.023$, obtained by experimentalists [9], and with theoretically improved extractions and $\mathrm{Bp}(Q^2 = 3\ \mathrm{GeV}^2) = 0.164 \pm 0.011$ [10], $\mathrm{Bp}(Q^2 = 3\ \mathrm{GeV}^2) = 0.177 \pm 0.018$ [11]. Other examples of the validity of the relations of Ref. [6] were presented in more detail work of Ref. [3]. It is also interesting to note, that the analysis of of existing experimental data, performed in Ref. [12], supports the relations between $1/Q^2$ corrections to xF_3 and $g_1^p - g_1^n$, which follow from the relations of Ref. [6], in the region $x > 0.2$, where the contributions of high-twist terms should be essentially important.

To conclude, we hope that the relation of Eq.(7) will allow to test self-consistency of extracting discussed sum rules values from the data of possible front-end DIS experiments at future Neutrino Factories.

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REFERENCES

1. S. Forte, M. L. Mangano and G. Ridolfi, Nucl. Phys. B **602** (2001) 585.
2. M. L. Mangano *et al.*, hep-ph/0105155.
3. A. L. Kataev, Mod. Phys. Lett. **A** (brief review, to appear), hep-ph/0505230.
4. E. Leader, A. V. Sidorov and D. B. Stamenov, Phys. Rev. D **67** (2003) 074017.
5. S. I. Alekhin and A. L. Kataev, J. Phys. G **29** (2003) 1993.
6. A. L. Kataev, JETP Lett. **83** (2005) 608, hep-ph/0505108.
7. D. J. Broadhurst and A. L. Kataev, Phys. Lett. B **544** (2002) 154.
8. J. H. Kim *et al.*, Phys. Rev. Lett. **81** (1998) 3595.
9. K. Abe *et al.* [E143 collaboration], Phys. Rev. D **58** (1998) 112003.
10. J. R. Ellis, E. Gardi, M. Karliner and M. A. Samuel, Phys. Lett. B **366** (1996) 268.
11. G. Altarelli, R. D. Ball, S. Forte and G. Ridolfi, Nucl. Phys. B **496** (1997) 337.
12. A. V. Sidorov, hep-ph/0505246.

[2]For the most recent review of the application of renormalon technique to DIS sum rules see Ref. [3].