

Right-handed Neutrino Fields are Real Spinors

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The ansatz that the right-handed neutrino fields are real spinors is proposed in this letter. We naturally explain why the right-handed neutrinos don't feel the electroweak interactions and why there is neutrino mixing. It is found that the Majorana representation of Dirac equation is uniquely permitted in our scenario. With this ansatz, we predict that: 1. there are at least four species of neutrinos; 2. the mass matrix of neutrinos must be traceless; 3. there is CP violation in the lepton sector. The difference between our scenario and the Zee model is discussed also.

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Till now there are enormous experiments on weak interactions. About half a century ago, Lee and Yang [1] proposed the violation of parity conservation in weak interactions. A short time after that, the experiments [2] revealed that only a single helicity appears in weak interactions: electrons and neutrinos are always created left-handed, positrons and antineutrinos are always created right-handed. Recently there are abundant data [3, 4, 5, 6] from atmospheric, solar, underground laboratory, and long baseline neutrino experiments on the neutrino mass and mixing. All of these experimental data, except for the results observed by LSND collaboration [6], can be explained by oscillations between three active neutrinos [7]. A light *sterile* neutrino has been assumed to interpret the LSND data. Neutrino oscillations provide direct evidence of nonzero neutrino masses and mixing between different mass eigen states of neutrinos. A massive neutrino must exist in both left-handed and right-handed states [8]. Furthermore, left-handed neutrinos traversing a strong gravitational field could be converted into right-handed ones [9]. Now that right-handed neutrinos must exist in universe, why don't they feel both the weak interactions and the electromagnetic interactions?

Now it is believed that massive neutrinos are either Dirac or Majorana [10] particles. But neither Dirac nor Majorana fields can embody the essential difference between the left-handed and right-handed neutrinos. In quantum field theory, the local gauge invariance, the spin and the charge possessed by a quantum field is deeply related with the algebra describing this field. For instance, the π^0 , being a neutral spin-0 boson, is characterized by a real scalar, whereas π^+ and π^- , being charged spin-0 bosons, have to be represented by complex scalar. In the stand electroweak model [11], we have to represent the charged intermediate vector bosons as follows

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(B_\mu^1 \mp iB_\mu^2), \quad (1)$$

where B_μ^1 and B_μ^2 are real SU(2) gauge fields introduced by Yang and Mills [12], whereas the Z^0 is described by a real vector. Hence a simple and natural ansatz to get

out of above mentioned dilemma is that the right-handed neutrino fields are real spinors.

The left-handed and right-handed fields for a spinor ψ are defined by

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi, \quad \psi_R = \frac{1}{2}(1 + \gamma_5)\psi. \quad (2)$$

In this case, our ansatz can be represented by

$$\psi_L = \frac{1}{2}(\psi - \psi^*), \quad \psi_R = \frac{1}{2}(\psi + \psi^*). \quad (3)$$

Obviously, comparing Eq.(3) with Eq.(2) directly yields

$$\gamma_5\psi = \psi^*. \quad (4)$$

When ψ is the mass eigen state of a fermion, then it is easy to prove that Eq.(4) can be satisfied if and only if that: 1. the Majorana representation of Dirac equation is adopted; 2. this fermion is massless and uncharged.

In the case of four generations of neutrinos, each of four neutrino mass eigen states shall be represented by ν_i ; $i = 1, 2, 3, 4$. Because of neutrino mixing, we have the linear superposition

$$\nu_\alpha = \sum_{i=1,2,3,4} U_{\alpha i} \nu_i, \quad (5)$$

where $U_{\alpha i}$ is a unitary matrix, $U^\dagger U = 1$, and $\alpha = e, \mu, \tau, s$ represents the weak flavor eigen states (corresponding to electron, muon, tauon and *sterile* neutrinos respectively), namely

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}, \quad \tilde{\nu} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix}. \quad (6)$$

Then Eq.(5) can be compactly rewritten as $\tilde{\nu} = U\nu$.

When there is neutrino mixing, our ansatz is thus expressed in mathematical language as

$$\begin{aligned} \nu_{\alpha R} &= \frac{1}{2}(1 + \gamma_5)\nu_\alpha = \frac{1}{2}(\nu_\alpha + \nu_\alpha^*), \\ \nu_{\alpha L} &= \frac{1}{2}(1 - \gamma_5)\nu_\alpha = \frac{1}{2}(\nu_\alpha - \nu_\alpha^*), \end{aligned} \quad (7)$$

or equivalently

$$\gamma_5 \nu_\alpha = \nu_\alpha^* . \quad (8)$$

Treating the ansatz as the first principle in this letter, we investigate the possible Lagrangian which make sure that the constraint of Eq.(8) is satisfied.

Consider the kinetic energy term of neutrinos in electroweak Lagrangian as follows($\hbar = c = 1$)

$$\begin{aligned} \mathcal{L}_k(\nu) &= \frac{i}{2} (\bar{\nu} \gamma^\mu \partial_\mu \nu + \bar{\nu}^* \gamma^\mu \partial_\mu \nu^*) \\ &= \frac{i}{2} \sum_{\alpha=e,\mu,\tau,s} (\bar{\nu}_\alpha \gamma^\mu \partial_\mu \nu_\alpha + \bar{\nu}_\alpha^* \gamma^\mu \partial_\mu \nu_\alpha^*) . \end{aligned} \quad (9)$$

It is obvious that the kinetic energy term keeps invariant under the transformation of Eq.(8). In the condition of Eq.(8), the above Lagrangian is represented in terms of left-handed and right-handed neutrino fields as follows

$$\mathcal{L}_k(\nu) = i \bar{\tilde{\nu}}_L \gamma^\mu \partial_\mu \tilde{\nu}_L + i \bar{\tilde{\nu}}_R^T \gamma_0 \gamma^\mu \partial_\mu \tilde{\nu}_R , \quad (10)$$

where $\tilde{\nu}_L \equiv \tilde{\nu}_L^\dagger \gamma_0$, and $\tilde{\nu}_R^T$ denotes $(\nu_{eR}^T, \nu_{\mu R}^T, \nu_{\tau R}^T, \nu_{sR}^T)$. Obviously the second term in the right hand side of above Lagrangian indicates that the right-handed neutrino fields can not keep both $U_Y(1)$ and $SU_R(2)$ local gauge invariance by introducing the $SU_R(2) \times U_Y(1)$ gauge fields. Hence our ansatz nicely explains why the right-handed neutrinos don't feel the electroweak interactions.

To investigate the antineutrinos, we introduce the charge conjugate fields

$$\tilde{\nu}_c = C \tilde{\nu}^* , \quad (11)$$

where C is the charge conjugation matrix, satisfying

$$C^2 = 1 , \quad C^\dagger = C , \quad C \gamma_\mu^* C = -\gamma_\mu . \quad (12)$$

After above definitions on the charge conjugation, our ansatz on neutrinos, namely Eq.(8), implies that

$$\gamma_5 \tilde{\nu}_c = -\tilde{\nu}_c^* , \quad (13)$$

because $C \gamma_5^* C = -\gamma_5$. This relation demonstrates that the left-handed antineutrinos defined by

$$\tilde{\nu}_{cL} = \frac{1}{2} (1 - \gamma_5) \tilde{\nu}_c = \frac{1}{2} (\tilde{\nu}_c + \tilde{\nu}_c^*) \quad (14)$$

don't feel the electroweak interactions either.

We have demonstrated that our ansatz doesn't break the $SU_L(2) \times U_Y(1)$ symmetry in the sector of left-handed neutrinos. The condition Eq.(8) keeps massless electroweak Lagrangian invariant, but our ansatz will give stringent constraints on neutrino mass matrix and its mixing matrix.

Assume that the ν_1, ν_2, ν_3 , and ν_4 correspond to four mass eigen states of masses m_1, m_2, m_3 , and m_4 respectively, then the addition of a mass term to total Lagrangian which is possibly invariant under the transformation of Eq.(8) should be

$$\begin{aligned} \mathcal{L}_m &= -\frac{1}{2} (\nu^\dagger \gamma_0 M \nu + \nu^{*\dagger} \gamma_0 M \nu^*) \\ &= -\frac{1}{2} (\tilde{\nu}^\dagger \gamma_0 U M U^\dagger \tilde{\nu} + \tilde{\nu}^{*\dagger} \gamma_0 U^* M U^{*\dagger} \tilde{\nu}^*) \end{aligned} \quad (15)$$

because $U^{-1} = U^\dagger$. Where M is the diagonal mass matrix, satisfying

$$M = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix} . \quad (16)$$

Making use of Eq.(8), the relation $\gamma_0 \gamma_5 = -\gamma_5 \gamma_0$, and the identity $(\gamma_5)^2 = 1$, we acquire

$$\begin{aligned} \mathcal{L}_m &= \frac{1}{2} (\tilde{\nu}^{*\dagger} \gamma_0 U M U^\dagger \tilde{\nu}^* + \tilde{\nu}^\dagger \gamma_0 U^* M U^{*\dagger} \tilde{\nu}) \\ &= \frac{1}{2} (\nu^{*\dagger} \gamma_0 U^{*\dagger} U M U^\dagger U^* \nu^* + \\ &\quad \nu^\dagger \gamma_0 U^\dagger U^* M U^{*\dagger} U \nu) . \end{aligned} \quad (17)$$

Comparing Eq.(17) with Eq.(15), one can find out that the Lagrangian \mathcal{L}_m keeps invariant if and only if the following identity is satisfied

$$U^{*\dagger} U M U^\dagger U^* = -M . \quad (18)$$

Define a new mass matrix

$$M_\nu = U M U^\dagger , \quad (19)$$

the identity (18) obviously means that

$$M_\nu = -M_\nu^* . \quad (20)$$

Since $M_\nu = M_\nu^\dagger$, then $M_\nu^T = -M_\nu$, therefore, the new mass matrix is imaginary and antisymmetric. Furthermore, $Tr(M_\nu) = 0$ makes sure that

$$Tr(M) = 0 . \quad (21)$$

Recently the traceless neutrino mass matrix has been systematically investigated in Refs.[13] in the case of three generations of neutrinos.

We would like to deduce the identity Eq.(18) from the viewpoint of Dirac equation again. The Dirac equation for free neutrinos is taken to be

$$(i\gamma^\mu \partial_\mu - M)\nu = 0 , \quad (22)$$

in terms of $\tilde{\nu}$, the above equation becomes

$$(i\gamma^\mu \partial_\mu - M)U^\dagger \tilde{\nu} = 0 . \quad (23)$$

Multiplying γ_5 in both sides of Eq.(23), and making use of Eq.(8) yields

$$(-i\gamma^\mu\partial_\mu - UMU^\dagger)\tilde{\nu}^* = 0, \quad (24)$$

Taking the complex conjugation for both sides of Eq.(23), we obtain

$$(-i\gamma^{*\mu}\partial_\mu - M)U^{*\dagger}\tilde{\nu}^* = 0, \quad (25)$$

There is a representation called the Majorana representation in which the γ s are all imaginary, so that $\gamma^{*\mu} = -\gamma^\mu$. Under this representation, Eq.(25) is simplified to

$$(i\gamma^\mu\partial_\mu - U^*MU^{*\dagger})\tilde{\nu}^* = 0, \quad (26)$$

Comparing the above equation with Eq.(24) directly yields the identity (20). Here we have also proved that the Majorana representation of Dirac equation must be adopted. Our ansatz automatically breaks the freedom of selecting the representation.

We explicitly write the mass matrix M_ν out

$$M_\nu = i \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} & m_{es} \\ -m_{e\mu} & 0 & m_{\mu\tau} & m_{\mu s} \\ -m_{e\tau} & -m_{\mu\tau} & 0 & m_{\tau s} \\ -m_{es} & -m_{\mu s} & -m_{\tau s} & 0 \end{pmatrix}. \quad (27)$$

The definition (19) shows that

$$\det |M| = \det |M_\nu|, \quad (28)$$

namely

$$m_1 m_2 m_3 m_4 = -2m_{e\mu} m_{\mu\tau} m_{\tau s} m_{es} + m_{e\tau}^2 m_{\mu s}^2 - m_{es}^2 m_{\mu\tau}^2 - m_{e\mu}^2 m_{\tau s}^2. \quad (29)$$

The equation (28) obviously indicates that the species of neutrinos must be no less than four to make sure the real neutrino masses.

In the Majorana representation, γ_0 is an imaginary matrix, then we can set

$$\gamma_0 = i\Gamma, \quad M_\nu = iN, \quad (30)$$

where Γ and N are real matrices. Since $\gamma_0^\dagger = \gamma_0$, then

$$\Gamma^T = -\Gamma, \quad N^T = -N. \quad (31)$$

Expressing the neutrino fields in real and imaginary parts respectively

$$\nu_\alpha = \Psi_{1\alpha} + i\Psi_{2\alpha}, \quad (32)$$

here $\Psi_{1\alpha}$ and $\Psi_{2\alpha}$ being real functions, then Eq.(7) is equivalently rewritten as

$$\nu_{\alpha R} = \Psi_{1\alpha}, \quad \nu_{\alpha L} = i\Psi_{2\alpha}. \quad (33)$$

After these definitions, we rewrite the Lagrangian (15) as follows

$$\begin{aligned} \mathcal{L}_m &= -\tilde{\nu}^\dagger \gamma_0 M_\nu \tilde{\nu} = \sum_{\alpha,\beta} \nu_\alpha^\dagger \Gamma N_{\alpha\beta} \nu_\beta \\ &= \sum_{\alpha,\beta} (\Psi_{1\alpha}^T \Gamma N_{\alpha\beta} \Psi_{1\beta} + \Psi_{2\alpha}^T \Gamma N_{\alpha\beta} \Psi_{2\beta} \\ &\quad + i\Psi_{1\alpha}^T \Gamma N_{\alpha\beta} \Psi_{2\beta} - i\Psi_{2\alpha}^T \Gamma N_{\alpha\beta} \Psi_{1\beta}) \\ &= \sum_{\alpha,\beta} (\nu_{\alpha R}^T \Gamma N_{\alpha\beta} \nu_{\beta R} + \nu_{\alpha L}^\dagger \Gamma N_{\alpha\beta} \nu_{\beta L}). \end{aligned} \quad (34)$$

It is astonishing that there are no left-handed and right-handed neutrino couplings in above mass term. We thus have systematically demonstrated that the left-handed neutrinos can independently keep local gauge invariance.

In the literature, the parametrization of neutrino mixing matrix has been studied extensively [14]. In our scenario, the neutrino mixing matrix is a 4×4 unitary matrix, which will be more complicated. The equation (18) indicates that our neutrino mixing matrix can not be real, therefore there is CP violation in the lepton sector.

In the Zee model [15], adding a charged +1 Higgs field which transforms as a singlet under SU(2) to the standard electroweak model leads to neutrino Majorana masses. The Zee mass matrix M_ν , generated by radiative correction at one loop level [16], is symmetric and has null diagonal entries. Thus the neutrino mixing matrix V is determined by $M = V^T M_\nu V$. The confrontation of Zee model with experimental data [17] and possible extended Zee models [18] have been extensively investigated. There are some similarity between the Zee model and our scenario since our mass matrix (27) has also null diagonal entries. But the form of our mass matrix is decided by the algebraic property of the neutrino fields. Obviously the neutrino described by our ansatz is neither Dirac nor Majorana particles. Furthermore, it is easy to incorporate our ansatz into the stand electroweak model without introducing an extra Higgs field.

In conclusion, after proposing an algebraic symmetry (8) of neutrino fields, we have naturally explained why the right-handed neutrinos don't feel the electroweak interactions and why there exists the neutrino mixing. We have shown that the Majorana representation of Dirac equation must be adopted. Our ansatz provide stringent constraints on neutrino mass matrix and its mixing matrix. The real mass condition needs a light *sterile* neutrino outside three active neutrinos. The complex mixing matrix has predicted the leptonic CP violation.

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