## Polarization transfer measurements of proton form factors: deformation by initial collinear photons

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## Abstract

It is demonstrated that an emission of collinear photons by the polarized initial electron in elastic electron-proton polarization transfer scattering leads to an apparent shifting of real events with small momentum transfer into the data sample with large momentum transfer. Effectively this shows a fictive enhancement of the cross section at large momentum transfer. However, the enhancement is different for transverse and longitudinal polarizations of the recoil proton. The former is responsible for a deformation of results when extracting the proton electromagnetic form factors ratio from the data on electronproton polarization transfer scattering. Nevertheless, this effect does not explain the suppression of the Dirac form factor at large momentum transfer completely.

In the past few years attention to the proton elastic form factors, which have always played an essential role in understanding of the nucleon electromagnetic structure, was reinforced. The reason for that was an appearance of JLab proton polarization data [1, 2, 3] on the ratio  $G_{Ep}(Q^2)/G_{Mp}(Q^2)$  measured by the method of polarization transfer [4, 5]. In contrast to the apparently well-established experimentally by the so-called Rosenbluth technique [6] ratio  $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2) \approx 1$ , which in a wide region of  $Q^2$  (0.1 GeV<sup>2</sup> <  $Q^2$  < 33.4 GeV<sup>2</sup>) is approaching one, the new results in the range 0.49 GeV<sup>2</sup>  $\leq Q^2 \leq 5.54$  GeV<sup>2</sup> reveal rapid fall of the ratio as  $Q^2$  increases.

The inconsistency in the results obtained by the above mentioned two different methods produced a broad discussion on the reliability of the methods and the accuracy of the one photon exchange approximation (see Refs. [7] and references therein) as well. As to the latter, it is known that in scattering of electrons and positrons on protons the contributions from two photon exchange diagram are small [8, 9]. Nevertheless, new results of two-photon exchange calculations [10] relying on the knowledge of the proton structure, which contain more detailed information than the form factors under consideration, demonstrate the situation to be more complicated. As a result there is a proposal [11] to reinvestigate the contribution of two photons exchange by a measurement of the difference of electron and positron cross sections on the proton more carefully.

The other source of corrections which can influence the extraction of the form factors from the polarization data is a radiation of photons by the initial electrons along the beam line. Such

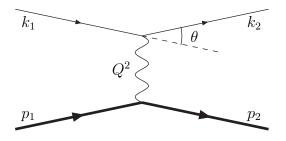


Fig. 1: Kinematics of  $ep \rightarrow e'p'$  reaction. No extra photon emission is assumed.

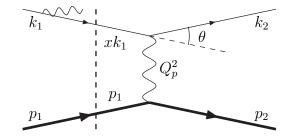


Fig. 2: Kinematics of  $ep \rightarrow e'p'$  reaction. Initial electron emits collinear photons.

photons are not therefore registered. They take away a part of the electron energy, then the genuine momentum transfer  $Q_p^2$  to the proton is less than the value obtained from the elastic electron scattering angle  $Q^2 = 4E_1E_2\sin^2\frac{\theta}{2}$ , where the energy  $E_2$  of the final electron is not measured and it is determined from the elastic scattering formula  $E_2 = ME_1/(M + 2E_1\sin^2\frac{\theta}{2})$ . The cross section of the process  $ep \to e'p'$  (no emission of extra photons) falls rapidly as  $Q^2$  increases. Therefore, if the energy of the final electron is not measured in order to check the elasticity of the event, the contribution of the inelastic events to the cross section can be significant since for them  $Q_p^2 < Q^2$  even though emission of an additional photon is suppressed by a factor of  $\alpha_{em}$ . Such corrections to the one-photon exchange can be calculated without model dependent assumptions. This task has been done in many works (see e.g. the original paper [12]) and the corrections were found to be small [9]. However, the corresponding calculations are quite complicated technically and no simple formula can be presented for their understanding.

In this letter we present a simple estimation of the corrections arising from the photon radiation by the initial electron.

Before, however, we briefly remind the formulae related to the method of polarization transfer assuming no emission of extra photons [4, 5]. The longitudinally polarized electron beam, with energy in lab. frame<sup>1</sup>  $E_1$  and polarization degree  $\lambda$ , scatters on unpolarized proton target (Fig. 1). In the final state, the knocked out proton is detected and its polarization is measured. The degree of transverse proton polarization is denoted by  $\mathcal{P}_x$ , the degree of its longitudinal polarization is denoted by  $\mathcal{P}_z$  and both depend on the electron scattering angle  $\theta$  as follows [4, 5]:

$$\mathcal{P}_x \frac{d\sigma}{d\Omega} = -\lambda \frac{\alpha^2}{Q^2} \left( \frac{M}{M + 2E_1 \sin^2 \frac{\theta}{2}} \right)^2 \frac{Q}{\sqrt{Q^2 + 4M^2}} \operatorname{ctg} \frac{\theta}{2} \ G_{Ep}(Q^2) G_{Mp}(Q^2), \tag{1}$$

$$\mathcal{P}_{z}\frac{d\sigma}{d\Omega} = -\lambda \frac{\alpha^{2}}{2M^{2}} \left(\frac{M}{M + 2E_{1}\sin^{2}\frac{\theta}{2}}\right)^{2} \sqrt{1 + \frac{4M^{2}}{Q^{2} + 4M^{2}}} \operatorname{ctg}^{2}\frac{\theta}{2} \ G_{Mp}^{2}(Q^{2}), \tag{2}$$

where  $Q^2 = -(k_1 - k_2)^2$ , *M* is the proton mass. The scattering angle  $\theta$  is related to  $Q^2$  by means of the expression

$$\sin^2 \frac{\theta}{2} = \frac{Q^2}{4E_1(E_1 - \frac{Q^2}{2M})}.$$
(3)

From Eqs. (1) and (2) the relation

$$R(Q^2) = \frac{G_{Ep}(Q^2)}{G_{Mp}(Q^2)} = \frac{\mathcal{P}_x \frac{d\sigma}{d\Omega}}{\mathcal{P}_z \frac{d\sigma}{d\Omega}} \frac{Q^2}{2M^2} \operatorname{tg} \frac{\theta}{2} \sqrt{1 + \frac{4M^2}{Q^2 \sin^2 \frac{\theta}{2}}}.$$
(4)

<sup>&</sup>lt;sup>1</sup>Throughout the letter, all non-covariant variables are written in the lab. frame.

follows, which is commonly used to extract the value of  $G_{Ep}(Q^2)/G_{Mp}(Q^2)$  from the polarization transfer data.

Let us now take into account emission of extra photons by the initial electron<sup>2</sup> (Fig. 2). The process can be split into two parts: the extra photon emission, described by  $\mathcal{D}$ -function [13] and the scattering of the slowed down electron on the proton described by Eqs. (1),(2). Since the extra photon is almost collinear to the initial electron, we take  $k'_1 = xk_1$  and it follows that the energy of the final electron  $E'_2$  and the momentum transferred to the proton  $Q_p^2 = -(p_1 - p_2)^2$ are given by

$$E_2' = \frac{xME_1}{M + 2xE_1\sin^2\frac{\theta}{2}},$$
(5)

$$Q_p^2 = \frac{4Mx^2 E_1^2 \sin^2 \frac{\theta}{2}}{M + 2x E_1 \sin^2 \frac{\theta}{2}} < Q^2 = \frac{4M E_1^2 \sin^2 \frac{\theta}{2}}{M + 2E_1 \sin^2 \frac{\theta}{2}}.$$
 (6)

Then the radiatively corrected expressions for the cross sections (1),(2) read

$$\left(\mathcal{P}_x \frac{d\sigma}{d\Omega}\right)_{corr} = -\lambda \int_{x_0}^1 dx \mathcal{D}(x) \frac{\alpha^2}{Q_p^2} \left(\frac{M}{M + 2xE_1 \sin^2 \frac{\theta}{2}}\right)^2 \frac{\sqrt{Q_p^2} \operatorname{ctg}\frac{\theta}{2}}{\sqrt{Q_p^2 + 4M^2}} R(Q_p^2)_{corr} G_{Mp}^2(Q_p^2)_{corr},$$
(7)

$$\left(\mathcal{P}_z \frac{d\sigma}{d\Omega}\right)_{corr} = -\lambda \int\limits_{x_0}^1 dx \mathcal{D}(x) \frac{\alpha^2}{2M^2} \left(\frac{M}{M + 2xE_1 \sin^2 \frac{\theta}{2}}\right)^2 \sqrt{1 + \frac{4M^2 \operatorname{ctg}^2 \frac{\theta}{2}}{Q_p^2 + 4M^2}} \ G_{Mp}^2 (Q_p^2)_{corr}, \quad (8)$$

where the  $\mathcal{D}$ -function is given by [13]

$$\mathcal{D}(x) = \frac{\beta}{2} \left[ \left( 1 + \frac{3}{8}\beta \right) (1-x)^{\frac{\beta}{2}-1} - \frac{1}{2}(1+x) \right] - \frac{\beta^2}{32} \left[ 4(1+x)\ln(1-x) + \frac{1+3x^2}{1-x}\ln x + 5 + x \right] + \mathcal{O}(\beta^3), \tag{9}$$

$$\beta = \frac{2\alpha}{\pi} \left[ \ln \left( \frac{Q_p^2}{m_e^2} \right) - 1 \right] \tag{10}$$

and  $x_0$  is determined by the minimal energy  $E_{2min}$  the final electron has to carry to be detected:

$$x_0 = \frac{ME_{2min}}{ME_1 - 2E_1E_{2min}\sin^2\frac{\theta}{2}} \sim \frac{E_{2min}}{E_1}.$$
 (11)

As it can be seen from Eq. (6), at small x and fixed  $\theta$  the integrand in Eq. (7) behaves like 1/x, while the integrand of Eq. (8) behaves like const(x). Therefore, extracted from the polarization data ratio of electric and magnetic form factors is sensitive to the value of  $x_0$ . The other very important issue is that the radiative correction is responsible for the effect of relative enhancement of the differential cross section at large  $Q^2$  with regard to the radiatively non corrected result [12, 13, 14]. Indeed, as the value of  $Q^2$  rises, the cross sections (1),(2) fall rapidly. However, the expressions for the radiatively corrected cross sections (7),(8) contain integration over the momentum transferred to the proton. At small  $x_0$ , the main contribution to the integrals (7),(8) comes from the region of small  $Q_p^2$ . Thus, even though the corrections

<sup>&</sup>lt;sup>2</sup>Photon emission by the final electrons is not important since the photon in that case should hit the same detector cell as the final electron. Thus in such case the entire energy of the electron after collision with the proton is detected.

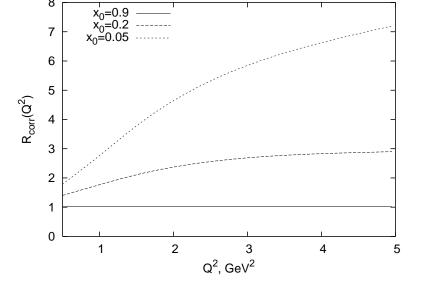


Fig. 3: Ratios  $R(Q^2)_{corr}$  for different  $x_0$ 

are suppressed by the electromagnetic coupling constant, they become relatively large as the value of  $Q^2$  rises.

To verify how large numerically these effects are, we have computed the corrected ratio (see Fig.3)

$$R(Q^2)_{corr} = \frac{\left(\mathcal{P}_x \frac{d\sigma}{d\Omega}\right)_{corr}}{\left(\mathcal{P}_z \frac{d\sigma}{d\Omega}\right)_{corr}} \left(\frac{Q^2}{2M^2} \mathrm{tg}\frac{\theta}{2} \sqrt{1 + \frac{4M^2}{Q^2 \sin^2\frac{\theta}{2}}}\right).$$
(12)

at different values of  $x_0$ .

For the sake of simplicity we have taken  $G_{Ep} = 1/(1+Q_p^2/0.71)^2$  and  $G_{Mp} = \mu_p/(1+Q_p^2/0.71)^2$  $Q_p^2/0.71)^2$ , i.e.  $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2) = 1$ . As it can be seen, the radiative corrections are very large if  $x_0$  is small. Therefore, extraction of the form factors requires a careful measurement of energy of the final electron. To make the extraction directly, the elastic scattering formula for  $E_2$  has to be checked carefully, that provides a firm cut off for the collinear photons emission. At the experiment, in which such cut off is used to indemnify elastic kinematics, the radiation of photons by the initial electrons along the beam line is suppressed. In this case two-photon exchange contributions can play an important role. However, their complete evaluation can not be carried out in a model-independent way [10] and one is forced to restrict himself to evaluation of box-diagrams. Here, one can estimate in a model-independent way diagrams with one-proton and also delta-resonance in the intermediate state. At the unpolarized process such contributions do not contain terms with  $Ln\frac{Q^2}{m_e^2}$  and they are finite for  $m_e \to 0$ . Investigations of such contributions are in progress.

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## References

[1] M. K. Jones *et al.* [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 84 (2000) 1398.

- [2] O. Gayou *et al.*, Phys. Rev. C **64** (2001) 038202.
- [3] O. Gayou et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 88 (2002) 092301.
- [4] A. I. Akhiezer and P.M. Rekalo, Sov. J. Part. Nucl. 3 (1974) 277.
- [5] R. G. Arnold, C. E. Carlson and F. Gross, Phys. Rev. C 23 (1981) 363.
- [6] M. N. Rosenbluth, Phys. Rev. **79** (1950) 615.
- [7] J. Arrington, Phys. Rev. C 68 (2003) 034325.
  S. Dubnička and A. Z. Dubničková, Fizika 13 (2004) 287.
  C. Adamuščin, S. Dubnička, A. Z. Dubničková, P. Weisenpacher, Prog. Part. Nucl. Phys. 55 (2005) 228.
- [8] J. Mar *et al.*, Phys. Rev. Lett. **21** (1968) 482.
   V. V. Bytev, E. A. Kuraev, B. G. Shaikhatdenov, JETP **122** (2002) 472.
- [9] A. Afanasev, I. Akushevich and N. Merenkov, Phys. Rev. D 64 (2001) 113009.
- [10] P. A. M. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. 91 (2003) 142303.
  P. G. Blunden, W. Melnitchouk and J. A. Tjon, Phys. Rev. Lett. 91 (2003) 142304.
  Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson and M. Vanderhaeghen, Phys. Rev. Lett. 93 (2004) 122301.
  A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y. C. Chen and M. Vanderhaeghen, arXiv:hep-ph/0502013.
  A. V. Afanasev and N. P. Merenkov, Phys. Lett. B 599 (2004) 48.
- [11] J. Arrington *et al.*, arXiv:nucl-ex/0408020.
   J. Arrington, Phys. Rev. C **71** (2005) 015202.
- [12] L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. **41** (1969) 205.
- [13] E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466 [Yad. Fiz. 41 (1985) 733].
   E. A. Kuraev, N. P. Merenkov and V. S. Fadin, Sov. J. Nucl. Phys. 47 (1988) 1009 [Yad. Fiz. 47 (1988) 1593].
- [14] V. N. Baier, V. S. Fadin and V. A. Khoze, Nucl. Phys. B 65 (1973) 381.
   V. N. Baier, E. A. Kuraev, V. S. Fadin and V. A. Khoze, Phys. Rept. 78 (1981) 293.