

Photon Irradiated Enhancement as a Tool of Investigating Fundamental Physics beyond Standard Model

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ABSTRACT

We clarify how intense laser irradiation leads to an enhancement of rare processes that may occur within atoms. Non-perturbative calculation using a coherent laser beam gives an exact, time dependent formula of the enhancement factor in the large power limit. A high quality laser may provide a new tool of experimentally investigating physics far beyond the standard model of particle physics such as lepton and baryon nonconservation.

In our recent paper [1] it was pointed out that intense laser beam, when irradiated to appropriate heavy atoms, can enhance rare processes related to atomic electron capture by nucleus, otherwise difficult to detect. In the present work we extend the formalism such that non-perturbative effects of laser-atom interaction are fully taken into account by directly solving dynamics of laser irradiation onto target atoms. In the dipole approximation of the two level problem of atoms the result suggests an onset of a repeated process of compression and expansion of the atomic electron cloud. Rare processes that may subsequently occur via the overlap of atomic electrons with nucleus may thus be enhanced. We propose to call this compression mechanism as PHIRAC to abbreviate PHoton IRrAdiated Compression of the electron cloud. For magnetic transitions such as those between hyperfine split levels this interpretation of electron cloud compression is not appropriate, but one can obtain a large enhancement of rare processes as well.

Our discussion in the present work is independent of any particular rare process X subsequent to photon absorption. It may thus provide a new method of exploring physics far beyond the standard model of particle physics. In this way one can experimentally explore lepton number nonconservation (LENNON) of the type $e^- \rightarrow e^+$, baryon number nonconserving (BARN-NON) process of the kind $e^- + N \rightarrow \text{many } \pi\text{'s}$, and hopefully many other fundamental processes that face physics beyond the standard model.

Consider laser irradiation which induces transitions between two electronic levels of a target atom, denoted by $|e\rangle$ and $|g\rangle$. We imagine that the target is irradiated continuously. For instance putting the target into a resonator may be useful. The system of laser and atom is then described by a Hamiltonian of the form [2],

$$H = \frac{\omega_{eg}}{2}\sigma_3 + \omega a^\dagger a + \tilde{s}(\sigma_+ a + \sigma_- a^\dagger). \quad (1)$$

Here the Pauli matrices σ_i act on two levels of the ground $|g\rangle$ and the excited $|e\rangle$ state, and $\omega_{eg} = \omega_e - \omega_g$ is the energy level difference. We made what is called the rotating wave approximation in the literature [2], by neglecting $\sigma_+ a^\dagger$ and its conjugate, which should be justified for mostly energy-conserving processes. The coupling strength s is given by

$$\langle e|\tilde{s}\sigma_+|g\rangle \equiv s = -\langle e|\vec{d}|g\rangle \cdot \vec{e}, \quad \vec{e} = i\frac{\omega}{\sqrt{2\omega V}}\vec{e}_k \quad (2)$$

for the dipole transtion, with \vec{d} the dipole operator of atomic electron and \vec{e} the electric field of a single photon beam with polarization \vec{e}_k . When

multiplied by a photon number N , this strength is expressed as $Ns^2 = \pi\gamma_d P/\omega^3$ where P is the laser power in the unit of energy/ (time *times* area).

The Hamiltonian is block-diagonal, hence the effect of laser irradiation can be solved by decomposing the infinite dimensional Fock space and using a mixture of two states, $|e, n\rangle = (a^\dagger)^n/\sqrt{n!}|e\rangle$ and $|g, n\rangle = (a^\dagger)^n/\sqrt{n!}|g\rangle$; $|\psi(t)\rangle = \sum_n (c_{g,n+1}(t)|g, n+1\rangle + c_{e,n}(t)|e, n\rangle)$. The Schroedinger equation to be solved is

$$i\frac{d}{dt} \begin{pmatrix} c_{e,n} \\ c_{g,n+1} \end{pmatrix} = \mathcal{H} \begin{pmatrix} c_{e,n} \\ c_{g,n+1} \end{pmatrix}, \quad \mathcal{H} = \begin{pmatrix} n\omega + \frac{\omega_{eg}}{2} & s\sqrt{n+1} \\ s\sqrt{n+1} & (n+1)\omega - \frac{\omega_{eg}}{2} \end{pmatrix}. \quad (3)$$

The solution may be written in terms of what is called dressed states denoted by $|\pm, n\rangle$; assuming a spacially constant (valid in the long wavelength approximation) laser field, the Hamiltonian diagonalization is possible with

$$|+, n\rangle = \cos \frac{\varphi_n}{2} |e, n\rangle + \sin \frac{\varphi_n}{2} |g, n+1\rangle, \quad (4)$$

$$|-, n\rangle = -\sin \frac{\varphi_n}{2} |e, n\rangle + \cos \frac{\varphi_n}{2} |g, n+1\rangle, \quad (5)$$

where

$$\tan \varphi_n = \frac{2s\sqrt{n+1}}{\omega - \omega_{eg}}, \quad \omega_{\pm} = (n + \frac{1}{2})\omega \pm \frac{\Omega_n}{2}, \quad (6)$$

with $\Omega_n = \sqrt{(\omega - \omega_{eg})^2 + 4s^2(n+1)}$ the Rabi frequency [2]. Unless the photon energy ω is very far from the resonance energy ω_{eg} , the mixing is nearly maximal; $\sin^2 \varphi_n \approx 1$. We thus assume the maximal mixing for simplicity; $\varphi_n = \pi/2$.

It is reasonable under the continuous laser irradiation to assume that the target is initially in a superposed state of two levels, $|\frac{1}{2}\rangle \equiv \frac{1}{\sqrt{2}}(e^{i\delta}|g\rangle + |e\rangle)$ with $\delta = \pi/2$. The result is insensitive to the choice of this phase and the initial condition as a whole.

Time evolution is then given by

$$\langle g, n+1 | \frac{1}{2}, n; t \rangle = ie^{-i(n+1/2)\omega t} \cos(\frac{\Omega_n t}{2} + \frac{\pi}{4}) c_n^{(\gamma)}(0). \quad (7)$$

In the rest of this paper we take the coherent state of laser, thus $c_n^{(\gamma)}(0) = e^{-N/2} N^n / \sqrt{n!}$, where the average photon number $\langle n \rangle = N$ and the dispersion $\langle (\Delta n)^2 \rangle = N$.

It is conceptually important to distinguish from the Schroedinger (S) picture and use the interaction (I) picture, since the subsequent rare process is treated in the perturbation theory. Fortunately, since the laser-atom system is exactly solvable, one may straightforwardly use the identity ${}_S\langle a|b\rangle_S = {}_I\langle a|b\rangle_I$, to simplify computation.

We imagine a circumstance under which rare processes occur via the ground state $|g\rangle$ of zero angular momentum, a ms state. It is assumed that at a time t the electron in the excited $|e\rangle$ goes to the ms $|g\rangle$ state due to the stimulated emission, and is subsequently captured by nucleus, with a probability proportional to the wave function factor $|\psi_{ms}(0)|^2$. An example of subsequent rare processes of this sort is LENNON electron capture of the kind $e^- \rightarrow e^+$, as discussed in [1]. The probability amplitude of laser irradiated rare process is then given by

$$\tilde{\mathcal{M}}_{1/2}(t) = e^{-(\gamma_e + \gamma_g)t/4} \sum_{n=0}^{\infty} \mathcal{M}_X(t) \frac{s\sqrt{n+1}}{\omega - \omega_{eg} + i\gamma/2} \langle g, n+1 | \frac{1}{2}, n; t \rangle \quad (8)$$

where $\mathcal{M}_X(t)$ is the amplitude for the rare X process from the ms state.

Computation of the discrete n sum (8) is well approximated in the large N limit by a continuous n integral. Using the large n limit formula of $n!$, and the coherent state expression for $c_n^{(\gamma)}(0)$, the integrand is found to change violently, and one may estimate the integral by a gaussian approximation around the stationary phase, or the saddle point. The saddle point n_0 of the integrand is determined by the minimal variation of the exponent of the integrand and is given by taking the n -derivative of the exponent to vanish. A complex saddle is thus obtained; $n_0 \approx N e^{-2i\omega t}$. Making the gaussian approximation around this saddle gives the rate formula,

$$|\tilde{\mathcal{M}}_{1/2}(t)|^2 \approx \left(\frac{\pi}{2}\right)^{1/2} N^{3/2} \frac{s^2 |\mathcal{M}_X(t)|^2}{(\omega - \omega_{eg})^2 + \gamma^2/4} \exp[-N(1 - \cos 2\omega t) - (\gamma_e + \gamma_g)t/2] [\sinh^2(\sqrt{N}st \sin \omega t) + \cos^2(\sqrt{N}st \cos \omega t)]. \quad (9)$$

As a function of time, the rate becomes very large of order $N^{3/2}$, periodically at $\cos 2\omega t = 1$. Outside regions of a time range $\approx 1/(\omega\sqrt{N})$ the rate is very small, of order $N^{3/2}e^{-N}$. Thus, the rate is sizable only around infinitely many discrete times of $t = k\pi/\omega$, with k any positive integer. In other words, the rate has a spiky time profile with a period π/ω . A formula

valid for $N \gg 1$

$$\exp[-N(1 - \cos 2\omega t)] \approx \sqrt{\frac{\pi}{2}} \frac{1}{\omega \sqrt{N}} \sum_k \delta(t - \frac{k\pi}{\omega}), \quad (10)$$

may then be used. For $\omega t \gg 1$, it is reasonable to take a time average over $\Delta t \approx \pi/\omega \times$ a few, which gives a time variant averaged rate,

$$\tilde{\mathcal{R}}_{1/2}(t) = \frac{N}{2} \frac{s^2 e^{-(\gamma_e + \gamma_g)t/2}}{(\omega - \omega_{eg})^2 + \gamma^2/4} \mathcal{R}_X(t), \quad (11)$$

with $\mathcal{R}_X(t) = d|\mathcal{M}_X(t)|^2/dt$.

The last factor $\mathcal{R}_X(t)$ may differ in rare processes in which one is interested. For example, the LENNON conversion of the type $e^- \rightarrow e^+$ has $\mathcal{R}_{e^- \rightarrow e^+}(t) = |\psi_{ms}(0)|^2 \sigma_{e^- \rightarrow e^+}$, where $\sigma_{e^- \rightarrow e^+}$ is the cross section of free electron capture, a virtual process considered for our gedanken experiment. This quantity is computed using the perturbation theory.

One may summarize arguments so far by defining a quality factor $Q(\omega)$ which signifies the rate enhancement,

$$Q \equiv r \frac{\pi \gamma_d}{2 \omega_0^2} \int d\omega \frac{\mathcal{P}(\omega)}{\omega[(\omega - \omega_0)^2 + \gamma^2/4]}, \quad (12)$$

where $\int d\omega \mathcal{P}(\omega) = P$ is the total laser power in the unit of energy / (time \times area). The factor r is the wave function ratio squared, for instance, for LENNON

$$r = \frac{|\psi_{ms}(0)|^2}{|\psi_{ns}(0)|^2} = \left(\frac{r_{ns}}{r_{ms}}\right)^3 \approx O\left[\left(\frac{n}{m}\right)^6\right], \quad n = 1. \quad (13)$$

For a laser beam of the energy resolution $\Delta E \gg \gamma$ we may replace $1/[(\omega - \omega_0)^2 + \gamma^2/4]$ by $\frac{2\pi}{\gamma} \delta(\omega - \omega_0)$ for a laser beam of Lorentzian energy distribution. When the laser tuning is perfect, $\mathcal{P}(\omega_0) \approx P/\Delta E$. It is thus found that

$$Q \approx r \frac{\pi^2 P}{\omega_0^4} \frac{\omega_0}{\Delta E} \approx 1.6 \times 10^6 r \frac{P}{W mm^2} \left(\frac{\omega_0}{eV}\right)^{-4} (10^{-9} \frac{\omega_0}{\Delta E}). \quad (14)$$

Note a strong dependence on the photon energy $\propto \omega_0^{-4}$, which should be important to get a large quality factor for BARRNON. This formula for the quality (14) agrees with the result of [1] after a minor correction [3].

The spiky time profile is however a result of the present nonperturbative formalism.

In order to advance an intuitive understanding of the enhancement mechanism, it might be instructive to compute the electron displacement squared

$$\begin{aligned}
(\delta\mathcal{D})^2(t) &= d^2 \sum_n \langle \frac{1}{2}, n; t | \sigma_- \sigma_+ a^\dagger a | \frac{1}{2}, n; t \rangle \\
&\approx \frac{d^2}{2} \sum_n (1 - \sin \Omega_n t) |c_n^{(\gamma)}(0)|^2.
\end{aligned}
\tag{15}$$

The leading term of $O[d^2 N/2]$ simply shows that quantum mechanics gives a result consistent with the classical Lorentz oscillator model.

The behavior of the next leading term in the large N limit of $(\delta\mathcal{D})^2(t)$ and the linear dipole $\mathcal{D}(t)$ is more complicated. They exhibit a spiky time profile in much the same way as the rate formula, but with an important difference of time scale; these quantities that have classical analogues vary in time much more slowly, the spike interval being given by $4\pi/\Omega_R$ with $\Omega_R = 2s\sqrt{N}$ the Rabi resonance frequency. We may only say that the spiky time profile observed in the rate enhancement is a signal of the onset of the oscillatory behavior of the electron displacement. The link is however indirect.

Baryon nonconservation may experimentally be investigated by searching for the atomic electron capture of the type [4], $e^- + N \rightarrow \pi + \pi$. Enhancement factor $\propto P$ may compensate the small nuclear overlap factor of order $(a_B m_\pi)^3 \approx 10^{-15}$ of atomic electrons that otherwise disfavors this process. A rough estimate of the rate gives the enhancement factor of order, $Q(r_A A^{1/3} m_\pi)^{-3}$, where the pion mass m_π times $A^{1/3}$ gives a measure of the inverse nuclear size, and r_A is the atomic size. It is important to go to a low frequency range for a large enhancement of order 10^{25} . Thus, the use of the Zeeman split hyperfine levels is promising, which involves the microwave region.

In summary, the prospect of search for new physics far beyond the standard model appears bright if one uses an appropriate high quality laser.

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References

- [1] M. Ikeda, I. Nakano, M. Sakuda, R. Tanaka, and M. Yoshimura, "New Method of Enhancing Lepton Number Nonconservation", hep-ph/0506062 v2.
- [2] For instance, L. Allen and J.H. Eberly, *Optical Resonance and Two-level Atoms*, John Wiley and Sons, New York (1975);
M. Sargent, M.O. Scully, and W.E. Lamb, *Laser Physics* Addison-Wesley, Reading (1974).
- [3] I have detected an error of double counting of the angular factor 4π in [1]. Thus, quoted numbers there should be divided by 4π .
- [4] The process of this type has been discussed with I. Nakano, which is much appreciated.