

# Gluonic phase in neutral two-flavor dense QCD

E.V. Gorbar,<sup>1,\*</sup> Michio Hashimoto,<sup>1,†</sup> and V.A. Miransky<sup>1,‡</sup>

<sup>1</sup> *Department of Applied Mathematics, University of Western Ontario, London, Ontario N6A 5B7, Canada*  
(Dated: April 21, 2018)

In the Ginzburg-Landau approach, we describe a new phase in neutral two-flavor quark matter in which gluonic degrees of freedom play a crucial role. We call it a gluonic phase. In this phase gluonic dynamics cure a chromomagnetic instability in the 2SC solution and lead to spontaneous breakdown of the color gauge symmetry, the electromagnetic  $U(1)$ , and the rotational  $SO(3)$ . In other words, the gluonic phase describes an anisotropic medium in which the color and electric superconductivities coexist. Because most of the initial symmetries in this system are spontaneously broken, its dynamics is very rich.

PACS numbers: 12.38.-t, 11.15.Ex, 11.30.Qc

## I. INTRODUCTION

It is natural to expect that cold quark matter may exist in the interior of compact stars. This fact motivated intensive studies of this system over the past few years (for a review, see Ref. [1]). While these studies firmly established that the cold and dense quark matter is a color superconductor, they also revealed a remarkably rich phase structure in this system, consisting of many different phases. The question which phase is picked up by nature is still open.

In this Letter we will describe a new phase in neutral and  $\beta$ -equilibrated two-flavor quark matter. We call it a gluonic phase. The name reflects a crucial role of gluonic degrees of freedom in the structure of its ground state. More precisely, besides the usual color superconducting condensate of quarks, there exist (vector) condensates of gluons in this phase. These vector condensates cure a chromomagnetic instability in the two-flavor superconducting (2SC) solution [2] and lead to a dramatic rearrangement of the 2SC ground state, most notably, to spontaneous breakdown of  $SU(2)_c \times \tilde{U}(1)_{em} \times SO(3)_{rot}$  symmetry down to  $SO(2)_{rot}$ . Here  $SU(2)_c$  and  $\tilde{U}(1)_{em}$  are the color and electromagnetic gauge symmetries in the 2SC medium, and  $SO(3)_{rot}$  is the rotational group (recall that in the 2SC solution the color  $SU(3)_c$  is broken down to  $SU(2)_c$ ). In other words, the gluonic phase describes an anisotropic medium in which the color and electric superconductivities coexist. As we will discuss below, there may exist a class of solutions with vector condensates of gluons. The solution described in this Letter is similar to that found in the gauged  $\sigma$ -model with a chemical potential for hypercharge [3], although physics in the present system is much richer. In particular, as will be shown in Sec. V, exotic hadronic states play an important role in its dynamics.

The framework of the present study is the Ginzburg-Landau (GL) approach. The basic point in our analysis is the inclusion of light gluonic degrees of freedom in the GL effective action. This leads us to revealing the gluonic phase in neutral two-flavor quark matter.

## II. EFFECTIVE POTENTIAL WITH VECTOR CONDENSATES

We study dense two-flavor quark matter in  $\beta$ -equilibrium. For our purpose, it is convenient to use a phenomenological Nambu-Jona-Lasinio (NJL) model, more precisely, a gauged NJL model including gluons. Although usually the NJL model is regarded as a low-energy effective theory in which massive gluons are integrated out, we introduce gluonic degrees of freedom because the gluons of the unbroken  $SU(2)_c$  subgroup of the color  $SU(3)_c$  are left as massless,

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\*Electronic address: egorbar@uwo.ca; On leave from Bogolyubov Institute for Theoretical Physics, 03143, Kiev, Ukraine

†Electronic address: mhashimo@uwo.ca

‡Electronic address: vmiransk@uwo.ca; On leave from Bogolyubov Institute for Theoretical Physics, 03143, Kiev, Ukraine

and, under certain conditions considered below, some other gluons can be also very light. As discussed in Ref. [4], the confinement scale  $\Lambda'_{\text{QCD}}$  in the two-flavor color 2SC phase is estimated as  $\mathcal{O}(10 \text{ MeV})$  or smaller. Thus it is not peculiar to consider gluonic degrees of freedom at energies less than the quark gap  $\Delta$ .

We neglect the current quark masses and, as usual in studying the 2SC phase, assume that the color superconducting condensate does not break parity. The Lagrangian density is then given by

$$\mathcal{L} = \bar{q}(i\not{D} + \hat{\mu}\gamma^0)q + G_\Delta \left[ (\bar{q}^C i\varepsilon^a \gamma_5 q)(\bar{q} i\varepsilon^a \gamma_5 q^C) \right] + \mathcal{L}_g, \quad (1)$$

where

$$\mathcal{L}_g = -\frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha\mu\nu} \quad (2)$$

and

$$D_\mu \equiv \partial_\mu - igA_\mu^\alpha T^\alpha, \quad F_{\mu\nu}^\alpha \equiv \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + gf^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma. \quad (3)$$

Here  $A_\mu^\alpha$  are gluon fields,  $T^\alpha$  are the  $SU(3)$  matrices in the fundamental representation, and  $\varepsilon^{ij}$  and  $\varepsilon^{acd}$  are the antisymmetric tensors in the flavor and color spaces, respectively. In  $\beta$ -equilibrium, the elements of the diagonal chemical potential matrix  $\hat{\mu}$  for up ( $u$ ) and down ( $d$ ) quarks are

$$\mu_{ur} = \mu_{ug} = \tilde{\mu} - \delta\mu, \quad \mu_{dr} = \mu_{dg} = \tilde{\mu} + \delta\mu, \quad (4a)$$

$$\mu_{ub} = \tilde{\mu} - \delta\mu - \mu_8, \quad \mu_{db} = \tilde{\mu} + \delta\mu - \mu_8, \quad (4b)$$

with

$$\tilde{\mu} \equiv \mu - \frac{\delta\mu}{3} + \frac{\mu_8}{3}, \quad \delta\mu \equiv \frac{\mu_e}{2}. \quad (5)$$

Here the subscripts  $r$ ,  $g$ , and  $b$  correspond to red, green and blue quark colors,  $\mu$  is the quark chemical potential (the baryon chemical potential  $\mu_B$  is given by  $\mu_B \equiv 3\mu$ ),  $\mu_e$  is the chemical potential for the electric charge, and  $\mu_8$  is the color chemical potential. The latter is simply connected with the vacuum expectation value (VEV) of the time component of the 8-th gluon [5]:

$$g\langle A_0^8 \rangle = \frac{2}{\sqrt{3}}\mu_8. \quad (6)$$

By using the auxiliary field  $\Delta^a \sim i\bar{q}^C \varepsilon^a \gamma_5 q$ , the Lagrangian density (1) can be rewritten as

$$\mathcal{L} = \bar{q}(i\not{D} + \hat{\mu}\gamma^0)q - \frac{1}{2}\Delta^a [i\bar{q}^C \varepsilon^a \gamma_5 q^C] - \frac{1}{2}[i\bar{q}^C \varepsilon^a \gamma_5 q]\Delta^{*a} - \frac{|\Delta^a|^2}{4G_\Delta} + \mathcal{L}_g. \quad (7)$$

We now introduce the Nambu-Gor'kov spinor,

$$\Psi = \begin{pmatrix} q \\ q^C \end{pmatrix}. \quad (8)$$

The inverse propagator  $S_g^{-1}$  of  $\Psi$  including gluons is written as

$$S_g^{-1} = \begin{pmatrix} [G_{0,g}^+]^{-1} & \Delta^- \\ \Delta^+ & [G_{0,g}^-]^{-1} \end{pmatrix}, \quad (9)$$

with

$$[G_{0,g}^+]^{-1} \equiv (p^0 + \tilde{\mu} - \delta\mu\tau^3 - \mu_8\mathbf{1}_b)\gamma^0 - \vec{\gamma} \cdot \vec{p} + gA^\alpha T^\alpha, \quad (10)$$

$$[G_{0,g}^-]^{-1} \equiv (p^0 - \tilde{\mu} + \delta\mu\tau^3 + \mu_8\mathbf{1}_b)\gamma^0 - \vec{\gamma} \cdot \vec{p} - gA^\alpha T^{\alpha T} \quad (11)$$

and

$$\Delta^- \equiv -i\epsilon^b \epsilon \gamma_5 \Delta, \quad \Delta^+ \equiv -i\epsilon^b \epsilon \gamma_5 \Delta^*. \quad (12)$$

Here the diquark condensate is chosen along the blue color direction, the constant fields  $A_\mu^\alpha$  represent possible vector gluonic condensates in the model, and we introduced matrices  $\tau^3 \equiv \text{diag}(1, -1)$  and  $\mathbf{1}_b \equiv \text{diag}(0, 0, 1)$  acting in the flavor and color spaces, respectively. The effective potential in this model includes both gluons and the scalar field  $\Delta$ . It is:

$$V_{\text{eff}} = \frac{|\Delta|^2}{4G_\Delta} + \frac{g^2}{4} f^{\alpha\beta\gamma} f^{\alpha\delta\sigma} A_\mu^\beta A_\nu^\gamma A^{\delta\mu} A^{\sigma\nu} - \frac{1}{2} \int \frac{d^4 p}{i(2\pi)^4} \ln \det S_g^{-1}. \quad (13)$$

We will utilize the hard dense loop approximation, in which only the dominant one-loop quark contribution is taken into account, while the contribution of gluon loops is neglected. On the other hand, we keep the tree contribution of gluons in the effective potential (13). This is because we want to compare this contribution with that of hard dense loops in order to check the consistency of the hard dense loop approximation.

### III. DYNAMICS OF GLUONS IN THE GINZBURG-LANDAU APPROACH

Let us consider dynamics of light gluonic degrees of freedom in this model. The two point function of gluons can be calculated from Lagrangian density (7). In Refs. [2, 6], the Debye and Meissner screening masses of the gluons in the 2SC phase were explicitly calculated. For the gluons of the unbroken  $SU(2)_c$ , i.e.,  $A^{(1)}$ ,  $A^{(2)}$ , and  $A^{(3)}$ , both Debye and Meissner masses vanish in the region  $\delta\mu < \Delta$  (henceforth, for clarity, we will put color indices of gluon fields in parentheses). For the gluons  $A^{(4)-(7)}$ , the Meissner mass is approximately

$$m_{M,4}^2 = \frac{g^2 \tilde{\mu}^2}{6\pi^2} \left( 1 - \frac{2\delta\mu^2}{\Delta^2} \right), \quad \delta\mu < \Delta. \quad (14)$$

Thus, near the critical point  $\delta\mu = \Delta/\sqrt{2}$ , the Meissner mass for  $A^{(4)-(7)}$  is very small. As  $\delta\mu$  exceeds the value  $\Delta/\sqrt{2}$ ,  $m_{M,4}^2$  becomes negative, thus signaling a chromomagnetic instability in the 2SC solution [2]. On the other hand, around the critical point  $\delta\mu = \Delta/\sqrt{2}$ , the  $SU(2)_c$  singlet gluon  $A^{(8)}$  is heavy. Actually, it has nonvanishing Debye and Meissner screening masses in the whole region  $\delta\mu < \Delta$ . This fact allows us to pick up the gluons  $A^{(1)-(7)}$  as relevant light degrees of freedom in the low energy effective theory around the critical point  $\delta\mu = \Delta/\sqrt{2}$ . Our goal is to describe the dynamics near this critical point in the GL approach.

The quark gap  $\Delta \neq 0$  breaks the QCD symmetry  $SU(3)_c$  down to  $SU(2)_c$ . With respect to  $SU(2)_c$ , the adjoint representation of  $SU(3)_c$  is decomposed as

$$\mathbf{8} = \mathbf{3} \oplus \mathbf{2} \oplus \bar{\mathbf{2}} \oplus \mathbf{1}, \quad (15)$$

i.e.,

$$\{A_\mu^{(\alpha)}\} = (A_\mu^{(1)}, A_\mu^{(2)}, A_\mu^{(3)}) \oplus K_\mu \oplus K_\mu^\dagger \oplus A_\mu^{(8)}, \quad (\alpha = 1, 2, \dots, 8). \quad (16)$$

Here we defined the complex doublet of the ‘‘matter’’ field describing color vector ‘‘kaons’’:

$$K_\mu \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu^{(4)} - iA_\mu^{(5)} \\ A_\mu^{(6)} - iA_\mu^{(7)} \end{pmatrix}. \quad (17)$$

We also define

$$f_{\mu\nu}^{(l)} \equiv \partial_\mu A_\nu^{(l)} - \partial_\nu A_\mu^{(l)} + g\epsilon^{lmn} A_\mu^{(m)} A_\nu^{(n)}, \quad (l, m, n = 1, 2, 3) \quad (18)$$

and

$$\mathcal{D}_\mu \equiv \partial_\mu - igA_\mu^{(l)} \frac{\sigma^l}{2}. \quad (19)$$

Then the building blocks of the effective action are the following six,

$$K_0, \quad K_j, \quad \mathcal{D}_0, \quad \mathcal{D}_j, \quad f_{0j}, \quad f_{jk}. \quad (20)$$

The effective action should be of course invariant under all initial symmetries, in particular, under the color  $SU(2)_c$  and the rotational  $SO(3)_{\text{rot}}$ .

Because of the chromomagnetic instability at  $\delta\mu > \Delta/\sqrt{2}$ , it is natural to study a spontaneous breakdown of the  $SU(2)_c$  via the formation of a vector condensate  $\langle K_\mu \rangle \neq 0$ . Because the instability is chromomagnetic, we assume that a spatial component of  $K_\mu$  has a VEV. By using the rotational symmetry  $SO(3)_{\text{rot}}$ , one can take  $\langle K_3 \rangle \neq 0$ . And because of the  $SU(2)_c$  symmetry, without loss of generality, we can choose  $\langle A_3^{(6)} \rangle \neq 0$ . This VEV breaks both the  $SU(2)_c$  and the  $SO(3)_{\text{rot}}$ .

The following remarks are in order. The complex doublet  $K_3$  plays here the role of a Higgs field responsible for spontaneous breakdown of the  $SU(2)_c$ . The situation is similar to that taking place in the electroweak theory. The essential difference however is that now the Higgs field is a spatial component of the vector field leading also to spontaneous breakdown of the rotational symmetry. In this paper, the unitary gauge will be used in which  $K_3^T = \frac{1}{\sqrt{2}}(0, \langle A_3^{(6)} \rangle + a_3^{(6)})$  where the real field  $a_3^{(6)}$  describes quantum fluctuations. The important point is that in the unitary gauge all auxiliary (gauge dependent) degrees of freedom are removed. *Therefore in this gauge the vacuum expectation values of vector fields are well-defined physical quantities.*

With a broken  $SU(2)_c$ , the  $SU(2)_c$  gluons could have VEVs. A similar situation takes place in the gauged  $\sigma$ -model with a chemical potential for hypercharge, where the gauge  $SU(2)_L$  is broken [3]. Motivating by that model, we assume

$$\langle A_3^{(1)} \rangle, \langle A_0^{(3)} \rangle \neq 0, \quad (21)$$

and use the following notation,

$$B \equiv g\langle A_3^{(6)} \rangle, \quad C \equiv g\langle A_3^{(1)} \rangle, \quad D \equiv g\langle A_0^{(3)} \rangle \quad (22)$$

(note that  $g\langle A_0^{(3)} \rangle$  can be considered as a chemical potential  $\mu_3$  related to the third component of the color isospin). As will be shown in Sec. IV, such a solution with nonzero  $B, C$  and  $D$  vector condensates exists in this model indeed.

We now describe a general symmetry breaking structure in this model. In the presence of  $\mu_e$ , the chiral symmetry is explicitly broken. Then the initial symmetry is

$$[SU(3)_c]_{\text{local}} \times [U(1)_{em} \times U(1)_{\tau_L^3} \times U(1)_{\tau_R^3}]_{\text{global}} \times SO(3)_{\text{rot}}, \quad (23)$$

where  $U(1)_{\tau_{L,R}^3}$  are  $U(1)$ -part of the chiral symmetry  $SU(2)_{L,R}$  (note that there is no photon field in the model). The baryon charge is incorporated in the subgroup,

$$\mathcal{B} = \frac{1}{3}\mathbf{1}_f \otimes \mathbf{1}_c = 2(Q - I_3), \quad (24)$$

where  $Q = \text{diag}(2/3, -1/3)$  and  $I_3 = \text{diag}(1/2, -1/2)$  acting on the flavor space. The quark gap  $\Delta$  breaks both  $SU(3)_c$  (down to  $SU(2)_c$ ) and  $U(1)_{em}$  but a linear combination of  $T^8$  and  $Q$  remains unbroken. The new electric charge of the unbroken  $\tilde{U}(1)_{em}$  is

$$\tilde{Q} = Q - \frac{1}{\sqrt{3}}T^8. \quad (25)$$

The baryon charge is also changed to

$$\tilde{\mathcal{B}} = 2(\tilde{Q} - I_3). \quad (26)$$

The VEV  $\langle A_3^{(6)} \rangle$  breaks  $SU(2)_c$ , but a linear combination of the generator  $T^3$  from the  $SU(2)_c$  and  $\tilde{Q}$ ,

$$\tilde{Q} = \tilde{Q} - T^3 = Q - \frac{1}{\sqrt{3}}T^8 - T^3, \quad (27)$$

determines the unbroken  $\tilde{U}(1)_{em}$  (the new baryon charge is  $\tilde{\mathcal{B}} = 2(\tilde{Q} - I_3)$ ). However, because  $T^1$  does not commute with  $T^3$ , the VEV  $\langle A_3^{(1)} \rangle$  breaks  $\tilde{U}_{em}(1)$ . The baryon charge is also broken.

After all, we have:

$$[SU(3)_c]_{\text{local}} \times [U(1)_{em} \times U(1)_{\tau_L^3} \times U(1)_{\tau_R^3}]_{\text{global}} \times SO(3)_{\text{rot}} \xrightarrow{\Delta} [SU(2)_c]_{\text{local}} \times [\tilde{U}(1)_{em} \times U(1)_{\tau_L^3} \times U(1)_{\tau_R^3}]_{\text{global}} \times SO(3)_{\text{rot}} \quad (28)$$

$$\xrightarrow{\langle A_3^{(6)} \rangle} [\tilde{U}(1)_{em} \times U(1)_{\tau_L^3} \times U(1)_{\tau_R^3}]_{\text{global}} \times SO(2)_{\text{rot}} \quad (29)$$

$$\xrightarrow{\langle A_3^{(1)} \rangle} [U(1)_{\tau_L^3} \times U(1)_{\tau_R^3}]_{\text{global}} \times SO(2)_{\text{rot}}. \quad (30)$$

Thus, this system describes an anisotropic medium in which both the color and electric superconductivities coexist.

Let us apply the GL approach to this system near the critical point  $\delta\mu \simeq \Delta/\sqrt{2}$ . The  $SU(2)_c$  and  $SO(3)_{\text{rot}}$  symmetries dictate that the general GL effective potential, made from building blocks (20) and including operators up to the mass dimension four, is

$$V_{\text{eff}} = V_{\Delta} + \frac{1}{2}M_B^2 B^2 + TDB^2 + \frac{1}{2}\lambda_{BC}B^2C^2 + \frac{1}{2}\lambda_{BD}B^2D^2 + \frac{1}{2}\lambda_{CD}C^2D^2 + \frac{1}{4}\lambda_B B^4, \quad (31)$$

where  $V_{\Delta}$  is the 2SC part of the effective potential. Here, while the coefficients  $\lambda_B$ ,  $\lambda_{BC}$ ,  $\lambda_{BD}$ , and  $\lambda_{CD}$  are dimensionless, the dimension (in mass units) of the coefficient  $T$  in the triple vertex is one. Expanding the potential (13) with respect to  $B$ ,  $C$ , and  $D$ , we can determine the potential (31). Amazingly, the structure of the gluonic part of effective potential (31) is quite similar to that of the potential in the gauged  $\sigma$ -model with the hypercharge chemical potential [3], although the present system is much richer.

Before realizing explicit calculations, we clarify the behavior of the effective potential (31) near the critical point. The stationary point of the effective potential (31) is given by equations

$$\frac{\partial V_{\text{eff}}}{\partial B} = B [M_B^2 + \lambda_B B^2 + 2TD + \lambda_{BC}C^2 + \lambda_{BD}D^2] = 0, \quad (32)$$

$$\frac{\partial V_{\text{eff}}}{\partial C} = C [\lambda_{BC}B^2 + \lambda_{CD}D^2] = 0, \quad (33)$$

$$\frac{\partial V_{\text{eff}}}{\partial D} = TB^2 + \lambda_{BD}DB^2 + \lambda_{CD}C^2D = 0, \quad (34)$$

and

$$\frac{\partial V_{\text{eff}}}{\partial \mu_e} = 0, \quad \frac{\partial V_{\text{eff}}}{\partial \mu_8} = 0, \quad \frac{\partial V_{\text{eff}}}{\partial \Delta} = 0. \quad (35)$$

We can expand  $\mu_e$ ,  $\mu_8$ , and  $\Delta$  around  $B = C = D = 0$ ,

$$\mu_e = \bar{\mu}_e + \xi_e, \quad (36)$$

$$\mu_8 = \bar{\mu}_8 + \xi_8, \quad (37)$$

$$\Delta = \bar{\Delta} + \xi_{\Delta}, \quad (38)$$

where the bar-quantities are the 2SC solution, when  $B = C = D = 0$ .

Let us assume that the origin (bifurcation point) of the solution with nonzero  $B$ ,  $C$ , and  $D$  corresponds to a second order phase transition (as will become clear in a moment, this assumption is self-consistent). Then, taking an infinitesimally small  $B$  near the critical point, we easily find that

$$\xi_e, \xi_8, \xi_{\Delta} \sim \mathcal{O}(B^2). \quad (39)$$

It implies that the difference of  $V_\Delta$  in the new solution and that in the 2SC one is

$$V_\Delta(\Delta^{\text{sol}}, \mu_e^{\text{sol}}, \mu_8^{\text{sol}}) - V_\Delta(\bar{\Delta}, \bar{\mu}_e, \bar{\mu}_8) \sim \mathcal{O}(B^4). \quad (40)$$

This fact will be useful in our analysis below.

From Eqs. (32)–(34), we find that when the 2SC solution becomes unstable ( $M_B^2 < 0$ ), a new solution occurs, if the parameters  $\lambda_{BC}$  and  $\lambda_{CD}$  satisfy

$$\lambda_{BC} > 0, \quad \lambda_{CD} < 0 \quad (41)$$

(in the next section, it will be shown that this constraint is satisfied indeed). The new solution is:

$$B_{\text{sol}} = \frac{-M_B^2}{3|T|} \sqrt{\frac{-\lambda_{CD}}{\lambda_{BC}}}, \quad C_{\text{sol}} = \sqrt{\frac{-M_B^2}{3\lambda_{BC}}}, \quad D_{\text{sol}} = \frac{-M_B^2}{3T}, \quad (42)$$

where we neglected higher order terms of  $M_B^2$ . In Eq. (42) the conventions  $B > 0$  and  $C > 0$  are chosen.

Near the critical point  $M_B^2 = 0$ , the solution behaves as

$$B_{\text{sol}} \propto -M_B^2, \quad C_{\text{sol}} \propto \sqrt{-M_B^2}, \quad D_{\text{sol}} \propto -M_B^2. \quad (43)$$

These scaling relations are quite remarkable. While the scaling relation for  $C$  is of engineering type, those for  $B$  and  $D$  are not (the origin of this is of course in the presence of the dimensional coefficient  $T$  in Eq. (42)). Such a scaling behavior implies that the  $B^4$  and  $B^2 D^2$  terms in the effective potential are irrelevant near the critical point  $M_B^2 = 0$ . Omitting them, we arrive at the reduced effective potential:

$$\tilde{V}_{\text{eff}} = V_\Delta + \frac{1}{2} M_B^2 B^2 + T D B^2 + \frac{1}{2} \lambda_{BC} B^2 C^2 + \frac{1}{2} \lambda_{CD} C^2 D^2. \quad (44)$$

Notice that  $(\tilde{V}_{\text{eff}} - V_\Delta) \sim \mathcal{O}(B^3)$ . This fact and Eq. (40) imply that in the leading approximation one can use the bar-quantities, defined in Eqs. (36)–(38), in calculating  $V_\Delta$ ,  $M_B^2$ ,  $T$ ,  $\lambda_{BC}$ , and  $\lambda_{CD}$  in the reduced potential. In other words, the effective potential can be decomposed into the ‘‘constant’’ 2SC part  $V_\Delta$ , with frozen fermion parameters, and the dynamical gluonic part:

$$\tilde{V}_{\text{eff}} \rightarrow \tilde{V}_{\text{eff}}(\bar{\Delta}, \bar{\mu}_e, \bar{\mu}_8; B, C, D) = V_\Delta(\bar{\Delta}, \bar{\mu}_e, \bar{\mu}_8) + \frac{1}{2} M_B^2 B^2 + T D B^2 + \frac{1}{2} \lambda_{BC} B^2 C^2 + \frac{1}{2} \lambda_{CD} C^2 D^2. \quad (45)$$

Then Eq. (42) is the exact solution of the potential (45) and the energy density at the stationary point is found as

$$\tilde{V}_{\text{eff}}(\bar{\Delta}, \bar{\mu}_e, \bar{\mu}_8; B_{\text{sol}}, C_{\text{sol}}, D_{\text{sol}}) = V_\Delta + \frac{1}{6} M_B^2 B_{\text{sol}}^2 = V_\Delta - \frac{(-M_B^2)^3}{54T^2} \left( -\frac{\lambda_{CD}}{\lambda_{BC}} \right) < V_\Delta. \quad (46)$$

Therefore the gluonic vacuum is more stable than the 2SC one.

By using the Gauss’s law constraint

$$T B^2 + \lambda_{CD} C^2 D = 0, \quad (47)$$

we find the true effective potential without the non-dynamical degree of freedom  $A_0^{(3)}$ :

$$\tilde{V}_{\text{eff}}^{\text{Gauss}} = V_\Delta + \frac{1}{2} M_B^2 B^2 + \frac{1}{2} \lambda_{BC} B^2 C^2 - \frac{T^2 B^4}{2\lambda_{CD} C^2}. \quad (48)$$

It is easy to show that solution (42) is a minimum by analyzing the curvature of  $\tilde{V}_{\text{eff}}^{\text{Gauss}}$ .

In the next section, we will calculate  $M_B^2$ ,  $T$ ,  $\lambda_{BC}$ , and  $\lambda_{CD}$ . In particular, it will be shown that constraint (41) is satisfied near the critical point.

#### IV. DYNAMICS IN ONE-LOOP APPROXIMATION

In this section, we determine the GL effective potential (45) in one-loop approximation and determine the dispersion relations for quarks in the gluonic phase. The 2SC  $V_\Delta$  part of the potential is known [7],

$$V_\Delta(\Delta, \mu_e, \mu_8) = \frac{\Delta^2}{4G_\Delta} - \frac{\mu_e^4}{12\pi^2} - \frac{\mu_{ub}^4}{12\pi^2} - \frac{\mu_{db}^4}{12\pi^2} - \frac{\tilde{\mu}^4}{3\pi^2} - \frac{\Delta^2}{\pi^2} \left[ \tilde{\mu}^2 - \frac{1}{4}\Delta^2 \right] \ln \frac{4\Lambda^2}{\Delta^2} - \frac{\Delta^2}{\pi^2} \left[ \Lambda^2 - 2\tilde{\mu}^2 + \frac{1}{8}\Delta^2 \right], \quad (\delta\mu < \Delta). \quad (49)$$

Here  $\Lambda$  is the ultraviolet cutoff in the NJL model and  $\mu_{ub}$ ,  $\mu_{db}$ , and  $\tilde{\mu}$  are given in Eqs. (4) and (5). For clarity of the presentation, the bars in  $\Delta$ ,  $\mu_e$  and  $\mu_8$  were omitted ( $\mathcal{O}(\tilde{\mu}^2/\Lambda^2)$  and  $\mathcal{O}(\Delta^2/\Lambda^2)$  and higher terms are neglected in this expression). Note that the color and electrical charge neutrality conditions in the 2SC solution yield [7]

$$\delta\mu = \frac{3}{10}\mu - \frac{1}{5}\mu_8, \quad (50)$$

and

$$(\tilde{\mu}^2 + \delta\mu^2)\mu_8 = -\tilde{\mu}\Delta^2 \left( \ln \frac{2\Lambda}{\Delta} - 1 \right) + \tilde{\mu}(\delta\mu^2 + \mu_8^2) - \frac{1}{3}\mu_8^3, \quad (51)$$

which is consistent with the result of Ref. [5],  $\mu_8 \sim \mathcal{O}(\Delta^2/\mu)$ , in the case of  $\delta\mu = 0$ . The size of  $\Delta$  is essentially determined by tuning the NJL coupling constant  $G_\Delta$  and cutoff  $\Lambda$ .

After straightforward but tedious calculations of relevant one-loop diagrams from the fermion determinant in Eq. (13), we find the following relations in the region  $\delta\mu < \Delta$ :

$$M_B^2 = -\frac{\mu_8^2}{g^2} + \frac{\tilde{\mu}^2}{6\pi^2} \left( 1 - \frac{2\delta\mu^2}{\Delta^2} \right), \quad (52)$$

$$\lambda_{BC} = \frac{1}{80\pi^2} \frac{\tilde{\mu}^2}{\Delta^2} \left[ -1 + 8 \frac{\delta\mu^2}{\Delta^2} \left( 1 - \frac{\delta\mu^2}{\Delta^2} \right) \right], \quad (53)$$

$$\lambda_{CD} = -\frac{1}{g^2} - \frac{1}{18\pi^2} \frac{\tilde{\mu}^2}{\Delta^2}, \quad (54)$$

$$T = \frac{\mu_8}{2g^2} + \frac{\mu_8}{24\pi^2} \frac{\tilde{\mu}^2}{\Delta^2} \left( -1 + 8 \frac{\delta\mu^4}{\Delta^4} \right) + \frac{\tilde{\mu}}{48\pi^2} \left( -1 + 4 \frac{\delta\mu^2}{\Delta^2} + 8 \frac{\delta\mu^4}{\Delta^4} \right). \quad (55)$$

Here the tree contribution of gluons

$$V_g \equiv -\mathcal{L}_g = -\frac{1}{2}F_{0j}^{(\alpha)}F_{0j}^{(\alpha)} = -\frac{1}{2g^2}\mu_8^2 B^2 + \frac{1}{2g^2}\mu_8 DB^2 - \frac{1}{8g^2}B^2 D^2 - \frac{1}{2g^2}C^2 D^2 \quad (56)$$

was also taken into account.

We see that the coefficient  $\lambda_{CD}$  is definitely negative. The parameter  $M_B^2$ , which is expressed through the Meissner mass (14), is negative when

$$\delta\mu > \delta\mu_{\text{cr}}, \quad \delta\mu_{\text{cr}} = \frac{\Delta}{\sqrt{2}} \sqrt{1 - \frac{3\pi}{2\alpha_s} \frac{\mu_8^2}{\tilde{\mu}^2}}, \quad \alpha_s \equiv \frac{g^2}{4\pi}. \quad (57)$$

Relation (50) and Eq. (5) yield

$$\tilde{\mu} = \frac{9}{10}\mu + \frac{2}{5}\mu_8, \quad (58)$$

and, at the critical point, we find from Eqs. (50), (51) and (57), (58) that  $\mu_8$  is approximately

$$\mu_8 = \frac{3 - \ln \frac{200\Lambda^2}{9\tilde{\mu}^2}}{12 + \frac{4}{9} \left( \ln \frac{200\Lambda^2}{9\tilde{\mu}^2} - 2 \right)} \mu. \quad (59)$$

For realistic values  $\Lambda = (1.5 - 2.0)\mu$  and  $\alpha_s = 0.75 - 1.0$ , we obtain numerically

$$\frac{3\pi}{2\alpha_s} \frac{\mu_8^2}{\tilde{\mu}^2} = 0.03 - 0.1. \quad (60)$$

This implies that the tree gluon contribution decreases the value of  $\delta\mu_{\text{cr}}$  by 1.5%–5% in comparison to its value in the (non-gauged) NJL model. The smallness of this correction is in accordance with the dominance of hard-dense-loop diagrams.

Let us now turn to the coefficient  $\lambda_{BC}$  (53). At the critical point  $\delta\mu = \delta\mu_{\text{cr}}$ , it is:

$$\lambda_{BC} = \frac{1}{80\pi^2} \frac{\tilde{\mu}^2}{\Delta^2} \left( 1 - \frac{9\pi^2}{2\alpha_s^2} \frac{\mu_8^4}{\tilde{\mu}^4} \right). \quad (61)$$

Because the  $\mu_8^4/\tilde{\mu}^4$ -term is negligibly small, we conclude that the coefficient  $\lambda_{BC}$  is positive near the critical point. Thus, constraint (41) is satisfied indeed.

Utilizing Eqs. (52)–(55) in Eq. (42), one can obtain the solutions for  $B$ ,  $C$ , and  $D$  in the near-critical region. Indeed, neglecting higher order terms in  $\mu_8/\mu$  in (52)–(55), we get the approximate relations

$$M_B^2 \simeq \frac{\tilde{\mu}^2}{6\pi^2} \left( 1 - \frac{\delta\mu^2}{\delta\mu_{\text{cr}}^2} \right), \quad \lambda_{BC} \simeq \frac{9}{160\pi^2}, \quad \lambda_{CD} \simeq -\frac{1}{4\pi\alpha_s} - \frac{1}{4\pi^2}, \quad T \simeq \frac{\tilde{\mu}}{16\pi^2} + \frac{\mu_8}{16\pi^2} \left( 3 + \frac{2\pi}{\alpha_s} \right), \quad (62)$$

which lead us to the near-critical solution:

$$B_{\text{sol}} = \frac{\delta\mu^2 - \delta\mu_{\text{cr}}^2}{\delta\mu_{\text{cr}}^2} \frac{16\tilde{\mu} \sqrt{10 \left( 1 + \frac{\pi}{\alpha_s} \right)}}{27 \left[ 1 + \frac{\mu_8}{\mu} \left( 3 + \frac{2\pi}{\alpha_s} \right) \right]}, \quad (63)$$

$$C_{\text{sol}} = \frac{\sqrt{\delta\mu^2 - \delta\mu_{\text{cr}}^2}}{\delta\mu_{\text{cr}}} \frac{4\sqrt{5}\tilde{\mu}}{9}, \quad (64)$$

$$D_{\text{sol}} = \frac{\delta\mu^2 - \delta\mu_{\text{cr}}^2}{\delta\mu_{\text{cr}}^2} \frac{8\tilde{\mu}}{9 \left[ 1 + \frac{\mu_8}{\mu} \left( 3 + \frac{2\pi}{\alpha_s} \right) \right]}. \quad (65)$$

It is noticeable that this solution describes nonzero field strengths  $F_{\mu\nu}^{(\alpha)}$  which correspond to the presence of *non-abelian* constant chromoelectric-like condensates in the ground state:

$$E_3^{(2)} = F_{03}^{(2)} = \frac{1}{g} C_{\text{sol}} D_{\text{sol}}, \quad (66)$$

$$E_3^{(7)} = F_{03}^{(7)} = \frac{1}{2g} B_{\text{sol}} (2\mu_8 - D_{\text{sol}}). \quad (67)$$

We emphasize that while an abelian constant electric field in different media always leads to an instability,<sup>1</sup> non-abelian constant chromoelectric fields do not in many cases: For a thorough discussion of the stability problem for

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<sup>1</sup> In metallic and superconducting media, such an instability is classical in its origin. In semiconductors and insulators, this instability is manifested in an creation of electron-hole pairs through a quantum tunneling process.



constant  $SU(2)$  non-abelian fields in theories with zero baryon density, see Ref. [8]. On a technical side, this difference is connected with that while a vector potential corresponding to a constant abelian electric field depends on spatial and/or time coordinates, a constant non-abelian chromoelectric field is expressed through constant vector potentials, as takes place in our case, and therefore momentum and energy are good quantum numbers in the latter.

In order to illustrate the stability issue in the gluonic phase, let us consider the dispersion relations for quarks there. Because the vacuum expectation values (63)-(65) are small near the critical point and because red and green quarks are gapped in the 2SC phase, the dispersion relations for gapless blue up and down quarks are of the most interest. From Eq. (9) we find that up to the first order in  $B^2$  they are

$$p_{ub}^0 = |\vec{p}| - \mu_{ub} + \frac{B_{\text{sol}}^2}{4} \frac{1}{2|\vec{p}| + \mu_8 + \frac{\Delta^2}{2\tilde{\mu} - \mu_8}} - \frac{B_{\text{sol}}^2}{4} \frac{(p^3)^2}{\vec{p}^2} \left( \frac{1}{2|\vec{p}| + \mu_8 + \frac{\Delta^2}{2\tilde{\mu} - \mu_8}} + \frac{2(|\vec{p}| - \tilde{\mu}) + \mu_8}{\Delta^2 - \mu_8^2 - 2\mu_8(|\vec{p}| - \tilde{\mu})} \right), \quad (68)$$

$$p_{db}^0 = |\vec{p}| - \mu_{db} + \frac{B_{\text{sol}}^2}{4} \frac{1}{2|\vec{p}| + \mu_8 + \frac{\Delta^2}{2\tilde{\mu} - \mu_8}} - \frac{B_{\text{sol}}^2}{4} \frac{(p^3)^2}{\vec{p}^2} \left( \frac{1}{2|\vec{p}| + \mu_8 + \frac{\Delta^2}{2\tilde{\mu} - \mu_8}} + \frac{2(|\vec{p}| - \tilde{\mu}) + \mu_8}{\Delta^2 - \mu_8^2 - 2\mu_8(|\vec{p}| - \tilde{\mu})} \right). \quad (69)$$

The  $B^2$ -terms in Eqs.(68) and (69) lead to non-spherical Fermi surfaces determined by the following equations:

$$|\vec{p}| = \mu_{ub} - \frac{B_{\text{sol}}^2 \sin^2 \theta}{4} \frac{1}{2\mu_{ub} + \mu_8 + \frac{\Delta^2}{2\mu_{ub} + \mu_e + \mu_8}} - \frac{B_{\text{sol}}^2 \cos^2 \theta}{4} \frac{\mu_e + \mu_8}{\Delta^2 + \mu_8 \mu_e + \mu_8^2}, \quad (\text{blue up}) \quad (70)$$

$$|\vec{p}| = \mu_{db} - \frac{B_{\text{sol}}^2 \sin^2 \theta}{4} \frac{1}{2\mu_{db} + \mu_8 + \frac{\Delta^2}{2\mu_{db} - \mu_e + \mu_8}} + \frac{B_{\text{sol}}^2 \cos^2 \theta}{4} \frac{\mu_e - \mu_8}{\Delta^2 - \mu_8 \mu_e + \mu_8^2}, \quad (\text{blue down}) \quad (71)$$

where we neglected higher order terms of  $B^2$  and defined the angle  $\theta$ ,

$$p^3 \equiv |\vec{p}| \cos \theta. \quad (72)$$

The dispersion relations (68) and (69) clearly show that there is no instability in the quark sector in this problem.

As to bosonic degrees of freedom (gluons and composite bosons), because it is very involved to derive their derivative terms from the fermion loop in the gluonic phase, this issue is beyond the scope of this letter. It is however noticeable that there are no instabilities for bosons in a phase with vector condensates in the gauged  $\sigma$ -model with a chemical potential for hypercharge [3]. Although that model is much simpler than the present one, its phase with vector condensates has many common features with the gluonic phase and this fact is encouraging.

We emphasize that these constant color condensates in the gluonic phase do *not* produce long range color *forces* acting on quasiparticles. This can be seen from the dispersion relations (68) and (69) for quarks in this model. They show that momentum and energy are conserved numbers. It would be of course impossible in the presence of long range forces. The role of these condensates is actually more dramatic: They change the structure of the ground state, making it anisotropic and (electrically) superconducting. Only in this sense, one can speak about a long range character of the condensates.

## V. MORE ABOUT DYNAMICS IN THE GLUONIC PHASE

In this section, we will describe some additional features of the gluonic phase. In particular, we will point out that a condensation of *exotic* vector mesons takes place in this dense medium.

The gluonic vector condensates are mostly generated at energy scales between the confinement scale in the 2SC state, which is  $\lesssim 10$  MeV, and the baryon chemical potential, which is about 300-500 MeV. It is the same region where the chromomagnetic instability in the 2SC phase is created and where the hard dense loop approximation is (at least qualitatively) reliable. At such scales, gluons are still appropriate dynamical degrees of freedom and utilizing the Higgs approach with color condensates in a particular gauge is appropriate and consistent: It is a region of hard physics. Because the gluonic phase occurs as a result of a conventional second order phase transition, the vector

condensates are very small only in the immediate surroundings of the critical point  $\delta\mu = \Delta/\sqrt{2}$ . Outside that region, their values should be of the order of the typical scale  $\delta\mu \sim \Delta \sim 100$  MeV.

These condensates represent hard dynamics connected with the appearance of a new parameter, chemical potential for the electric charge  $\mu_e \sim 100$  MeV. As a result, the  $SU(2)_c$  gauge symmetry becomes completely broken and the strong coupling (confinement) dynamics presented in the 2SC solution at the scale of order 10 MeV is washed out. In other words, a conventional Higgs mechanism is realized in the gluonic phase. In this respect, the gluonic phase is similar to the color-flavor locked (CFL) phase, where the constant color condensates (although not vector ones) completely break the  $SU(3)_c$  color gauge symmetry [1].

It is easy to check that the electric charge  $\tilde{Q}$  and the baryon number  $\tilde{B}$  introduced in Section 3 are integer both for gluons and quarks. Do they describe hadronic-like excitations? We believe that the answer is “yes”. The point is that in models like this one, with a Higgs field in the fundamental representation of the gauge group, there is no phase transition between Higgs and confinement phases [9]. These two phases provide dual, and physically equivalent, descriptions of dynamics (the complementarity principle). In particular, they provide two complementary descriptions of a spontaneous breakdown of global symmetries, such as the rotational  $SO(3)$  and the electromagnetic  $U(1)$  in the present case. Following Ref. [9], one can apply the dual gauge invariant approach in this model and show that all the gluonic and quark fields can be replaced by colorless composite fields. The flavor numbers of these fields are described by the conventional electric and baryon charges  $Q$  and  $B$ . They are integer and coincide with those the operators  $\tilde{Q}$  and  $\tilde{B}$  yield for gluonic and quark fields.

While these issues will be considered in more detail elsewhere, here we would like to point out the following noticeable feature of these states: *some of them are exotic*. For example, the electric and baryon charges  $\tilde{Q}$  and  $\tilde{B}$  of  $A_\mu^{(+)} = A_\mu^{(1)} + iA_\mu^{(2)}$  gluons are equal to +1 and +2, respectively. Because  $A_3^{(+)}$  gluons are condensed in the gluonic phase, we conclude that in the dual gauge invariant description this corresponds to a condensation of *exotic* vector mesons. In this regard, it is appropriate to mention that some authors speculated about a possibility of a condensation of vector  $\rho$  mesons in dense baryon matter [10]. The dynamics in the gluonic phase yield a scenario even with a more unexpected condensation.

## VI. SUMMARY AND DISCUSSIONS

The gluonic phase whose existence was shown in this paper is very different from all known phases in dense quark matter discussed in the literature. Also, to the best of our knowledge, no phase like that has been considered in condensed matter physics. One of its features is the presence of *non-abelian* constant chromoelectric condensates in the ground state. They make the dynamics of the gluonic phase to be manifestly non-abelian. <sup>2</sup>

Because most of the initial symmetries in this system (including the rotational  $SO(3)_{\text{rot}}$  and the electromagnetic  $U(1)$ ) are spontaneously broken, the spectrum of excitations in the gluonic phase should be very rich. In particular, there should be two gapless Nambu-Goldstone (NG) modes connected with the two broken generators of the rotational group and one NG mode corresponding to the broken electric charge (the latter mode will be absorbed into photon field neglected in our model). <sup>3</sup> Another interesting feature of the gluonic phase is that there are excitations corresponding to exotic hadrons. We are planning to return to this issue elsewhere.

The solution (63)–(65) described in this paper corresponds to a minimum of the effective potential. Whether or not this minimum is global is an open question. We suspect that there may exist a class of solutions with vector condensates. In this regard, it is instructive to describe the Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) phase [12] from this point of view. It is easy to show that the LOFF solution with one plane wave along, say, third spatial coordinate can be gauge transformed into a solution with a usual (homogeneous) diquark condensate *and* a vector

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<sup>2</sup> The role of constant chromomagnetic non-abelian fields in the dynamics of color superconductivity was studied in Ref. [11]. It was shown that they could enhance the value of a diquark condensate. Unlike the gluonic phase, where non-abelian chromoelectric condensates are solutions of dynamical equations, the chromomagnetic fields in [11] are external.

<sup>3</sup> The spectrum of excitations in the gauged  $\sigma$ -model with hypercharge chemical potential [3], where vector condensates also occur, strongly supports these expectations.

condensate  $\langle A_3^{(8)} \rangle$ . [Note that there is no such a transformation in the case of the LOFF solution with two or more plane waves [13].] Unlike the present gluonic solution, there are no non-abelian chromoelectric condensates in that case, and, therefore, the LOFF dynamics is not genuinely non-abelian. Still, because the LOFF solution has been recently used to cure a chromomagnetic instability in the neutral 2SC phase [14], it would be worth to study a possibility of extending the present solution by including the VEV  $\langle A_3^{(8)} \rangle$ .<sup>4</sup>

Last but not least, it would be interesting to check the possibility of the existence of a gluonic phase in three-flavor dense quark matter. It is especially interesting because a chromomagnetic instability has been recently revealed also in that case [16].

### Acknowledgments

We thank K. Rajagopal and I. Shovkovy for useful discussions. The work was supported by the Natural Sciences and Engineering Research Council of Canada.

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<sup>4</sup> A model for describing dynamics for larger  $\delta\mu \simeq \Delta$ , close to the edge of the gapless phase [7], has been recently considered in Ref. [15]. It is an abelian gauge model with a photon gauge field having a constant VEV. By using an appropriate gauge transformation, one can remove this VEV, with a cost of introducing a phase factor in the order parameter, as it takes place in the LOFF state.