

Isospin odd πK scattering length

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Abstract

We make use of the chiral two-loop representation of the πK scattering amplitude [J. Bijnens, P. Dhonte and P. Talavera, JHEP 0405 (2004) 036] to investigate the isospin odd scattering length at next-to-next-to-leading order in the SU(3) expansion. This scattering length is protected against contributions of m_s in the chiral expansion, in the sense that the corrections to the current algebra result are of order M_π^2 . In view of the planned lifetime measurement on πK atoms at CERN it is important to understand the size of these corrections.

Key words: Chiral symmetries, Meson-meson interactions

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1 Introduction

In the sixties and seventies a set of experiments was performed on πK scattering [1]. To obtain predictions for the low-energy parameters, the measured πK phases had to be extrapolated using dispersion relations and crossing symmetry [2], since the region of interest is not directly accessible by scattering experiments. The most precise values for the πK scattering lengths were obtained only recently from an analysis of Roy-Steiner equations [3,4]. Alternatively, particular combinations of πK scattering lengths may be extracted from experiments on πK atoms [5,6,7]. The πK atom decays due to the strong interactions into $\pi^0 K^0$ and a lifetime measurement will allow one to determine the isospin odd S-wave πK scattering length $a_0^- = 1/3(a_0^{1/2} - a_0^{3/2})$. Such a measurement is planned at CERN [8]. Particularly interesting about

the isospin odd πK scattering length is that there exists a low-energy theorem due to Roessl [9]. Based on SU(2) chiral perturbation theory (CHPT) [9,10,11,12], where the strange quark mass is treated as a heavy partner, it is valid to all orders in powers of m_s . It states that Weinberg's current algebra result [13,14] receives corrections of order M_π^2 only,

$$a_0^- = \frac{M_\pi M_K}{8\pi F_\pi^2 (M_\pi + M_K)} \left\{ 1 + \mathcal{O}(M_\pi^2) \right\}. \quad (1)$$

Here M_π , M_K and F_π denote the physical meson masses and the physical pion decay constant. In view of this low-energy theorem, one would expect higher order corrections to the scattering length to be relatively small. These days, the πK scattering amplitude is available at next-to-next-to-leading order [15,16,17,18] in SU(3) CHPT [19]. The one-loop corrections [15,16,17] to a_0^- turn out as expected, they change the current algebra value at the 11% percent level. Surprisingly, for the two-loop corrections this seems not to be the case. According to the numerical study performed in Ref. [18], the scattering length a_0^- receives at order p^6 a 14% correction. The aim of the present article is to understand the nature of these rather substantial contributions at two-loop order. Other recent work on πK scattering makes use of resonance chiral Lagrangian predictions [20] together with resummations [21]. There were also earlier attempts at unitarisation of current algebra for this process, see Ref. [22] and references therein.

We use the chiral two-loop representation for the πK amplitude [18] to investigate the order p^6 corrections to a_0^- . In Section 2, we extract the contributions from the low-energy constants and determine the double chiral logs as well as the $\log \times L_i^r$ terms by means of the renormalization group equations for the renormalized coupling constants [23]. Further, we specify the 1-loop $\times L_i^r$ terms in an expansion in powers of M_π/M_K . The numerical analysis is carried out in Section 3 and the results for the partial two-loop contributions are collected in Table 2.

2 'Low cost' terms at two-loop order

The SU(3) chiral expansion of the isospin odd πK scattering length looks as follows

$$a_0^- = \frac{M_\pi M_K}{8\pi F_\pi^2 (M_\pi + M_K)} \left\{ 1 + \delta^{(2)} + \delta^{(4)} + \mathcal{O}(p^6) \right\}, \quad (2)$$

where $\mathcal{O}(p^6) = \{\hat{m}^3, \hat{m}^2 m_s, \hat{m} m_s^2\}$. The scattering length is expressed in terms of the physical meson masses M_π and M_K and the physical pion decay constant F_π [24]. The next-to-leading order contribution $\delta^{(2)}$ [16,17] depends on one

single low-energy constant L_5^r [19] only,

$$\begin{aligned} \delta^{(2)} = & \frac{M_\pi^2}{32\pi^2 F_\pi^2} \left[256\pi^2 L_5^r - 3 \ln \frac{M_K^2}{\mu^2} - \frac{3(2M_K^2 - M_\pi^2)}{M_K^2 - M_\pi^2} \ln \frac{M_\pi^2}{M_K^2} \right. \\ & \left. - \frac{4M_K^2 - M_\pi^2}{2(M_K^2 - M_\pi^2)} \ln \frac{4M_K^2 - M_\pi^2}{3M_\pi^2} \right] + \frac{M_\pi M_K}{3F_\pi^2} \\ & \times \left[\bar{J}(s_{\text{thr}}, M_K^2, \frac{1}{3}(4M_K^2 - M_\pi^2)) - \bar{J}(u_{\text{thr}}, M_K^2, \frac{1}{3}(4M_K^2 - M_\pi^2)) \right], \quad (3) \end{aligned}$$

where $s_{\text{thr}} = (M_\pi + M_K)^2$, $u_{\text{thr}} = (M_K - M_\pi)^2$ and the function \bar{J} is defined as follows

$$\begin{aligned} \bar{J}(p^2, m_1^2, m_2^2) &= J(p^2, m_1^2, m_2^2) - J(0, m_1^2, m_2^2), \\ J(p^2, m_1^2, m_2^2) &= -i \int \frac{d^d q}{(2\pi)^d} (m_1^2 - q^2)^{-1} (m_2^2 - (p+q)^2)^{-1}. \quad (4) \end{aligned}$$

Note that at the order considered it makes a difference whether we represent $\delta^{(2)}$ as a function of the physical pion, kaon and η masses or express one of them through the other two¹. In Eq. (3), we choose to describe $\delta^{(2)}$ in terms of the physical pion and kaon mass only, because this ensures that both $\delta^{(2)}$ and $\delta^{(4)}$ are independently scale invariant.

The two-loop order correction can be decomposed as

$$\delta^{(4)} = \delta_{L_i=C_i=0}^{(4)} + \delta_{1\text{-loop}L_i}^{(4)} + \delta_{L_i L_j}^{(4)} + \delta_{C_i}^{(4)}. \quad (5)$$

The first term contains the two-loop functions, the second one-loop functions with insertions of $\mathcal{O}(p^4)$ coupling constants and the last two terms consist of counter term contributions. Some of the two-loop functions in $\delta_{L_i=C_i=0}^{(4)}$ are very demanding to analyze analytically. For the moment, we thus restrict ourselves to the chiral double logs,

$$\delta_{L_i=C_i=0}^{(4)} = \delta_{\log^2}^{(4)} + \delta_{\text{rem}}^{(4)}, \quad (6)$$

and neglect the remainder $\delta_{\text{rem}}^{(4)}$ which is given numerically in Table 2. In a first step, we extract the contributions from the p^6 low-energy constants (C_i^r) [23,25] from the representation of the πK scattering amplitude in Ref. [18],

$$\begin{aligned} \delta_{C_i}^{(4)} = & \frac{16M_\pi^2}{F_\pi^2} \left[-2M_K^2 (C_1^r - 2C_3^r - 4C_4^r - C_{14}^r - C_{15}^r + 2C_{22}^r \right. \\ & \left. - 2C_{25}^r - C_{26}^r + 2C_{29}^r) + M_\pi^2 (C_{15}^r + 2C_{17}^r) \right], \quad (7) \end{aligned}$$

¹ This will generate a correction proportional to $\Delta_{\text{GMO}} \equiv (4M_K^2 - M_\pi^2 - 3M_\eta^2)/(M_\eta^2 - M_\pi^2)$ [19] which contributes to $\delta^{(4)}$.

as well as products of two p^4 constants ($L_i^r \times L_j^r$),

$$\delta_{L_i L_j}^{(4)} = \frac{64 M_\pi^2 L_5^r}{F_\pi^4} \left\{ M_\kappa^2 [2(L_4^r - 2L_6^r) - L_5^r] + M_\pi^2 [L_4^r - 2L_6^r + 2(L_5^r - L_8^r)] \right\}. \quad (8)$$

In order to determine the chiral double logs and the $\log \times L_i^r$ terms, we consider the renormalization group equations of the renormalized order p^4 and p^6 low-energy constants [23],

$$\mu \frac{dL_i^r(\mu)}{d\mu} = -\frac{1}{(4\pi)^2} \Gamma_i, \quad \mu \frac{dC_i^r(\mu)}{d\mu} = \frac{1}{(4\pi)^2} [2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu)]. \quad (9)$$

The coefficients $\Gamma_i^{(L)}$ are linear combinations of p^4 constants which satisfy the following differential equations,

$$\mu \frac{d\Gamma_i^{(L)}(\mu)}{d\mu} = -\frac{\Gamma_i^{(2)}}{8\pi^2}, \quad (10)$$

in accordance with Weinberg's consistency conditions [10]. The coefficients $\Gamma_i^{(1)}$, $\Gamma_i^{(2)}$ and $\Gamma_i^{(L)}(\mu)$ are listed in Table II of Ref. [23]. The solutions of the renormalization group equations read [26]

$$\begin{aligned} L_i^r(\mu) &= L_i^r(\mu_0) - \frac{\Gamma_i}{2} L(\mu/\mu_0), \\ C_i^r(\mu) &= C_i^r(\mu_0) - \frac{1}{4} \Gamma_i^{(2)} L(\mu/\mu_0)^2 + \frac{1}{2} [2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu_0)] L(\mu/\mu_0), \end{aligned} \quad (11)$$

with the chiral logarithm

$$L(\mu/\mu_0) = \frac{1}{(4\pi)^2} \ln \frac{\mu^2}{\mu_0^2}. \quad (12)$$

As a two-loop order quantity $\delta^{(4)}$ consists of

$$\delta^{(4)} = \hat{a}(\mu) + \sum_i b_i C_i^r(\mu) + \sum_{i,j} b_{ij} L_i^r(\mu) L_j^r(\mu), \quad (13)$$

where $\hat{a}(\mu)$ is scale dependent and contains one-loop functions with insertions of p^4 constants as well as two-loop functions. In order to extract the double log and $\log \times L_i^r$ contributions from $\hat{a}(\mu)$, we insert the solutions for the renormalized coupling constants into the latter equation,

$$\begin{aligned} \delta^{(4)} &= \hat{a}(\mu_0) + \sum_i b_i C_i^r(\mu_0) + \sum_{i,j} b_{ij} L_i^r(\mu_0) L_j^r(\mu_0), \\ \hat{a}(\mu_0) &= \hat{a}(\mu) - \frac{1}{4} L(\mu/\mu_0)^2 [b_i \Gamma_i^{(2)} - b_{ij} \Gamma_i \Gamma_j] \\ &\quad + \frac{1}{2} L(\mu/\mu_0) [b_i (2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu_0)) - 2b_{ij} \Gamma_i L_j^r(\mu_0)]. \end{aligned} \quad (14)$$

Now, the scale dependence of $\hat{a}(\mu_0)$ becomes apparent and we may read off the wanted \log^2 and $\log \times L_i^r$ terms. The solutions of the renormalization group equations thus allow us to determine the double log and $\log \times L_i^r$ contributions from Eqs. (7) and (8).

The double chiral logs (\log^2) amount to

$$\delta_{\log^2}^{(4)} = \frac{M_\pi^2}{F_\pi^4} \left[\frac{37M_K^2}{8} + \frac{59M_\pi^2}{24} \right] L(M_\chi/\mu)^2, \quad (15)$$

while the single logarithms times p^4 constants ($\log \times L_i^r$) yield

$$\begin{aligned} \delta_{\log L_i}^{(4)} = \frac{-2M_\pi^2}{3F_\pi^4} \{ & M_K^2 [84L_1^r + 114L_2^r + 53L_3 - 96L_4^r - 28L_5^r \\ & + 48(3L_6^r + L_7 + 2L_8^r)] - M_\pi^2 [12L_1^r + 30L_2^r \\ & + 19L_3 - 64L_5^r + 24(2L_7 + L_8^r)] \} L(M_\chi/\mu). \end{aligned} \quad (16)$$

Here M_χ stands for a characteristic meson mass.

In the remaining part of this section, we investigate Roessl's low-energy theorem [9] at next-to-next-to-leading order in SU(3) CHPT. More precisely, we specify the order M_π^2 and order M_π^4 corrections to Eq. (1). To approach the SU(2) chiral expansion, we regard the kaon mass as heavy and expand a_0^- in powers of M_π/M_K ,

$$a_0^- = \frac{M_\pi M_K}{8\pi F_\pi^2 (M_\pi + M_K)} \left\{ 1 + M_\pi^2 c_2 + M_\pi^4 c_4 + \mathcal{O}(M_\pi^6) \right\}. \quad (17)$$

Again, the quantities M_π , M_K and F_π stand for the physical masses and the physical pion decay constant [24]. At next-to-next-to-leading order in SU(3) CHPT, the coefficient c_2 depends on L_5^r [16,17],

$$\begin{aligned} c_2 |_{1\text{-loop}} = \frac{1}{F_\pi^2} \left\{ & 8L_5^r - \frac{1}{32\pi^2} \left[3 \ln \frac{M_K^2}{\mu^2} + 4 \ln \frac{M_\pi^2}{M_K^2} \right] \right. \\ & \left. + \frac{1}{144\pi^2} \left[-12 + 10\sqrt{2} \arctan \sqrt{2} - 7 \ln \frac{4}{3} \right] \right\}, \end{aligned} \quad (18)$$

while the one-loop contributions to c_4 do not contain any low-energy constants and can safely be neglected numerically. At next-to-next-to-leading order in the chiral SU(3) expansion, the contributions from counter terms, double chiral logs and $\log \times L_i^r$ terms to the coefficients c_2 and c_4 are specified in Eqs. (7), (8), (15) and (16). In addition, we list the expansion of the one-loop functions

	CA	SU(2) [9]	p^4 SU(3) [16]	p^6 SU(3) [18]	Ref. [4]
$M_\pi a_0^-$	0.071	$0.077 \pm 0.003^*$	0.0793 ± 0.0006	0.089	0.090 ± 0.005

Table 1

Isospin odd scattering length a_0^- : CA current algebra value, SU(2) prediction [9], chiral SU(3) prediction at order p^4 [16] and order p^6 [18], dispersive analysis from Roy-Steiner equations [4]. *Note that in Ref. [9] $M_\pi = 137.5$ MeV and $M_K = 495.5$ MeV, while all other references use $M_\pi \doteq M_{\pi^+}$ and $M_K \doteq M_{K^+}$ for the pion and kaon masses in the isospin symmetry limit.

with insertions of p^4 couplings in powers of M_π/M_K . We have

$$\begin{aligned}
c_2 |_{1\text{-loop}L_i} = & \frac{M_K^2}{12\pi^2 F_\pi^4} \left\{ -\frac{1}{2} [84L_1^r + 114L_2^r + 53L_3 - 96L_4^r - 28L_5^r \right. \\
& + 48(3L_6^r + L_7 + 2L_8^r)] \ln \frac{M_K^2}{\mu^2} \\
& - \frac{4}{27} L_3 \left[56\sqrt{2} \arctan \sqrt{2} - 5 \ln \frac{4}{3} \right] \\
& - \frac{1}{3} [L_5^r - 6(2L_7 + L_8^r)] \left[13\sqrt{2} \arctan \sqrt{2} + 2 \ln \frac{4}{3} \right] + 93L_1^r \\
& \left. + \frac{189}{2} L_2^r + \frac{2045}{36} L_3 - 16 [L_5^r + 6(L_4^r - L_6^r + L_7)] \right\}, \quad (19)
\end{aligned}$$

and

$$\begin{aligned}
c_4 |_{1\text{-loop}L_i} = & \frac{1}{8\pi^2 F_\pi^4} \left\{ \frac{1}{3} [12L_1^r + 30L_2^r + 19L_3 - 64L_5^r \right. \\
& + 24(2L_7 + L_8^r)] \ln \frac{M_K^2}{\mu^2} + 4[8L_1^r + 12L_2^r + 6L_3 - 8L_4^r \\
& - 9L_5^r + 6(2L_6^r + L_8^r)] \ln \frac{M_\pi^2}{M_K^2} - \frac{\sqrt{2}}{8} \left[\frac{1840}{81} L_3 - \frac{1415}{18} L_5^r \right. \\
& \left. + 45(2L_7 + L_8^r) \right] \arctan \sqrt{2} + \frac{4}{9} \left[\frac{2}{9} L_3 - 17L_5^r \right. \\
& \left. + 18(2L_7 + L_8^r) \right] \ln \frac{4}{3} - \frac{1}{4} \left[8L_1^r + 4L_2^r - \frac{410}{27} L_3 \right. \\
& \left. + \frac{323}{6} L_5^r - 67(2L_7 + L_8^r) \right] \left. \right\}, \quad (20)
\end{aligned}$$

where we have checked that the $\log \times L_i^r$ terms agree with Eq. (16). Here both the contributions to $M_\pi^2 c_2$ and $M_\pi^4 c_4$ are numerically sizeable, see Table 2.

a	$\delta_a^{(4)}$	$M_\pi^2 c_2 _a$	$M_\pi^4 c_4 _a$	$\beta _a$
$L_i = C_i = 0$	0.05*	-	-	-
\log^2	0.010	0.010	0.0004	3.7
1-loop L_i	0.013	0.007	0.006	2.9
$L_i L_j$	-0.004	-0.004	0.0002	-1.5
C_i	0.08 [†]	0.08	0	30.6
rem	0.04	-	-	-

Table 2

Numerical results for the p^6 contributions at the scale $\mu = 770$ MeV: * pure loop contributions and [†] resonance estimate are taken from Ref. [18]. The notation is understood as in Eq. (5). For instance the contributions of the 1-loop $\times L_i^r$ terms to $\delta^{(4)}$ is given by $\delta_{1\text{-loop}L_i}^{(4)} = 0.013$.

3 Numerical analysis

In the following, we present the numerical results for the partial p^6 corrections to $\delta^{(4)}$. The pion and kaon mass in the isospin symmetry limit are identified with their charged masses $M_\pi \doteq M_{\pi^+}$ and $M_K \doteq M_{K^+}$. To be consistent with the numerical analysis performed in Ref. [18], we use for the pion decay constant ² $F_\pi = 92.4$ MeV. In Table 1, we list the various numerical results for a_0^- available in the literature. The first row contains the current algebra value, the next number is the SU(2) prediction at next-to-leading order [9], row three and four display the order p^4 [16] and order p^6 [18] SU(3) predictions and the last value is based on a phenomenological analysis from Roy-Steiner equations [4]. As can be read off, the SU(3) prediction at order p^6 is in good agreement with the Roy-Steiner value. The SU(3) chiral expansion of the scattering length a_0^- looks as follows

$$\begin{aligned} \frac{8\pi F_\pi^2 (M_\pi + M_K)}{M_K M_\pi} a_0^- &= 1 + \delta^{(2)} + \delta^{(4)} + \dots \\ &= 1 + 0.11 + 0.14 + \dots \end{aligned} \quad (21)$$

The one-loop contribution $\delta^{(2)}$ changes the current algebra result at the 11% level, while the two-loop contributions $\delta^{(4)}$ amount to a 14% correction. The aim was to understand this rather large order p^6 correction and our insights are collected in Table 2 which contains a splitting up of the various contributions at two-loop order.

For the low-energy constants L_i^r at the scale $\mu = 770$ MeV (M_ρ), we use fit

² Recently, a new value was obtained $F_\pi = 92.2 \pm 0.2$ MeV [27].

	$L_i = C_i = 0$	1-loop L_i	$L_i L_j$	C_i
$\Delta\delta_a^{(4)}$	-0.03	0.02	-0.01	0.02

Table 3

Variations of the partial p^6 contributions to $\delta^{(4)}$ for $M_\eta \leq \mu \leq 770$ MeV (M_ρ). More precisely, we display the difference $\Delta\delta_a^{(4)} = \delta_a^{(4)}|_{\mu=M_\eta} - \delta_a^{(4)}|_{\mu=M_\rho}$. For the notation, see Table 2.

10 of Ref. [28]. The double chiral logs are evaluated for a characteristic meson mass³ $M_\chi = M_K$ and the size of the remainder $\delta_{\text{rem}}^{(4)}$ is estimated by the use of Eq. (6). Row two and three of Table 2 contain the partial order p^6 corrections to the coefficients c_2 and c_4 , respectively. Note that for the double chiral logs as well as for the products of p^4 constants their contribution to c_4 can be neglected while for the one-loop functions with insertions of L_i^r 's, both $M_\pi^2 c_2$ and $M_\pi^4 c_4$ are numerically sizeable. The enhancement of the coefficient c_4 is mainly due the contributions proportional to $\ln M_\pi/M_K$, see Eq. (20).

As one can read off from Table 2, more than half of the contributions to $\delta^{(4)} = 0.14$ stem from the resonance estimate for the p^6 constants which includes effects of the lowest-lying vector and scalar resonances [18]. We checked that with this procedure the meson resonance exchange contributions to C_{15}^r and C_{17}^r vanish which implies that $c_4|_{C_i}$ is equal to zero. Further, for the combination of p^6 constants occurring in $c_2|_{C_i}$, the contributions from scalar resonances do not play a dominant role: They amount to 0.03 of the 0.08 generated by the C_i^r 's in total. It would be instructive to see whether these features persist in an improved estimate for the p^6 constants which respects the constraints that follow by imposing the proper asymptotic behaviour for massless QCD [29].

The splitting of the order p^6 contributions in Table 2 is scale dependent. Table 3 displays the scale dependence of the various contributions to $\delta^{(4)}$. The values for the 1-loop $\times L_i^r$, $L_i^r \times L_j^r$ and C_i^r terms at the scales $\mu = 770$ MeV and $\mu = M_\eta$ allow us to read off the scale dependence of the pure loop contributions $\delta_{L_i=C_i=0}^{(4)}$.

Finally, we sum up the various SU(3) one- and two-loop contributions to c_2 and c_4 and get for the expansion of a_0^- in powers of M_π/M_K ,

$$\begin{aligned} \frac{8\pi F_\pi^2(M_\pi + M_K)}{M_\pi M_K} a_0^- &= 1 + M_\pi^2 c_2 + M_\pi^4 c_4 + \dots \\ &= 1 + 0.2 + 0.01 + \delta_{\text{rem}}^{(4)} + \dots \end{aligned} \quad (22)$$

³ The choice $M_\chi = \sqrt{M_\pi M_K}$ leads to an unnatural large number for the double logs $\delta_{\log^2}^{(4)} = 0.058$, to be compared with the full pure loop corrections $\delta_{L_i=C_i=0}^{(4)} = 0.05$ [18]. For $M_\chi = M_\pi$ the value becomes even more unreasonable.

Note that this decomposition is valid up to the contribution of $\delta_{\text{rem}}^{(4)} = 0.04$ only. Compared to the chiral SU(3) expansion in Eq. (21), the series in M_π/M_K converges much more rapidly. The correction $M_\pi^2 c_2$ consists of

$$M_\pi^2 c_2 = \frac{M_\pi^2}{(4\pi F_\pi)^2} \left[\alpha + \frac{M_K^2}{(4\pi F_\pi)^2} \beta + \dots \right], \quad (23)$$

where the coefficients α and β contain the one-loop and two-loop contributions, respectively. Numerically, we have $\alpha = 7.6$, where the dominant part stems from the term proportional to $\ln M_\pi/M_K$ in Eq. (18). The contributions from double logs, 1-loop $\times L_i$ terms and p^6 constants to β are listed in Table 2. Here the bulk part comes from the resonance estimate for the p^6 constants [18].

4 Conclusions

In the present work, we used the chiral two-loop representation for the πK amplitude available in the literature [18] to investigate the isospin odd S-wave scattering length a_0^- . This scattering length differs from other low-energy parameters in πK scattering in the sense that contributions of m_s in the chiral expansion are suppressed by powers of \hat{m} . Based on SU(2) CHPT [9], there exists a low-energy theorem (1) which states that the current algebra result for a_0^- receives corrections of order M_π^2 only. It was therefore expected that the one-loop result [15,16,17] in SU(3) CHPT represents a decent estimate for the scattering length. However, the dispersive analysis from Roy-Steiner equations [4] and the chiral two-loop calculation [18] are not in agreement with this expectation. In fact, the numerical analysis performed in Ref. [18] showed that the two-loop order corrections to a_0^- are of the same order of magnitude as the one-loop contributions.

In order to understand this rather substantial next-to-next-to-leading order correction, we determined analytically the contributions containing p^6 constants (7), products of two p^4 constants (8), double chiral logs (15) and single logarithms times p^4 constants (16). We further expanded the one-loop functions with insertions of p^4 constants in powers of M_π/M_K , see Eqs. (19) and (20). The expansion of the pure two-loop functions in powers of M_π/M_K was beyond the scope of this work. The numerical values of the partial p^6 contributions are collected in Table 2.

In the remaining part of this work, we investigated the low-energy theorem for a_0^- at next-to-next-to-leading order in the SU(3) expansion. While it is true that the corrections are of order M_π^2 , the chiral expansion of the accompanying coefficient proceeds in powers of M_K and is not protected against

sizeable contributions. At two-loop accuracy in the SU(3) expansion, the order M_π^2 correction roughly amounts to about 20%, see Eq. (22). Note that this number depends on the resonance estimate [18] for the p^6 constants. If we compare this result with Roessl's value [9], the SU(2) prediction for the scattering length a_0^- seems to be underestimated. At first surprisingly, we have to keep in mind that the numerical estimates for the low-energy constants in SU(2) CHPT were obtained through matching the scattering amplitude with the corresponding SU(3) CHPT result at one-loop order. It would be very interesting to estimate these low-energy constants using a resonance saturation approach in the context of SU(2) CHPT with strangeness number 1.

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