Thermalisation in Thick Wall Electroweak Baryogenesis

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In models of thick wall electroweak baryogenesis a common assumption is that the plasma interacting with the expanding Higgs bubble wall during the electroweak phase transition is in kinetic equilibrium (or close to it). We point out that, in addition to the requirement of low wall velocity, kinetic equilibrium requires that the change in the momentum of the particles due to the force exerted by the wall should be much less than that due to scattering as the plasma passes through the wall. We investigate whether this condition is satisfied for charginos and neutralinos participating in thick wall supersymmetric electroweak baryogenesis

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Standard electroweak baryogenesis models attempt to create the observed baryon asymmetry of the universe during the electroweak phase transition by the interaction of the expanding Higgs bubble wall with the ambient plasma. In models in which the thickness of the bubble wall is greater than the mean free path of particles passing through the bubble wall the Higgs field is treated as a classical background field for the plasma traversing the wall. These thick wall electroweak baryogenesis models make certain assumptions regarding the thermal nature of the plasma. In particular, it is assumed that the plasma is in kinetic equilibrium (or close to it) as it passes through the wall. In this Brief Report we shall highlight a condition for the validity of this assumption that has not been discussed elsewhere while estimating the baryon asymmetry.

The thermal assumption in thick wall electroweak baryogenesis models is indicated by the form of the perturbed thermal particle distribution function substituted in the kinetic equation in the semi-classical force mechanism of Refs. [1, 2, 3, 4, 5], in the adoption of a thermal density matrix (for calculating various sources) and the diffusion approximation ($J = -D(T)\nabla n$) in the wall in Ref. [6], and in the choice of thermal equilibrium Green's functions while evaluating the source in the Closed Time Path formalism of Refs. [7, 8, 9, 10, 11]. The thermal assumption is justified by arguing that departures from kinetic equilibrium are small for $v_w \ll 1$ or $\Gamma l_w/v_w \gg 1$, where l_w and v_w are the wall width and velocity and Γ is the rate of interactions that maintain kinetic equilibrium. This picture of maintaining kinetic equilibrium inside the bubble wall is valid only if the momentum transferred to the particles due to their interaction with the wall is very small compared to the effect of collisions. Below we argue that satisfying this requirement provides an additional condition, *independent of the wall velocity*.

Unless the change in momentum of the particles due to scattering in the plasma is much greater than the change due to the action of the background Higgs field, the particles in the bubble wall will acquire a directed velocity and will no longer have a randomly directed particle velocity distribution, as in kinetic equilibrium. In plasmas where a background electric field acts over the entire bulk of the plasma, this happens for a fraction of the particles and they get constantly accelerated by the background field. A thermal distribution or a perturbed thermal distribution can not be used to describe the distribution of these particles. This phenomenon is known as 'runaway' [12, 13]. For our case, the background Higgs field acts over a small region (the wall) and particles will not be indefinitely accelerated. Nevertheless, if scattering does not dominate over the background field kinetic equilibrium will not be maintained in the bubble wall. Below we discuss whether this may occur for charginos and neutralinos interacting with the Higgs wall in supersymmetric electroweak baryogenesis. [1]

The Lagrangian describing the interaction of particles (fermions) with the Higgs bubble wall can be modeled by

$$\mathcal{L} = i\bar{\psi}\,\partial\!\!\!/\,\psi + \frac{1}{2}\partial_\mu\theta\bar{\psi}\gamma^\mu\gamma^5\psi - \frac{m}{\hbar}\bar{\psi}\psi \qquad (1)$$

The Higgs bubble wall is treated as a background field which provides a spatially varying mass for the particles, and a term associated with the axial current, in the bubble wall frame. In the limit of large bubbles the wall can be treated as planar and m and θ are functions of

[1] After completing this work we found a similar discussion of thermalisation of top quarks in the bubble wall in the context of an analysis of the electroweak phase transition [14].

z. This gives rise to a z dependent force F on particles in the wall given to $O(\hbar^0)$ by $dp_z/dt = -m^{2'}/2E$, where $E = (p^2+m^2)^{1/2}$. (The wall distinguishes between left- and right-handed particles only at $O(\hbar)$ [2, 15, 16, 17].) We shall assume that in the plasma frame the wall is moving to the right in the positive z direction, and so in the wall frame the plasma is moving to the left. Rewriting the force as $v\, dp_z/dz$, where $v = dz/dt = p_z/E$, and assuming that the mass changes uniformly across the wall, the force equation can be integrated to give

$$\Delta p_z^2 = -m^{2'} \Delta z\,. \tag{2}$$

To later facilitate comparison with the change in momentum due to scattering, we shall take particles to be moving in the z direction and $\Delta z = l$, where $l(p)$ is the mean free path. We take $m^{2'} = -M^2/l_W$, where M^2 is the change in the mass squared across the wall. Therefore, the magnitude of change in the momentum of a particle over one mean free path due to the background field in the wall is

$$(\Delta p_z)_F = \frac{M^2}{p_1+p_2}\frac{l(p_1)}{l_W} \tag{3}$$

where $p_{1,2}$ are the initial and final momenta for the motion over $l(p_1)$.

Let us take the change in the momentum of a particle due to scattering that maintains kinetic equilibrium to be $(\Delta p_z)_{sc}$. Runaway, as discussed in Refs. [12, 13], is generally obtained for high momentum particles. For such particles, their initial momentum in a scattering event is greater than the typical momentum of the particle they scatter off and the change in momentum is taken to be of the order of the initial momentum. Below we shall consider particles with the mean thermal momentum and they largely scatter off particles with momentum $\sim T$. We take $(\Delta p_z)_{sc} \sim T$. [2] Therefore,

$$\mathcal{R} \equiv \frac{(\Delta p_z)_F}{(\Delta p_z)_{sc}} = \frac{M^2}{T(p_1+p_2)}\frac{l(p_1)}{l_W}\,. \tag{4}$$

If $\mathcal{R} \ll 1$ then the thermalisation assumption in the wall is valid. In such a case one may assume that kinetic equilibrium is established in the wall. However, if this condition is not satisfied then the thermalisation assumption is not valid.

If one models the bubble wall profile by a tanh function, i.e., $m^2(z) = m_0^2 + M^2/2 - M^2/2\tanh(z/l_W + 1/2)$, then our expression for $\mathcal{R}$ is multiplied by a factor of $0.5\,\mathrm{sech}^2(z/l_W + 1/2)$ which is of the order of 1/2 in the region of the wall ($-l_W \le z \le 0$). (m_0 is any mass in the unbroken phase.)

We are studying thick wall electroweak baryogenesis for which $l_W = (10-100)/T$. We consider a particle moving in the direction of increasing mass, and so $p_2 < p_1$. Below we take $p_2 = p_1$ in Eq. (4) for the most conservative test for thermalisation. Our strategy is to presume p_1 is the mean thermal momentum and to check for consistency of the thermalisation condition $\mathcal{R} \ll 1$. We now calculate $l(p_1)$.

The mean free path is given by $1/(n\sigma)$, where n is the number density of the species participating in electroweak baryogenesis (charginos and neutralinos) and σ is the dominant cross-section. It is the Higgsino components of the charginos and neutralions that plays an important role in electroweak baryogenesis. Below we first ignore mixing and work with charged and neutral Higgsino eigenstates of mass μ_T. This is equivalent to presuming that the contribution of the Higgs vev dependent terms in the mass matrix of charginos and neutralinos (Eqs. (C9) and (C38) of Ref. [18]) are small which should be valid in the outer regions of the wall or if the μ term contribution in the mass matrix is dominant. For us μ_T includes the vacuum Higgsino mass, μ, and thermal corrections. The further change in mass squared across the wall is M^2, not to be confused with the gaugino mass M in Ref. [18]. The Higgs vev dependent terms in the mass matrix which are responsible for M^2 will also mix the Higgsinos with gauginos. While considering pure Higgsino states we ignore this mixing. Later we shall include large mixing between Higgsinos and gauginos and consider whether thermalisation is valid.

Working with Higgsino eigenstates, the mean free path for relativistic particles may be taken to be the inverse of the damping rate γ for Higgsinos in the thermal plasma as given in Ref. [19] and references therein. This is obtained from the imaginary part of the two-point Green's function (in the unbroken phase). The calculation includes resummation of hard thermal loops and provides the rate for absorption or emission of gauge bosons, B and $W^{\pm,0}$, in the thermal background. The damping rate is the same for charged and neutral Higgsinos and is $0.025T = T/40$. For non-relativistic particles, $l = v/\gamma$ but the damping rate is suppressed by a factor of v in comparison with the expression for relativistic particles [20] (ignoring logarithmic corrections, as in Ref. [19]). Therefore the mean free path is similar to that for relativistic particles.

The quantities in Eq. (4) are in the wall frame while the mean free path, or the damping rate, given above is in the plasma frame. But for typical wall velocities v_w less than $0.1c$ [21] the Lorentz factor $1/\sqrt{1-(v_w/c)^2} \sim 1$ and can be ignored in the expressions for n and σ in l. Therefore $l = 40/T$. We take $l_W = 100/T$. For relativistic Higgsinos p_1 in the wall frame is practically the same as in the plasma frame and we take $p_1 \sim T$.

[2] The momentum lost due to scattering is estimated in Ref. [14] using the stopping power expression which may not be appropriate for a free streaming plasma and for particles with energies close to the mean thermal temperature.

Then

$$\mathcal{R} = 0.2\,\frac{M^2}{T^2}\,. \tag{5}$$

For relativistic Higgsinos, $M \ll T$, and $\mathcal{R} \ll 0.2$.

For non-relativistic Higgsinos, the mean momentum $p_1 = (3\mu_T T)^{\frac{1}{2}}$ in the plasma frame. We may rewrite the wall frame p_1 appearing explicitly in Eq. (4) as $(3\mu_T T)^{\frac{1}{2}} + \mu_T v_w \sim (3\mu_T T)^{\frac{1}{2}}$, for low wall velocities. Therefore,

$$\mathcal{R} = 0.1\left(\frac{T}{\mu_T}\right)^{\frac{1}{2}}\,\frac{M^2}{T^2}\,. \tag{6}$$

However, $\frac{3}{2}T \ll \mu_T$. Thus, for non-relativistic Higgsinos, $\mathcal{R} \ll 0.08M^2/T^2$. If $M < 3T$, thermalisation occurs.

We now consider non-trivial mixing of Higgsinos and gauginos, which is also important for CP violation. If this mixing between Higgsinos and gauginos is large then μ_T may dominate only in the outer regions of the wall and M can be large. But now as one goes deeper into the wall from outside the bubble the physical eigenstates are no longer Higgsinos but mixed with winos and binos. We shall assume that the rate of change of the mass of the physical states is still smooth so that $m^{2'} = -M^2/l_W$ is still valid. The damping rate for winos is given in Ref. [19] to be $T/15$. Gaugino damping rates are proportional to the gauge coupling constant squared, and so the bino damping rate should be lower. For the case where the physical eigenstates were largely Higgsinos we used $l_W/l = 2.5$. Ignoring the variation in the mean free path in the wall as the particle composition changes, we use a constant value of l_W/l and set it equal to 2.5, and use the equations obtained above.

If the charginos and neutralinos are relativistic in the entire wall then, as before, $\mathcal{R} \ll 0.2$. If the particles are non-relativistic in the entire wall then Eq. (6) may be rewritten as

$$\mathcal{R} = 0.1\left(\frac{T}{m(z)}\right)^{\frac{1}{2}}\frac{M^2}{T^2}\,. \tag{7}$$

For a species that is non-relativistic in the entire wall $(3/2)T \ll m(z)$, and $\mathcal{R} \ll 0.08\,M^2/T^2$. Again the thermalisation condition is valid if $M < 3T$.

We now consider the scenario where the particles become non-relativistic at some z in the interior of the wall. In this case, $\mathcal{R}$ will be given by Eqs. (5) and (7) in the outer and inner regions of the wall respectively. Now, $M \gg (3/2)T$. Combining this with Eq. (5) we obtain $\mathcal{R} \gg 0.5$ in the outer regions of the bubble wall. In the inner regions of the bubble wall where the particles become non-relativistic, $m(z) = m_0 + \Delta m(z) \approx \Delta m(z) \leq M$. Therefore Eq. (7) implies that $\mathcal{R} > 0.1(M/T)^{3/2}$ and, for $M \gg (3/2)T$, $\mathcal{R} \gg 0.2$. Therefore, in this scenario, the thermalisation condition, $\mathcal{R} \ll 1$, may not be satisfied in the bubble wall.

Thus, our analysis above indicates that if the mass barrier is larger than $3T$ then kinetic equilibration may not occur in certain circumstances. Lighter charginos are preferred for electroweak baryogenesis but the parameter space for sufficient baryon asymmetry includes heavy particles of masses upto 500 GeV [11, 22] for which the thermalisation assumption may not hold. As discussed earlier, this is important as it is a key assumption in the estimation of the baryon asymmetry in many models of thick wall electroweak baryogenesis. Moreover, as pointed out in Sec. 5.3 of Ref. [23], a CP-even deviation from kinetic equilibrium as discussed above can also act as a source of asymmetry, independent of the CP-odd sources typically calculated in the literature [3]. Thus the issue of kinetic equilibration has significant consequences for the generation of the baryon asymmetry during the electroweak phase transition.

In conclusion, we have argued that kinetic equilibration of the plasma as it passes through the Higgs bubble wall during the electroweak phase transition requires that scattering effects should dominate over the force due to the background Higgs field. A priori one may not know which effect dominates and hence we have investigated whether this condition is satisfied for charginos and neutralinos participating in supersymmetric electroweak baryogenesis. We find that the thermalisation condition is satisfied for relativistic charginos and neutralinos, and is valid for non-relativistic charginos and neutralinos if the height of the mass barrier is less than 3T. If, however, the particles are relativistic outside the bubble and become non-relativistic in the wall then the thermalisation condition may not be satisfied.

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