W-exchange/Annihilation amplitudes in LCSR - $B_d^0 \to D_s^- K^+$ as an example

Namit Mahajan*
Harish-Chandra Research Institute,
Chhatnag Road, Jhunsi, Allahabad - 211019, India.
and
High Energy Section,
The Abdus Salam International Centre for Theoretical Physics,
Strada Costiera, 11-34014 Trieste, Italy.

Abstract

We evaluate the W-exchange diagram within the framework of light cone sum rules (LCSR), taking $B_d^0 \to D_s^- K^+$ as an example. This decay mode, though proceeding via the W-exchange diagram only and therefore expected to be highly suppressed, has the branching ratio 3.2×10^{-5}. We estimate the W-exchange amplitude within LCSR, including soft gluon corrections, to twist-3 accuracy. The calculation naturally brings out the features which suggest that such an amplitude is expected to be small in cases when both the final state mesons are light, without relying on any kind of an assumption. With minor changes, it is also possible to have a rough estimate of the exchange/annihilation type contributions to other processes like $B \to \pi\pi,\ \pi K, KK$. We find that though it appears as if the sum rule method yields a fair agreement with the observed value, a careful analysis of individual terms shows that the method, in its present form, is inadequate to capture the correct physical answer for the case of heavy final states.

1 Introduction

There is a compelling need to have an unambiguous quantitative estimate of the two body hadronic B decay amplitudes. It is now well established that the naive factorization approach has to be abandoned and one has to rely on one of the QCD based approaches, like QCD factorization (QCDF) [1], perturbative QCD (PQCD) [2], soft collinear effective theory (SCET) [3] or light cone sum rules (LCSR) [4], to calculate hadronic decay amplitudes. These differ in their treatment of one or more hadronic quantities/parameters and therefore the results are quite approach dependent and at times very different.

*E–mail : nmahajan@mri.ernet.in

The rare decay $B_d^0 \to D_s^- K^+$ proceeds solely via the W-exchange diagram. In general, certain diagrams, which go under the name of *annihilation (A), W-exchange (E)and penguin-annihilation (PA)* - hereafter generically referred to as annihilation diagrams or contributions, are neglected on the ground of being much lower in the hierarchy of contributing diagrams. These are the diagrams where the lighter quark present in the B-meson also participates in the weak process as opposed to *tree (T) or color-suppressed tree (C) or penguin (P)* type diagrams. The annihilation contributions are most often either set to zero or become parameters of the model. Since the mode $B_d^0 \to D_s^- K^+$ has only the E-type contribution, it offers a unique opportunity to confront theoretical predictions against the experimental observations. Also, it can be used to extract and understand the general structure of such an amplitude. This decay mode has been investigated in the past within various calculational approaches. If factorization holds, then the exchange amplitude will have the form

$$E = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb}^* \left(\frac{C_1}{3} + C_2 \right) f_B F_0^{0 \to D_s K} \tag{1}$$

ie the initial B-meson annihilates into vacuum and the $[D_s K]$ pair is created from the vacuum (represented by the time-like form factor). Within naive factorization, the time-like form factor is expected to be small at $q^2 = m_B^2$ and also the combination of the Wilson coefficients appearing is a small number. Therefore, this amplitude is negligible and thus justifies the neglect in such a scheme. Using the BSW model [5] the branching ratio is predicted to be 6.5×10^{-8} [6]. In order to have a sizeable branching ratio within such a picture, the possible explanation is to consider final state rescattering effects which can lift the suppression [7]. An early attempt to predict the branching fraction based upon PQCD [8] yielded a value $(4.7 - 6.6) \times 10^{-6}$ - an increase by two orders of magnitude compared to the BSW based prediction.

The decay has been experimentally observed both by BaBar [9] and Belle [10] and the observed branching ratio is

$$BR(B_d^0 \to D_s^- K^+) = \begin{cases} (3.2 \pm 1.0 \pm 1.0) \times 10^{-5} & (BaBar) \\ (4.6 \pm 1.2 \pm 1.3) \times 10^{-5} & (Belle) \end{cases} \tag{2}$$

The observed branching ratio is larger than the predicted value. It has been re-examined within PQCD [11] and it is found that theoretical prediction matches well with the experimental value. However, any such calculation requires precise knowledge of heavy meson wave functions, which is lacking at the moment. Also, the employed wave functions do not satisfy equation of motion constraints. Therefore, once that is taken into account, some differences may arise. In [12], within SCET, it is noted that the C- and E-type contributions are of similar size and are both suppressed compared to T- contributions and it is expected that the $B_d^0 \to D_s^- K^+$ amplitude will have a suppression factor of about 3 compared to $B_d^0 \to D^0 \pi^0$ to account for the experimental numbers. However, a complete calculation within SCET is still missing. Further, since such contributions are free parameters in QCDF or schemes relying on $SU(3)$ classification, it is desirable to have an independent check on such results employing some other method. This is further required by the need to verify the presence (or absence) of possible large final state interactions in the channel. It should be noticed that the effect of such rescatterings can be to enhance the rates and also to bring in extra contributions with different CKM elements - thus providing with an opportunity to look for CP violation which will unambiguously confirm such rescatterings. However, in the present case, the dominant rescattering contributions come from $\pi^{+(0)}D^{-(0)}$ intermediate states. These amplitudes have the same weak phase as the E-type contribution and therefore there is no possibility of CP violation. Therefore, it becomes even more important to have independent checks to be sure of the results. Moreover, an unambiguous estimation of such contributions is necessary in order to faithfully extract CKM parameters, since such contributions are present in the decay modes often employed for extracting CKM angles.

Recently, a modified light cone sum rule method has been proposed [13] and has been employed to estimate emission [13] and penguin contributions [14], both hard and soft as well as factorizable and non-factorizable, to $B \to \pi\pi$ mode as well as to evaluate soft non-factorizable contributions in case of $B \to J/\psi K$ [15], $B \to D\pi$ [16], $B \to \eta_c K,\ \chi_c K$ [17], $B \to$ K + charmonium [18] and $B \to K\pi$ [19]. It has been shown in these studies that the soft gluon contributions are equally important as the hard gluon ones, if not dominant. However, none of these studies addresses the issue of annihilation type diagrams within this modified LCSR approach. It is the aim of this study to focus on the evaluation of such contributions employing LCSR.

We evaluate the W-exchange diagram and work to twist-3 accuracy. The calculation explicitly brings out the features which clearly show why the exchange/annihilation type contributions are generally small. The leading soft gluon corrections turn out to be proportional to $q^2 = m_K^2$, which vanish in the chiral limit. Such soft gluon corrections may not be completely insignificant numerically due to the presence of large multiplicative factors. The main aim of the present study is to present an unambiguous and clear way of estimating annihilation type contributions, without having to rely on any dynamical assumptions. Using the computed amplitude, and not relying on general expectations, we argue that the annihilation type amplitudes are $1/m_B$ suppressed compared to the tree contributions. However, a careful analysis of individual terms in the final expression reveals that the sum rule method in its present form is not suitable for final states containing heavy quark like charm.

The paper is organised as follows. In the next section we outline the method and introduce the basic correlation function. In Section 3 we present the calculation of the correlation function, including the leading soft gluon corrections. Section 4 contains the numerical results and discussions. Conclusions are summarised in Section 5.

2 Modified LCSR framework and relevant correlation function

We are interested in evaluating the amplitude for the decay $B_d^0 \to D_s^- K^+$. It proceeds via the W-exchange diagram, with the $s\bar{s}$ pair attached to any of the quark legs via a gluon. The effective Hamiltonian relevant for the process is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb}^* \ [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] \tag{3}$$

where

$$O_1 = (\bar{b}\Gamma_\mu c)(\bar{u}\Gamma^\mu d) \qquad\qquad O_2 = (\bar{u}\Gamma_\mu c)(\bar{b}\Gamma^\mu d) \tag{4}$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$ The above effective Hamiltonian can be cast into the following form (suppressing the scale μ in operators as well as coefficients)

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb}^* \left[\left(\frac{C_1}{3} + C_2 \right) O_2 + 2C_1 \tilde{O}_2 \right] \tag{5}$$

where

$$\tilde{O}_2 = \left(\bar{u} \frac{\lambda^a}{2} \Gamma_\mu c \right) \left(\bar{b} \frac{\lambda^a}{2} \Gamma^\mu d \right) \tag{6}$$

and λ^a are the Gell-Mann matrices with normalization $Tr(\lambda^a \lambda^b) = 2\delta^{ab}$. The amplitude that we are interested in is

$$\mathcal{A}(B_d^0 \to D_s^- K^+) = \langle D_s^-(p) K^+(q) | \mathcal{H}_{eff} | B_d^0(p+q) \rangle \tag{7}$$

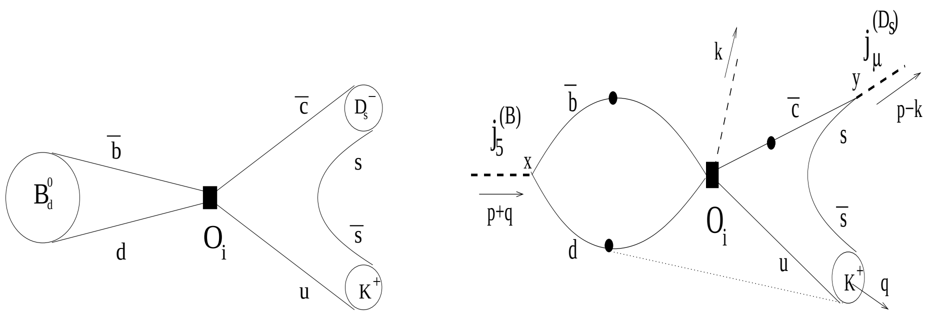

Figure 1: Schematic representation of the exchange diagram (Left). O_i represents one of the operators of the effective Hamiltonian. LCSR picture (Right) with B and D_s interpolating currents. k is the artificial four momentum introduced at the weak vertex. The thick dot indicates possible lines to which a soft gluon (dotted line) from the kaon distribution amplitude can be attached.

The starting point of any LCSR calculation is the specification of the relevant vacuum-meson correlation function. In the present case, the correlator of interest is the following

$$F_\mu^{(O)}(p,q,k) = -\int d^4x e^{i(p-q)x} \int d^4y e^{i(p-q)y} \langle K^+(q) | T[j_\mu^{(D_s)}(y)\ O(0)\ j_5^{(B)}(x)] | 0 \rangle \tag{8}$$

where O is one of the operators present in the effective Hamiltonian and $j_\mu^{(D_s)}$ and $j_5^{(B)}$ are the interpolating currents for D_s and B-meson respectively. Explicitly we have, $O = (\bar{u}^i \Gamma_\mu c^j)(\bar{b}^k \Gamma^\mu d^l)\ A^{ij} A^{kl}$, where $A^{ij}A^{kl} = \delta^{ij}\delta^{kl}$ for $O = O_2$ and $A^{ij}A^{kl} = \left(\frac{\lambda^a}{2}\right)^{ij} \left(\frac{\lambda^a}{2}\right)^{kl}$ for $O = \tilde{O}_2$ and $j_5^{(B)} = i m_b \bar{d}\gamma_5 b$, $j_\mu^{(D_s)} = \bar{c}\gamma_\mu\gamma_5 s$.

As proposed and explained in [13], an unphysical, artificial four momentum k is introduced in the problem. This is done in order to avoid unwanted contributions to the dispersion relation. This is ensured with the introduction of the unphysical momentum because now the total momentum of the final state mesons, $P = (p-k)+q$, is different from the momentum of the initial B-meson, $p+q$. In the final physical matrix elements, no trace of this fictitious momentum k remains. The six invariants are chosen to be p^2, q^2, k^2, $(p-k)^2$, $(p+q)^2$

and P^2. The kaon is on-shell. In the chiral limit, the kaon can be treated as massless and thus $q^2 = 0$. We do not assume such a limit for the time being and shall only consider it towards the end. The correlation function is evaluated in the deep Euclidean region and then analytically continued to the time-like region. The complete kinematical region in which the light cone expansion is applicable, relevant to this case is

$$k^2 = 0 \qquad q^2 = m_K^2 \qquad p^2 = m_{D_s}^2$$

$$|(p+q)|^2, |(p-k)|^2, |P|^2 >> \Lambda_{QCD}^2$$

The correlation function, Eq(8), can be expressed in terms of the four independent tensor structures, namely

$$F_\mu^{(O)}(p,q,k) = F_0^{(O)}\ (p-k)_\mu + F_1^{(O)}\ q_\mu + F_2^{(O)}\ k_\mu + F_3^{(O)}\ \epsilon_{\mu\nu\alpha\beta}p^\nu q^\alpha k^\beta \tag{9}$$

For the present case, $F_0^{(O)}$ is the only object of interest to us.

The above correlation function is evaluated to the desired accuracy, both in the strong coupling, g_s, and the kaon distribution amplitudes governed by the twist expansion. The evaluated result is then expressed as a double dispersion integral with respect to the variables $(p-k)^2$ and $(p+q)^2$ and is subsequently matched to the corresponding hadronic double dispersion integral. In the intermediate steps, Borel transformations are applied in both the variables and quark-hadron duality is invoked to approximate the excited state contributions. It is found that the ground state B-meson contribution is independent of the auxiliary momentum k. Thus the final physical matrix elements do not have any dependence on the fictitious momentum introduced. The end result of such a matching has the following structure

$$\begin{aligned}\mathcal{A}(B_d^0 \to D_s^- K^+) \ &\equiv\ [..]\ \langle D_s^-(p)K^+(q)|O|B_d^0(p+q)\rangle \\ &=\ [..]\ \left(-\frac{i}{\pi^2 f_{D_s} f_B m_B^2}\right)\int_{m_c^2}^{s_{th}^{D_s}} ds_1\ e^{(m_{D_s}^2-s_1)/M_1^2}\int_{m_b^2}^{R_b} ds_2\ e^{(m_B^2-s_2)/M_2^2} \\ &\qquad Im_{s_1} Im_{s_2} F_{0,QCD}^{(O)}(s_1,s_2)\end{aligned} \tag{10}$$

where [..] represents the overall multiplicative factors like CKM elements and Wilson coefficients relevant to the operator inserted.

3 QCD calculation of correlation function and hadronic matrix elements

The possible contributions to the correlation function up to $\mathcal{O}(g_s)$ can be diagrammatically represented as shown in Fig.1. These come in two forms - contributions without any soft gluon and with one soft gluon connecting the kaon to one of the remaining three quark lines. The soft gluon contributions are obtained by expanding the relevant propagator(s) around the light cone and picking the first non-trivial terms containing one gluon field [20]:

$$S^{jk}(x,y:m) \ \equiv\ -i\langle 0|T[q^j(x)\bar{q}^k(y)|0\rangle \tag{11}$$

$$= \int \frac{d^4k}{2\pi^4}\, e^{-i(x-y)k}\left[\frac{\not{k}+m}{k^2-m^2}\delta^{jk} - \int_0^1 dv g_s G_a^{\mu\nu}(vx+(1-v)y)\left(\frac{\lambda^a}{2}\right)^{jk}\right.$$
$$\times \left.\left\{\frac{1}{2}\frac{\not{k}+m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x-y)_\mu\gamma_\nu\right\} + \mathcal{O}(g_s^2)\right]$$

We discuss and evaluate these contributions below. To this end, we note that various contributions are labeled according to the operator from the effective Hamiltonian and the order in strong coupling. We restrict ourselves to twist-3 order in the kaon distribution amplitudes. Incorporation of higher twist effects is fairly straightforward, though cumbersome. The contribution due to insertion of operator O_2 is labeled as $F_0^{(O_2)}$. Further, it is explicitly split into terms according to the order in strong coupling and subsequently the contributions are labeled as $F_0^{(O_2,g_s^0)}$ and $F_0^{(O_2,g_s^1)}$. The former has no gluon line while in the latter, the gluon from the kaon amplitude is hooked on to the charm quark line. The last contribution is a sum of two diagrams where the soft gluon gets attached to either the bottom or down quark line. These are the soft gluon contributions arising due to $\tilde{O}_2$ and are labeled as $F_0^{(\tilde{O}_2,g_s^1)}$. The calculation is done in the NDR scheme. We note that though all the diagrams are divergent, the divergent terms vanish on Borel transforming. Further, we do not show finite terms which disappear after Borel transformation in either or both the variables or the terms which are not proportional to the four vector $(p-k)$.

We start by evaluating, $F_0^{(O_2,g_s^0)}$, the leading factorizable contribution to the correlation function. In the correlation function Eq(8), use $O = O_2$ and make the relevant Wick contractions. For the quark-anti-quark T-ordered products, pick the trivial terms. It is straightforward to evaluate the one-loop Feynman integrals. The end result is

$$\begin{aligned} F_0^{(O_2,g_s^0)} &= -\frac{f_K m_b^2}{4\pi^2}\int [D\alpha_i]\int_0^1 dx \frac{1}{[(p'-k)^2-m_c^2]}\Big\{[(xQ^2-q.Q)\phi_K(\alpha_i) \\ &- (x-1)q.(p'-k)\phi_K(\alpha_i) + m_c\mu_K(x-1)\phi_p(\alpha_i)]\ln(m_b^2-(1-x)Q^2) \\ &+ \frac{m_c\mu_K}{3}(1-x)^2\frac{q.Q}{m_b^2-(1-x)Q^2}\phi_\sigma(\alpha_i)\Big\} \end{aligned} \tag{12}$$

where $[D\alpha_i] = (\Pi_i d\alpha_i)\delta(1-\sum_i\alpha_i)$, $\mu_K = m_K^2/(m_u+m_s)$, $Q = (p+q)$ and $p' = p+\alpha_s q$. The light quark masses are all set to zero. We choose to label the α_i's by the parton they correspond to. The different ϕ's represent different kaon distribution amplitudes defined through the following relations [21]:

$$\begin{aligned} \langle K^+(q)|\bar{u}(0)\gamma_\mu\gamma_5 s(y)|0\rangle &= -iq_\mu f_K\int[D\alpha_i]e^{i\alpha_s y.q}\phi_K(\alpha_i) \\ \langle K^+(q)|\bar{u}(0)\gamma_5 s(y)|0\rangle &= -if_K\mu_K\int[D\alpha_i]e^{i\alpha_s y.q}\phi_p(\alpha_i) \\ \langle K^+(q)|\bar{u}(0)\sigma_{\mu\nu}\gamma_5 s(y)|0\rangle &= i(q_\mu y_\nu - q_\nu y_\mu)\frac{f_K\mu_K}{6}\int[D\alpha_i]e^{i\alpha_s y.q}\phi_\sigma(\alpha_i) \end{aligned} \tag{13}$$

ϕ_K is twist-2 kaon distribution amplitude while ϕ_p and ϕ_σ are twist-3 distribution amplitudes.

Next we consider the $\mathcal{O}(g_s)$ soft gluon contribution, $F_0^{(O_2,g_s^1)}$. This contribution arises when the soft gluon connects the final state kaon to the charm quark line. Following exactly the same procedure as above, and considering the $\mathcal{O}(g_s)$ term of the propagator, we obtain

$$F_0^{(O_2,g_s^1)} = 16 i m_c m_b^2 f_{3K} q^2 \int_0^1 dv \int [D\alpha_i] \frac{\phi_{3K}(\alpha_i)}{[(k-p'')^2 - m_c^2]^2} \int \frac{d^4 k_d}{(2\pi)^4} \frac{k_{d\mu}}{k_d^2[(k_d - Q)^2 - m_b^2]} \quad (14)$$

where $p'' = p + (\alpha_s + v\alpha_g)q$ and ϕ_{3K} is the twist-3 three particle distribution amplitude defined via the relation

$$\begin{aligned} \langle K^+(q)|\bar{u}(0)\sigma^{\alpha\gamma}\gamma_5 \mathcal{G}^{\lambda\sigma}(vx)s(y)|0\rangle &= -i f_{3K}[(q^\lambda q^\alpha \eta^{\sigma\gamma} - q^\sigma q^\alpha \eta^{\lambda\gamma}) - (q^\lambda q^\gamma \eta^{\sigma\alpha} - q^\sigma q^\gamma \eta^{\lambda\alpha})] \\ &\quad \int [D\alpha_i] e^{i(\alpha_s y + v\alpha_g x).q} \phi_p(\alpha_i) \qquad (15) \end{aligned}$$

where $\mathcal{G}^{\lambda\sigma}(vx) = g_s G_a^{\lambda\sigma}(vx)\left(\frac{\lambda^a}{2}\right)$.

The last contribution to be evaluated is the soft gluon contribution to the correlation function due to $\tilde{O}_2$. There are two diagrams to this piece - with the gluon attaching to the bottom or down quark line. The sum of these two diagrams yields

$$\begin{aligned} F_0^{(\tilde{O}_2,g_s^1)} &= -2 i m_c m_b^2 f_{3K} q^2 \int_0^1 dv \int [D\alpha_i] \frac{\phi_{3K}(\alpha_i)}{[(k-p')^2 - m_c^2]} \\ &\quad \int \frac{d^4 k_d}{(2\pi)^4} \frac{k_{d\mu}}{k_d^2[(k_d - Q')^2 - m_b^2]} \left[\frac{1}{k_d^2} + \frac{1}{(k_d - Q')^2 - m_b^2}\right] \qquad (16) \end{aligned}$$

where $Q' = p + (1 - v\alpha_g)q$.

Before proceeding further with the calculation of hadronic matrix elements, we would like to discuss some of the features that are evident from the above computation. In particular, the following is noteworthy. The soft contributions, Eq(14) and Eq(16), are proportional to the momentum squared of the on-shell kaon. If we work in the chiral limit, where $q^2 = m_K^2 = 0$, then these contributions vanish identically. Moreover, both these contributions are proportional to the charm quark mass. These are two of the most important features of the present calculation. Imagine computing the annihilation type contribution for the case of two light mesons in the final state. Then the charm quark mass would have been replaced by the corresponding light quark mass which would have been set to zero without introducing any new assumption. This would have implied that the soft gluon corrections vanish, even if the corresponding q^2 is not set to zero. Also, some terms drop out from the factorizable contribution, Eq(12), on similar arguments. We therefore have a natural explanation for expecting the annihilation type diagrams in most of the cases to be rather small.

It is worthwhile to mention that setting the charm mass to zero would imply vanishing of the amplitude arising due to O_2 because of current conservation. However, the sum rule calculation yields a non vanishing result and this point must be understood. Consider computing annihilation amplitude for B-meson decay into light hadrons. In that case, the light quark masses would have all been set to zero and the only non vanishing contribution stems from quark mass independent terms in Eq(12). However, this particular contribution will be $\mathcal{O}(s_{th}/m_B^2)$, where s_{th} is the corresponding threshold in the light hadron channel. Such a contribution is to be neglected in the approximation we are working with, and therefore, the sum rule calculation conforms with the expectation of vanishing contribution. Let us now consider what happens in our case. We set the charm mass to zero. The contribution in the limit of zero charm mass is $\mathcal{O}(s_{th}^{D_s}/m_B^2) \sim (20-30)\%$ for $s_{th}^{D_s} = 6$ GeV^2. Such a contribution, $\mathcal{O}(s_{th}/m_B^2)$, is actually an artefact of the sum rule method and must be looked upon as an error in the prediction. In contrast to the case of light final states, this induced error is quite large. We take this as a hint that the sum rule method, in the present form and within the approximations employed, is too crude to capture the physically correct answer for the decay into heavy final state(s). However in order to estimate the numerical error due to such a contribution, we must evaluate the matrix elements and study individual pieces.

Having discussed the important features emerging from the structure of various contributions to the correlation function, we proceed to the evaluation of hadronic matrix elements. We choose to work in the chiral limit ie. we set $q^0 = m_k^2 = 0$. In physical terms, this means that we have neglected $\mathcal{O}(q^2/m_B^2)$ terms in the analysis. Note however that $\mu_K \neq 0$ in this limit. This implies that the soft gluon corrections are neglected altogether and we are left with $F_0^{(O_2,g_s^0)}$ only. [1] Define two new variables

$$s_1 = \frac{m_c^2 - \alpha_s P^2}{1-\alpha_s} \qquad\qquad s_2 = \frac{m_b^2}{1-x} \tag{17}$$

It is fairly straightforward to make the change of variables in the expression for $F_0^{(O_2,g_s^0)}$ to arrive at the following

$$\begin{aligned} F_0^{(O_2,g_s^0)} &= \frac{f_K m_b^2}{4\pi^2}\int_{m_c^2}^{\infty} ds_1 \int_{m_b^2}^{\infty} ds_2 \left[\frac{m_b^2}{s_2^2(s_1-P^2)}\right]\left[\frac{1}{s_1-(p-k)^2}\right] \\ &\times \left[\left\{\left[\left(\frac{1}{2}-\frac{m_b^2}{s_2}\right)P^2 - \frac{m_b^2 s_1}{2s_2}\right]\phi_K(\alpha_s) - \frac{m_c\mu_K m_b^2}{s_2}\phi_p(\alpha_s)\right\}\ln(s_2-Q^2)\right. \\ &+ \left.\left(\frac{1}{2}-\frac{m_b^2}{s_2}\right)\phi_K(\alpha_s)[Q^2\ln(s_2-Q^2)] + \frac{m_c\mu_K m_b^2}{6s_2}(s_2-p^2)\phi_\sigma(\alpha_s)\left(\frac{1}{s_2-Q^2}\right)\right] \end{aligned} \tag{18}$$

Borel transforming twice with respect to $(p-k)^2$ and $Q^2 = (p+q)^2$ leads to the hadronic matrix element

$$\begin{aligned} \mathcal{A}(B_d^0 \to D_s^- K^+) &= \frac{G_F}{\sqrt{2}}V_{ud}V_{cb}^*\left(\frac{C_1}{3}+C_2\right)\ \langle D_s^-(p)K^+(q)|O_2|B_d^0(p+q)\rangle \\ &= \frac{G_F}{\sqrt{2}}V_{ud}V_{cb}^*\left(\frac{C_1}{3}+C_2\right)\ \left(-\frac{i}{\pi^2 f_{D_s} f_B m_B^2}\right)\int_{m_c^2}^{s_{th}^{D_s}} ds_1\ e^{(m_{D_s}^2-s_1)/M_1^2} \\ &\times \int_{m_b^2}^{R_b} ds_2\ e^{(m_B^2-s_2)/M_2^2}\ Im_{s_1}Im_{s_2}F_{0,QCD}^{(O_2,g_s^0)}(s_1,s_2) \end{aligned} \tag{19}$$

with

$$\begin{aligned} Im_{s_1}Im_{s_2}F_{0,QCD}^{(O_2,g_s^0)}(s_1,s_2) &= \left(\frac{f_K m_b^4}{4}\right)\left[\frac{M_2^2}{s_2^2(s_1-m_B^2)}\right] \\ &\times \left\{\left[\frac{m_b^2}{s_2}(m_B^2+M_2^2+\frac{s_1}{2}) - \frac{m_B^2+M_2^2}{2}\right]\phi_K(\alpha_s)\right. \\ &+ \left.\frac{m_c\mu_K m_b^2}{s_2}\phi_p(\alpha_s) + \frac{m_c\mu_K m_b^2}{6s_2M_2^2}(s_2-m_{D_s}^2)\ \phi_\sigma(\alpha_s)\right\} \end{aligned} \tag{20}$$

[1] We would like to mention a word of caution at this point regarding the neglect of the soft gluon contribution due to $\tilde{O}_2$, $F_0^{(\tilde{O}_2,g_s^1)}$. Although, the contribution is proportional to q^2, it comes with factors of m_c and $2C_1$ (recall that $C_1 \sim 1$) and thus it is possible that the q^2/m_B^2 suppression is partially lifted.

In the above, we have analytically continued $P^2 \to m_B^2$ and $p^2 \to m_{D_s}^2$. This then completes the LCSR calculation of the W-exchange contribution to twist-3 accuracy in the kaon distribution amplitude in the chiral limit, $q^2 = m_K^2 = 0$. Note the absence of any imaginary/absorptive parts indicating that there are no rescatterings. Although not shown explicitly, it is easy to conclude from the structure of the soft corrections that they also do not introduce any additional phases.

From the structure of the amplitude, Eq(19), it is clear that we can not write the amplitude in a factorizable form ie as a product of B-meson decay constant and a form factor. This thus provides us with a rigorous argument that factorization seizes to hold in such amplitudes and thereby offers the justification for naive factorization failing so badly in predicting the decay rate.

The scaling behaviour of various terms of the amplitude in the large m_B limit can be easily obtained. From the above expression of the hadronic amplitude, it is very much evident that the twist-3 terms are suppressed by one or two powers of m_B compared to the twist-2 leading term. We treat the charm quark also as a light quark. Further, a quick comparison with the heavy mass behaviour studied in [13] confirms the expectation that the annihilation or W-exchange type contributions are $\mathcal{O}(1/m_B)$ of the tree/emission type contributions. The soft gluon contributions are $1/m_B^2$ suppressed, which further justifies their neglect.

4 Numerical estimates

We begin by specifying the input parameters. The NLO values for Wilson coefficients in NDR [22] are $C_1 = 1.082$ and $C_2 = -0.1.85$. The absolute values of the CKM elements are taken to be $|V_{cb}| = 0.043$ and $|V_{ud}| = 0.974$. For the form of various distribution amplitudes and other sum rule parameters we rely on [21, 23, 24]. We only specify the central values of various parameters that we have taken into account for the numerical estimation: $m_{D_s} = 1.968$ GeV, $m_c = 1.3$ GeV, $f_{D_s} = 0.22$ GeV, $s_{th}^{D_s} = 6$ GeV^2, $M_1^2 = 1.5$ GeV^2, $m_B = 5.28$ GeV, $m_b = 4.8$ GeV, $f_B = 0.18$ GeV, $\bar{R} = s_{th}^B = 35$ GeV^2, $M_2^2 = 10$ GeV^2, $f_K = 0.16$ GeV, $f_{3K} = 0.0026$ GeV^2, $\mu_b = \sqrt{m_B^2 - m_b^2} = 2.4$ GeV, $\mu_K(\mu_b) = 2.5$ GeV. To have an estimate of the variation of the results with the sum rule parameters, we check for the variation in the following interval: $0.8 \leq M_1 \leq 1.5$ and $2.8 \leq M_2 \leq 3.5$.

For the two body decay into two pseudoscalars, the decay rate is simply given as

$$\Gamma(B \to P_1 P_2) = \left(\frac{1}{16\pi m_B}\right) |\mathcal{A}(B \to P_1 P_2)|^2 \lambda^{1/2}\left(1, \frac{m_{M_1}^2}{m_B^2}, \frac{m_{M_2}^2}{m_B^2}\right) \tag{21}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. Using this expression, with $m_K^2 = 0$, the absolute value of the amplitude required by the experimental data Eq(2) is (in GeV) (we quote the numbers corresponding to the central values of BaBar and Belle results)

$$|\mathcal{A}(B_d^0 \to D_s^- K^+)|_{expt} \sim (6.5 - 9) \times 10^{-8} \tag{22}$$

Using the input values of the parameters listed above, to the twist-3 accuracy, we obtain the following value for the amplitude

$$|\mathcal{A}(B_d^0 \to D_s^- K^+)|_{Tw3,LCSR} = \left(3.6^{+2.1}_{-1.9}\right) \times 10^{-8} \tag{23}$$

where the upper and lower values correspond to the two extreme values for the sum rule parameters, M_1 and M_2. The central value is about half the experimental value. However, we must remember that the present analysis only takes into account terms up to twist-3 in the kaon distribution amplitude. We can expect that twist-4 contributions will partially take care of the discrepancy. Also, recall that some contribution is expected from the soft gluon corrections, even at twist-3 level, because the q^2/m_B^2 suppression is lifted due to the large multiplicative factors, $2C_1 m_c$. Therefore, it is tempting to conclude that the experimental values will be completely saturated once these extra contributions are accounted for.

However, in view of the discussion related to the charm mass independent terms in the previous section, it is very important to investigate the individual contributions separately. Let us recall some of the important points related to the individual contributions. There are two type of terms - charm mass independent (twist-2), $\mathcal{A}^{ind}\mathcal{O}(s_{th}^{D_s}/m_B^2) \sim (20-30)\%$, and terms proportional to the charm quark mass (twist-3), $\mathcal{A}^{dep}$. A quick check on the relative size of the two contributions reveals that the bulk of contribution arises from the charm mass independent terms. Although, this agrees with the expected scaling behaviour of the individual terms, this observation implies that the sum rule method yields a much larger theoretical error than we had anticipated. This thus forces us to conclude that the method is not suitable for the decay of B-mesons into final states containing a heavy quark like charm. Further, we do not expect the higher twist contributions to change the picture drastically as the dominant term in the present case is twist-2, which will continue to dominate even in the presence of twist-4 or higher terms. In Table 1 we summarize the values of the two contributions for different choices of the sum rule parameters - M_1 and M_2.

Table 1: Individual contributions to the amplitude for different choices of M_1 and M_2. The middle row corresponds to the central value of the parameters.

(M_1, M_2) (GeV)	$\mathcal{A}^{ind}$	$\mathcal{A}^{dep}$
(0.8,2.8)	3.16×10^{-8}	-1.4×10^{-8}
(1.22,3.16)	4.26×10^{-8}	-6.9×10^{-9}
(1.5,3.5)	6.53×10^{-8}	-8.1×10^{-9}

From the table it is clear that the two contributions come with opposite sign and it is only for the lower values of M_1 and M_2 that the two are almost equal in magnitude. Also clear from the table is the fact that the expected error, an artefact of the method, overwhelms the total contribution. Although we had anticipated that the error in the present case is going to be large, the values in the table are at complete variance with our naive expectations. Therefore, in the present form, the sum rule method is not reliable for the case of final states involving a heavy quark.

5 Results and discussion

Employing the modified light cone sum rule method [13], we have computed the hadronic matrix element for the rare decay $B_d^0 \to D_s^- K^+$. This decay channel falls under the very special class of decays which receive contribution only from annihilation/W-exchange diagrams. Such diagrams lie way below in the usual hierarchy of diagrams/topologies and are generally neglected. In such a situation, the only way to have a sizeable branching ratio is to expect large final state interactions. Moreover, annihilation type diagrams contribute to many other channels like $\pi\pi$ or πK, to mention the obvious. It is rather difficult to cleanly extract various standard model parameters if a precise knowledge of these amplitudes is not known.

Motivated by all these factors, we have evaluated the W-exchange diagram to twist-3 accuracy within the LCSR method. We find that naively it appears that the sum rule method is more or less successful in explaining the observed branching ratio, while a careful analysis of individual terms leads us to the conclusion that such an expectation is completely wrong. Within the accuracy of the sum rule method, we neglect $\mathcal{O}(s_{th}/m_B^2)$ terms. However, in the present case such terms are already $\mathcal{O}(20\%)$. Therefore a large error is expected. We find that bulk of the contribution to the amplitude actually arises from the terms that should be viewed as theoretical error. We therefore take this as a clear indication of the fact that the sum rule method, in the present form and within the adopted accuracy, is unsuitable for explaining the decay amplitudes when there is a massive quark in the final state, though the method seems to work for light final states. The only consistent way out would be to try to extend the method beyond the adopted approximations and carefully investigate whether such an extension leads to more meaningful results and predictions. This would require systematically incorporating, at least the leading, $\mathcal{O}(s_{th}/m_B^2)$ terms. However, it is not clear if such a modification will be easy to implement or if the whole approach is to be changed.

In view of the above discussion, it is tempting to use the above results to have a crude estimate of annihilation type diagrams for the decay to light mesons, like $B \to \pi\pi$. This can be easily achieved by appropriately changing various parameters. A crude estimate for the ratio of the annihilation type amplitudes to the factorizable amplitude in the $\pi\pi$ channel turns out to be at the percent level at best. However, this must include the higher order effects also and should be investigated in detail. The annihilation type diagrams may contribute subdominantly to the decay rate, but still can have significant impact on CP asymmetries. A systematic study is thus called for in such cases and will be reported elsewhere.

We have argued above that the method can not be trusted for explaining the annihilation diagrams in case of final state(s) involving a heavy quark. However, also clear from the above discussion is the fact that the method is well suited for light final states. It may be worthwhile to present the summary of the main features which emerge from the analytic structure of the expressions. For this discussion we do not bother about the numerical values of the individual pieces as we hope that such a discussion is useful in understanding the basic structure of a generic annihilation diagram - of course we keep in mind all the discussion and conclusions relevant for a massive final state.

- Factorizable contribution to twist-3 order is proportional to the final state quark mass (the quark mass independent term is an error due to the method itself). Such a contribution thus vanishes for light final states and one can hope that the twist-4 contributions will yield the leading non-vanishing contribution.

- The soft gluon non-factorizable contributions turn out to be proportional to q^2, the mass squared of the meson described by the distribution amplitudes and mass of the (anti-)quark emerging out of the weak vertex from the bottom (anti-)quark. In the present case, the proportionality factor thus is $m_c q^2$, with $q^2 = m_K^2$. Such contributions are expected to vanish in the chiral limit. However, for cases where the final state contains a heavy and a light meson, these contributions can become significant as there is an additional enhancement factor, $2C_1$, yielding a net enhancement factor of $2m_c C_1$ for some of the soft gluon contributions.

- The annihilation/W-exchange amplitudes are $\mathcal{O}(1/m_B)$ compared to the tree/emission type amplitudes. Further, the twist-3 contributions are suppressed by additional powers of the large mass in comparison to the twist-2 terms.
- It is not possible to write the amplitude in a factorized form.
- The amplitudes are all real to twist-3 accuracy, implying absence of rescattering.

To conclude, we have described the evaluation of annihilation type amplitudes within the framework of light cone sum rules and applied to the case of $B_d^0 \to D_s^- K^+$. Our results indicate that the modified sum rule method is not in a form which can be applied to the case of B-meson decaying into a heavy final state. The numerical results clearly show that the dominant contribution stems from terms that are an artefact of the sum rule method and strictly speaking, should be considered a part of the theoretical error. Only for some particular choices of the sum rule parameters, the different contributions approach each other in magnitude. Although, the present study shows that the method fails when trying to explain the mode $B_d^0 \to D_s^- K^+$ (similar conclusion will hold for any other heavy state), the calculation brings out some generic features of a typical annihilation type diagram. The present computation, in principle, completes the computation of all types of quark level diagrams within LCSR. It is hoped that with straightforward modifications and improvements, the results of this study and the ones already existing can be combined to obtain a clear and consistent picture of the two body hadronic B decays.

Acknowledgements I would like to thank the High Energy Section, The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, for hospitality during the final stages of the work.

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