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Matching meson resonances to OPE in QCD

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We investigate the possible corrections to the linear Regge trajectories for the light-quark meson sector by matching two-point correlators of quark currents to the Operator Product Expansion. We find that the allowed modifications to the linear behavior must decrease rapidly with the principal quantum number. After fitting the lightest states in each channel and certain low-energy constants the whole spectrum for meson masses and residues is obtained in a satisfactory agreement with phenomenology. The perturbative corrections to our results are discussed.

Keywords: QCD; sum rules; large- N_c .

The observed masses squared of mesons with given quantum numbers form linear trajectories^{1,2} depending on the number of radial excitation n . This is a strong indication that QCD admits an effective string description, as this type of spectrum is characteristic e.g. of the bosonic string. In the bosonic string model the slope of all trajectories must be equal since this quantity is proportional to the string tension depending on gluodynamics only. However, there exist sizeable deviations from the string picture. In the present analysis we examine possible corrections to the linear trajectories in the vector (V), axial-vector (A), scalar (S), and pseudoscalar (P) channels³. Our method is based on the consideration of the two-point correlators of V,A,S,P quark currents in the large- N_c limit of QCD⁴. On the one hand, by virtue of confinement they are saturated by an infinite set of narrow meson resonances, that is, they can be represented by the sum of related meson poles in Euclidean space:

$$\Pi_J(Q^2) = \int d^4x \exp(iQx) \langle \bar{q} \Gamma q(x) \bar{q} \Gamma q(0) \rangle_{\text{planar}} = \sum_n \frac{2F_J^2(n)}{Q^2 + m_J^2(n)}, \quad (1)$$

expressing the quark-hadron duality⁵. Here $J \equiv S, P, V, A$; $\Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5$. Further we denote $F_{S,P} \equiv G_{S,P} m_{S,P}$. On the other hand, their high-energy asymptotics

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is provided by the perturbation theory and the Operator Product Expansion (OPE) with condensates⁶. Matching these two approaches results in the Chiral Symmetry Restoration sum rules representing a set of constraints on meson mass parameters⁷. We performed the further analysis in the chiral limit and firstly in the leading order of perturbation theory.

Let us consider the linear ansatz for the meson mass spectra with a non-linear correction δ :

$$m_J^2(n) = m_{0,J}^2 + a n + \delta_J(n), \quad J \equiv V, A, S, P. \quad (2)$$

The last term in Eq. (2) signifies a possible deviation from the string picture in QCD. The condition of convergence for the generalized Weinberg sum rules⁹ imposes the equality of slopes and a falloff of non-linear correction.

The asymptotic freedom leads to the relation between residues and masses: $F^2(n) \sim \frac{dm^2(n)}{dn}$. Our analysis showed that the analytical structure of the OPE admits, however, exponentially small deviations from this relation (in contrast to^{9,10}),

$$m_J^2(n) = M_J^2 + a n + A_m^J e^{-B_m \cdot n}, \quad (3)$$

$$F_{V,A}^2(n) = a \left(\frac{1}{8\pi^2} + A_F^{V,A} e^{-B_F \cdot n} \right), \quad G_{S,P}^2(n) = a \left(\frac{3}{16\pi^2} + A_G^{S,P} e^{-B_G \cdot n} \right) \quad (4)$$

with certain constants $A_{m,F,G}^{V,A,S,P}$ and $B_{m,F,G} > 0$ to be fitted. It is plausible to suppose that, for masses, the dynamics under these exponentially small corrections is governed mostly by gluons and thereby does not depend on flavor. Thus, we keep the exponent B_m the same for all channels. For the same reason we regard $B_{F,G}$ as independent of parity.

In Tables 1 and 2 we show an example of numerical fits resulting from our approach where in the S,P-channels two possibilities are considered: π -meson belongs to the radial Regge trajectory (' π -in') and does not (' π -out'). The inputs general for all tables (if any) are: $a = (1120 \text{ MeV})^2$, $\langle \bar{q}q \rangle = -(240 \text{ MeV})^3$, $\frac{\alpha_s}{\pi} \langle (G_{\mu\nu}^a)^2 \rangle = (360 \text{ MeV})^4$, $f_\pi = 103 \text{ MeV}$, $Z_\pi = 2 \frac{\langle \bar{q}q \rangle^2}{f_\pi^2}$, $\alpha_s = 0.3$. The units are: $m(n), F(n), G(n) — \text{MeV}$; $A_m — \text{MeV}^2$; $A_F, A_G, B_{F,G,m} — \text{MeV}^0$.

Let us discuss the impact of running coupling constant at next-to-leading order of perturbation theory. We have in this case the contribution to the imaginary part of correlator which is related to the full correlator through the dispersion relation. In the large- N_c limit for the vector and axial-vector case this leads to:

$$\text{Im}\Pi(t) = \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s(t)}{\pi} \right) + \mathcal{O}(\alpha_s^2) = \pi \sum_{n=0}^{\infty} 2F^2(n) \delta(t - m^2(n)). \quad (5)$$

We perform summation in Eq. (5) by applying the Euler-Maclaurin summation formula. This provides smoothness of this expression. Finally we obtain the relation:

$$F^2(n_0) = \frac{dm^2(n)}{dn} \frac{1}{8\pi^2} \left(1 + \frac{\alpha_s(t(n_0))}{\pi} \right). \quad (6)$$

Table 1. An example of parameters for the mass spectra. The existing experimental values ^{2,11} are displayed in brackets.

Case	Inputs	Fits and constants
VA	$m_V(0) = 770 (769.3 \pm 0.8)$, $m_A(0) = 1200 (1230 \pm 40)$,	$M = 920$, $B_m = 0.97$, $B_F = 0.72$, $A_m^V = -500^2$, $A_m^A = 770^2$, $A_F^V = 0.0012$, $A_F^A = -0.0031$, $L_{10} = -6.5 \cdot 10^{-3}$, $\Delta m_\pi = 2.3$
SP (π -in)	$m_S(0) = 1000$, $m_P(0) = 0$, $m_P(1) = 1300 (1300 \pm 100)$, $B_m = 0.97$	$\bar{M} = 840$, $B_G = 0.42$, $A_m^S = 550^2$, $A_m^P = -840^2$, $A_G^S = -0.0009$, $A_G^P = 0.0004$, $L_8 = 1.0 \cdot 10^{-3}$
SP (π -out)	$m_S(0) = 1000$, $m_P(0) = 1300 (1300 \pm 100)$, $m_P(1) = 1800 (1801 \pm 13)$, $B_m = 0.97$	$\bar{M} = 1470$, $B_G = 1.27$, $A_m^S = -1080^2$, $A_m^P = -690^2$, $A_G^S = 0.0213$, $A_G^P = 0.0067$, $L_8 = 0.9 \cdot 10^{-3}$

Table 2. Mass spectrum and residues for the parameter sets of Table 1.

n	out	0	1	2	3
$m_V(n)$		770 (775.8 \pm 0.5)	1420 (1465 \pm 25)	1820 (1900?)	2140 (2149 \pm 17)
$F_V(n)$		138 (154 \pm 8)	135	133	133
$m_A(n)$		1200 (1230 \pm 40)	1520 (1640 \pm 40)	1850 (1971 \pm 15)	2150 (2270 \pm 50)
$F_A(n)$		116 (123 \pm 25)	125	128	130
$m_S(n)$		1000 (980 \pm 10)	1440 (1507 \pm 5)	1800 (1714 \pm 5)	2100
$G_S(n)$		176	178	178	179
$m_P(n)$		0 (π -in)	1300 (1300 \pm 100)	1760 (1801 \pm 13)	2100 (2070 \pm 35)
$G_P(n)$		-	179	179	179
$m_S(n)$		1000 (980 \pm 10)	1730 (1714 \pm 5)	2120	2420
$G_S(n)$		243	199	185	181
$m_P(n)$	0	1300 (1300 \pm 100)	1800 (1812 \pm 14)	2150 (2070 \pm 35)	2430 (2360 \pm 30)
$G_P(n)$	-	201	186	181	180

Eq. (6) can be approximated by finite differences. For the first two states this reads:

$$\frac{F_\rho^2}{m_{\rho'}^2 - m_\rho^2} \approx \frac{F_{a_1}^2}{m_{a_1'}^2 - m_{a_1}^2} \approx \frac{1}{8\pi^2} \left(1 + \frac{\alpha_s(1 \text{ GeV})}{\pi} \right). \quad (7)$$

Substituting experimental values ¹¹ into Eq. (7) one arrives at the estimate: $1.5 \pm 0.2 \approx 1.3 \pm 0.6 \approx 1.4$ in a good agreement with the phenomenology.

In order to reproduce the running coupling behaviour in Eq. (5) one should accept the following ansatz for the residues:

$$F^2(n) \simeq \frac{dm^2(n)}{dn} \frac{1}{8\pi^2} \left(1 + 4 \left(\beta_0 \ln \frac{m^2(n)}{\Lambda_{\text{QCD}}^2} \right)^{-1} \right) + \text{exp. corr.} \quad (8)$$

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However this ansatz introduces into the OPE the additional terms $\sim \frac{\ln Q^2}{Q^2}$ (and powers of these terms) which are absent in the standart OPE. Thus ansatz (7) must be improved. This problem is under our studies now.

Let us summarize the results of our analysis:

- 1) The convergence of the generalized Weinberg sum rules requires the universality of slopes and intercepts for parity conjugated trajectories.
- 2) The matching to the OPE cannot be achieved by a simple linear parameterization of the mass spectrum, the linear trajectory ansatz. There must exist deviations from the linear trajectory ansatz triggered by chiral symmetry breaking. These deviations must decrease at least exponentially with n .
- 3) For heavy states, the D-wave vector mesons have to decouple from asymptotic sum rules. This fact implies the exponential (or faster) decreasing the corresponding decay constants $F_D^2(n)$.
- 4) Our results seem to exclude a light $\sigma(600)$ particle as a quarkonium state and rather favor the non-linear realization of chiral symmetry with the lightest scalar of mass ~ 1 GeV, its chiral partner being the $\pi'(1300)$.
- 5) In our approach the quantities L_8, L_{10} and Δm_π are obtained, in satisfactory agreement with the phenomenology.
- 6) Perturbative corrections can be systematically treated making, nevertheless, small effects on the fits presented.

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