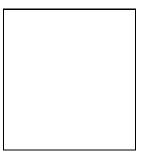
## FORWARD HADRONIC SCATTERING FROM FEW GeV TO MULTI TeV WITHIN REGGE THEORY

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The Regge description of forward hadronic scattering is extended from  $\sim 1 \text{GeV}$  above each threshold up to the multi TeV range. This is achieved with a simple parametrization, that includes mass effects, a logarithmic growth based on an improved unitarity bound at intermediate energies and a separate factorization of singularities. This parametrization can be easily implemented for phenomenological use but also sheds light on subdominant trajectories and the Pomeron logarithmic growth law.

## 1 Introduction

In recent works<sup>1</sup> we have shown how it was possible to extend down to roughly 1 GeV above threshold the Regge description of those combinations of total cross sections that involved the Pomeron, P' and  $\rho$  Regge trajectories only. In particular we provided several fits to the  $(\bar{p}p+pp)$ ,  $(K^+p+K^-p)$ ,  $\pi^{\pm}N$  and  $\pi\pi$  cross sections.

I report here on preliminary results<sup>2</sup> that include the a and  $\omega$  trajectories and extend the analysis to the total hadronic cross sections of  $\bar{p}p$ , pp,  $\bar{p}n$ , pn,  $K^{\pm}p$ ,  $K^{\pm}n$ ,  $\pi^{\pm}N$  and  $\pi\pi$ , as well as the ImF/ReF ratios of  $\bar{p}p$ , pp, pn,  $\pi^{\pm}N$  and  $K^{\pm}p$  forward elastic amplitudes, F. The data has been obtained from the extensive compilation of the COMPAS group. Given the fact that the original references did not treat the systematic uncertainties uniformly, we have followed two fitting strategies: First, we keep the original uncertainties as such. This allows for an easy comparison with previous works, including the PDG and the extensive ones of the COMPETE group<sup>3</sup>. However, this introduces a bias toward those data sets, usually the oldest, that do not provide systematic uncertainties. Indeed, many of these data are incompatible within their statistical errors, and cannot be described simultaneously. This produces an artificially large  $\chi^2/d.o.f$ . no matter what function is fitted. For that reason, in our second strategy, we have added a systematic error, but only to those data without it, of 0.5% for pp, 1% for  $\bar{p}p$  and 1.5% for other processes. The size of these additional errors has been chosen of the same order of magnitude given by other experiments for the same process, so that we give similar weight to all sets without discarding any point. To account for different ways of combining statistical and systematic errors, in the first strategy we have added them in quadrature and linearly in the second. In addition, we use  $\sigma^{total}$  data<sup>10</sup> on  $\pi^+\pi^-$ ,  $\pi^-\pi^-$ ,  $\pi^+\pi^0$ , above 1.42GeV, plus one data point per channel reconstructed from phase shift analysis<sup>1</sup> at 1.42GeV. If using  $\pi\pi$  low energy information<sup>1,2</sup>, the  $\rho$  residue and intercept come out somewhat smaller and larger, respectively.

The contributions to the  $F_{AB\to AB}$  amplitudes from the Pomeron P, the f (or P'),  $\rho$ ,  $\omega$  and a trajectories, are

$$F_{p^{\pm}p} = (P_{NN} + f_{NN} + a_{NN} \mp \omega_{NN} \mp \rho_{NN})/2, \quad F_{p^{\pm}n} = (P_{NN} + f_{NN} - a_{NN} \mp \omega_{NN} \pm \rho_{NN})/2,$$

$$F_{K^{\pm}p} = (P_{KN} + f_{KN} + a_{KN} \mp \omega_{KN} \mp \rho_{KN})/2, \quad F_{K^{\pm}n} = (P_{KN} + f_{KN} - a_{KN} \mp \omega_{KN} \pm \rho_{KN})/2,$$

$$F_{\pi^{\pm}p} = (P_{\pi N} + f_{\pi N})/\sqrt{6} \mp \rho_{\pi N}/2, \quad F_{\pi^{\pm}\pi^{-}} = (P_{\pi\pi} + f_{\pi\pi})/3 \pm \rho_{\pi\pi}/2, \quad F_{\pi^{0}\pi^{-}} = (P_{\pi\pi} + f_{\pi\pi})/3$$

where  $N = p^{\pm}$ , *n* and isospin Clebsch Gordan coefficients have been extracted to simplify the factorization<sup>8</sup> relations  $R_{AB}(\nu) = f_A^R f_B^R R(\nu)$ , where for  $R = \rho, f, a, \omega$ 

$$R(\nu) = \beta_R \left(\frac{1 + \tau e^{-i\pi\alpha}}{\sin\pi\alpha}\right) \nu^{\alpha_R},\tag{1}$$

where  $\tau$  is the signature of the trajectory. If we want to take account of the masses, the energy dependence should appear through  $\nu = (s - u)/2$ , instead of the usual choice of s. Note that for forward scattering  $\nu = s - m_a^2 - m_b^2 > s - s_{th}$ . For the intercepts  $\alpha_R$  we will assume, following the QCD version of Regge theory, and the recent analysis by the COMPETE group<sup>3</sup>, that the f/a and  $\rho/\omega$  trajectories are degenerate, that is, that  $\alpha_a = \alpha_f$  and  $\alpha_\omega = \alpha_\rho$ . In addition, some factors are redundant and can be absorbed by setting  $f_{\pi}^R = 1$ , for  $R = P, f, \rho$  and  $\beta_R = 1$  for  $R = a, \omega$ . This choice eases the notation and the comparison with previous works<sup>1</sup>.

For the Pomeron, we propose the use of a "constant plus logarithm", i.e.,

$$P_{AB} = C_{AB} + L_{AB}, \quad \text{Im} P(\nu) = \nu \left(\beta_P + A \log^2 \left[\frac{\nu - \nu_{th}}{\nu_1 \log^{7/2}(\nu/\nu_2)}\right]\right), \tag{2}$$

where the logarithmic law follows an improved unitarity bound<sup>9</sup>. The choice  $\nu - \nu_{th}$  yields slightly better fits. Note that  $\nu_{th}$  is  $\nu$  at the branch point of the right cut for each amplitude. Indeed, it has been confirmed <sup>3</sup> that the  $\sigma^{tot}$  growth and the Re F/Im F ratios, are better described with  $s \log s$  or, slightly better <sup>3</sup>, with  $s \log^2 s$  terms rather than with  $\alpha_P > 1$ . Our Eq.(2) grows faster than  $s \log s$  but slower at intermediate energies than the  $s \log^2 s$  Froissart bound, which is nevertheless recovered at very high s. The recent generalization of the "factorization theorem"<sup>6</sup> requires each singularity to factorize separately. Thus, as a first approximation, and for the Pomeron, we use separated factors:  $f_A^C$  and  $f_A^L$ .

Finally, the real parts of amplitudes are obtained from the dispersive representation, whereas total cross sections are given by:

$$\sigma_{ab} = 4\pi^2 \operatorname{Im} F_{a+b\to a+b}(s,0) / \lambda^{1/2}(s,m_a^2,m_b^2), \quad \lambda(s,m_a^2,m_b^2) = s^2 + (m_a^2 - m_b^2)^2 - 2s(m_a^2 + m_b^2)$$

However,  $\lambda$  is usually approximated by  $s^2$ . Only very recently<sup>7</sup> a slight improvement in  $\chi^2/dof$  has been found by keeping the whole  $\lambda$ , instead of  $s^2$ , down to  $\sqrt{s} = 5$  GeV. Let us remark that, if  $E_{kin} \simeq 1$  GeV, the effect of using  $s^2$ , instead of  $\lambda$ , yields a 30% overestimation for NN.

## 2 Results

We show in Table 1 the parameters of the fits to data for different strategies and different  $E_{kin}^{min}$ , including the nominal uncertainty obtained from the  $\chi^2/dof$  minimization routine MINUIT. Since the parameters are very strongly correlated these errors should be taken only nominally around the central values provided for strategy 2. Since there are flat directions in parameter space it is possible to get somewhat different parameter sets with almost the same  $\chi^2/dof$  but beyond those nominal errors. An indication of the systematic error in the determination of single parameters can be obtained from the difference between strategies.

	strategy 2	strategy 1	Minuit		strategy 2	strategy 1	Minuit
$E_{kin}^{min}$	$1\text{-}1.3~\mathrm{GeV}$	$1\text{-}1.3~\mathrm{GeV}$	errors	$E_{kin}^{min}$	$1\text{-}1.3~\mathrm{GeV}$	$1\text{-}1.3~\mathrm{GeV}$	errors
$\beta_P$	0.746	0.937	0.003	$f_K^f$	0.30	0.32	0.01
$egin{array}{c} f^P_N \ f^P_K \ f^P_K \end{array}$	1.792	1.705	0.007	$\alpha_f$	0.646	0.640	0.002
$f_K^P$	0.731	0.714	0.004	$f_N^a$	-0.24	0.25	0.04
A	0.043	0.050	0.001	$f_K^a$	-0.55	0.5	0.1
$\nu_1$	0.0005	0.001	0.0001	$\beta_{\rho}$	1.28	1.34	0.11
$\nu_2$	0.676	0.633	0.001	$f_N^{\dot{ ho}}$	0.51	0.46	0.04
$f_N^{log}$	1.02	0.993	0.001	$f_K^{\rho}$	0.49	0.54	0.04
$f_K^{log}$	0.723	0.733	0.012	$\alpha_{ ho}$	0.464	0.464	0.003
$\hat{\beta}_f$	1.70	1.77	0.014	$f_N^{\omega}$	1.97	1.98	0.015
$f_N^f$	1.78	1.75	0.01	$f_K^{\omega}$	0.66	0.65	0.01
				$\sigma_{LHC}$	$109 \mathrm{~mb}$	110 mb	1mb

**Table 1.** Fit parameters with different strategies. The Minuit errors are *just statistical*, and nominal, since the parameters are strongly correlated and can only be used with the central values of *strategy 2*.

$E_{kin}^{min}$ (GeV)	1-1.3	1.5	2	3					
# data points	1186	1002	895	768					
Parametrization	$\chi^2$ /d.o.f. for strategy 2 / 1								
Ours	0.85/1.56	0.63/1.14	0.57/1.05	0.52/0.95					
$\nu_1 \equiv 0.01 \text{ GeV}^2$	0.85/1.57	0.63/1.26	0.58/1.06	0.52/0.97					
powers of $s^{\alpha}$	1.58/2.87	1.16/2.11	0.99/1.80	0.78/1.42					
Pomeron logarithmic term									
u log( u)	1.01/1.83	0.69/1.26	0.59/1.09	0.52/0.97					
$ u \log( u -  u_{th}) $	1.03/1.83	0.69/1.27	0.59/1.12	0.52/0.98					
$ u \log^2( u) $	0.97/1.79	0.68/1.24	0.59/1.10	0.52/0.95					
$ u \log^2( u -  u_{th}) $	0.91/1.68	0.65/1.18	0.58/1.06	0.52/0.95					
Factorization of Pomeron logarithms									
$f_a^L \equiv 1 \text{ (as PDG)}$	0.92/1.70	0.66/1.23	0.59/1.10	0.54/1.01					
$f_a^C = f_a^L$	0.89/1.67	0.64/1.41	0.60/1.14	0.58/1.02					

**Table 2.**  $\chi^2$ /d.o.f. for several  $E_{kin}^{min}$  and different modifications of our parametrization.

In Table 2 we compare the  $\chi^2/d.o.f.$  of parameterizations where we have not implemented some of our suggestions. Let us recall that these are: i) using  $\nu$  instead of s, ii) the Eq.(2), iii) the separate factorization, i) and the correct flux factor. For each one of them we find an improvement in  $\chi^2/d.o.f.$ , with both fitting strategies.

In Figure 1 we show the curves obtained from our parametrization including, as gray bands, the nominal uncertainties from strategy 2. The dashed lines correspond to the PDG2004 parametrization <sup>4</sup>, valid above  $\sqrt{s} = 5$  GeV, but naively extrapolated below. We see that the simple parametrization reported here describes remarkably well 20 observables extending

from several TeV down to  $\sim 1 \text{GeV}$  above the threshold of each reaction. Further details will be given in a forthcoming publication<sup>2</sup>, but apart from establishing the logarithmic growth of the Pomeron, we hope it could be easily used for dispersive studies in hadronic physics that involve integrals from the resonance region to infinity.

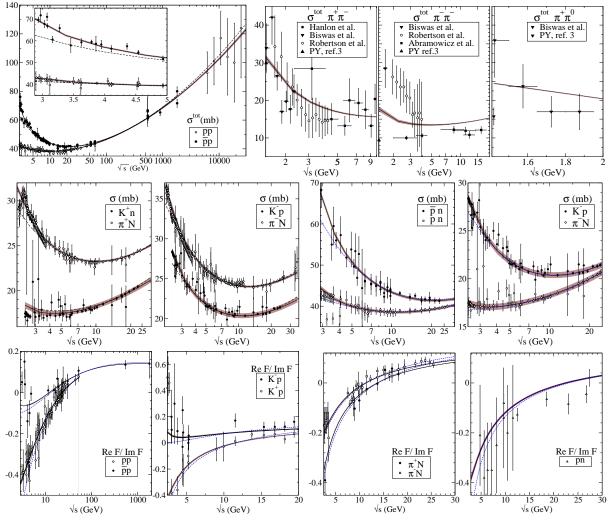


Figure 1: Results from our fit down to 1 GeV above threshold. Total NN,  $\pi\pi$ ,  $\pi^{\pm}N$ ,  $K^{\pm}p$  and  $K^{\pm}n$  cross sections as a function of  $\sqrt{s}$ , with detail of pp and  $\bar{p}p$  at low energies. In the bottom row we show the results for ReF/ImF. The bands cover the nominal uncertainties in the parameters.

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