

## The Kaon $B$ -parameter in quenched QCD\*

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I report on a recent determination by the ALPHA collaboration of the kaon  $B$ -parameter using lattice QCD with Wilson type quarks. An effort is made to control all systematic errors except for the quenched approximation. The preliminary result for the renormalization group invariant parameter is  $\hat{B}_K = 0.834(37)$ , which translates to  $B_K^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.604(27)$  in the  $\overline{\text{MS}}$  scheme with anticommuting  $\gamma_5$ .

### 1. Introduction

The kaon  $B$ -parameter remains an important ingredient in current analyses of CP violation in the Standard Model <sup>1</sup>. It is defined in QCD with up, down and strange quarks by the matrix element of a four-quark operator between kaon states,

$$\begin{aligned} \langle \bar{K}^0 | O_{(V-A)(V-A)}^{\Delta S=2} | K^0 \rangle &= \frac{8}{3} F_K^2 m_K^2 B_K, \\ O_{(V-A)(V-A)}^{\Delta S=2} &= \sum_{\mu} [\bar{s} \gamma_{\mu} (1 - \gamma_5) d]^2. \end{aligned} \quad (1)$$

While this matrix element is well-defined, its relation to the full amplitude in the Standard Model relies on the hypothesis that the charm quark can be treated as a heavy particle. While the quality of this approximation is difficult to assess, it is typically assumed to be valid at the five percent level, which sets the scale for the precision to be attained by a lattice determination of  $B_K$ .

### 2. $B_K$ and Wilson type quarks

Despite recent progress with other fermion formulations, lattice QCD with Wilson type quarks remains attractive because it is computationally cheap and does not suffer from mixing between flavour and spin degrees of freedom (unlike staggered fermions). On the other hand, all axial symmetries are explicitly broken, which leads to mixing of operators with opposite chirality, and the possible occurrence of unphysical fermion zero modes in the quenched approximation. In order to avoid the latter problem in typical quenched simulations, the masses of pseudoscalar mesons are typically heavier than the physical kaon mass. Concerning the

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calculation of  $B_K$ , the mixing problem is particularly annoying. Decomposing the operator  $O_{(V-A)(V-A)} = O_{VV+AA} - O_{VA+AV}$  the relevant piece for  $K^0-\bar{K}^0$  mixing is the parity even component, which renormalizes as follows <sup>2</sup>:

$$[O_{VV+AA}]_R = Z_{VV+AA} \left[ O_{VV+AA} + \sum_{i=1}^4 z_i O_i^{d=6} \right]. \quad (2)$$

While the mixing problem can be solved by imposing axial continuum Ward identities <sup>3</sup>, it represents a major obstacle for precision results. Note that the parity-odd operator component  $O_{VA+AV}$  does indeed renormalize multiplicatively <sup>2</sup>. This can be exploited for the computation of  $B_K$  as will be explained shortly.

### 3. QCD with chirally rotated mass terms

Axial and vector symmetries can be distinguished according to whether or not the quark mass term is left invariant. Hence, a non-standard form of the quark mass term also modifies the form of the symmetry transformations. Let us consider the continuum theory for a light quark doublet  $\psi$  and the  $s$  quark including a chirally twisted mass term,

$$\mathcal{L}_f = \bar{\psi} (\not{D} + m + i\mu_q \gamma_5 \tau^3) \psi + \bar{s} (\not{D} + m_s) s. \quad (3)$$

After a chiral rotation of the doublet fields

$$\psi' = \exp\left(i\alpha\gamma_5\frac{\tau^3}{2}\right)\psi, \quad \bar{\psi}' = \bar{\psi}\exp\left(i\alpha\gamma_5\frac{\tau^3}{2}\right), \quad (4)$$

and with  $\alpha$  chosen such that  $\tan\alpha = \mu_q/m$ , the Lagrangian reads

$$\mathcal{L}'_f = \bar{\psi}' (\not{D} + m') \psi' + \bar{s} (\not{D} + m_s) s, \quad m' = \sqrt{m^2 + \mu_q^2}. \quad (5)$$

The field rotation re-defines the symmetries and maps composite fields, e.g.

$$O'_{VV+AA} = \cos(\alpha)O_{VV+AA} - i\sin(\alpha)O_{VA+AV} = -iO_{VA+AV} \quad (\alpha = \pi/2). \quad (6)$$

In particular, the operators are mapped to each other at “maximal twist”  $\alpha = \pi/2 \Leftrightarrow m = 0$ , where the quark mass is determined entirely by the chirally twisted mass parameter  $\mu_q$ . Hence, by using Wilson quarks with a maximally twisted light quark doublet and a standard  $s$  quark, the complicated operator mixing problem can be by-passed <sup>4</sup>. An additional benefit consists in the elimination of unphysical zero modes by the twisted mass term. This allows for numerical simulations to get close to the physical situation: light, mass degenerate  $u, d$  quarks and a heavier  $s$  quark. However, most results in lattice QCD are currently obtained in the limit where the kaon is made out of mass-degenerate  $d$  and  $s$  quarks. In this way a quenched artefact is avoided <sup>5</sup>, but it implies that the zero mode problem is back for the standard Wilson  $s$  quark. In order to cope with mass-degenerate quarks we also consider a different set-up, where the rôles of  $s$  and  $u$  quarks are interchanged. The chirally twisted doublet is now  $\psi = (s, d)$  and after the chiral rotation one finds

$$O'_{VV+AA} = \cos(2\alpha)O_{VV+AA} - i\sin(2\alpha)O_{VA+AV} = -iO_{VA+AV} \quad (\alpha = \pi/4). \quad (7)$$

Again the operators are mapped to each other provided  $\alpha = \pi/4 \Leftrightarrow m = \mu_q$ . For obvious reasons we will refer to the two set-ups as  $\pi/2$  and  $\pi/4$  scenarios, respectively. The latter has the advantage that both  $d$  and  $s$  quark masses can be decreased simultaneously without encountering unphysical zero modes. Setting  $\alpha$  to  $\pi/2$  or  $\pi/4$  requires some parameter tuning using known results for finite renormalization constants. In addition one needs the multiplicative operator renormalization constant. The renormalization problem has been solved non-perturbatively in <sup>6</sup>, using a finite volume scheme based on the Schrödinger functional and recursive finite size techniques <sup>7</sup>.

#### 4. Numerical simulations

The numerical simulations have been performed using the  $O(a)$  improved Wilson quark action and the Wilson's plaquette action for the gauge fields. 4-5  $\beta$ -values have been chosen in the interval [6.0 – 6.45], corresponding to lattice spacings  $a = 0.05$ – $0.1$  fm, if the scale is set by  $r_0 = 0.5$  fm. The lattice volumes range from  $16^3 \times 48$  to  $32^2 \times 72$ . Quark masses are tuned to achieve  $\alpha = \pi/2$  or  $\alpha = \pi/4$  and pseudoscalar masses around or above  $m_K$ . The analysis of excited states determined safe plateaux regions where  $B_K$  could be extracted from suitable ratios of correlation functions. Finite volume effects were checked at the coarsest lattice spacing ( $\beta = 6.0$ ), and found to be below the statistical errors. In the  $\pi/2$  scenario chiral extrapolations were performed linearly to the physical kaon mass. In the  $\pi/4$  scenario the kaon mass could be reached by *interpolation* except at  $\beta = 6.45$  where finite volume effects at the physical kaon mass would have been non-negligible. The resulting values for the  $B_K$  are given in figure 1 as a function of  $a/r_0$ . Discarding the values at the coarsest lattice the data seem to scale very well. A continuum extrapolation of  $\pi/4$  data linear in  $a$  seems adequate.

#### 5. Conclusions

We have performed a benchmark calculation of  $B_K$  where, apart from the quenched approximation, all systematic errors are under control. These include cutoff effects, finite volume effects and the contamination by excited states. A non-perturbative renormalization procedure has been employed, and the continuum limit has been taken. The preliminary result

$$\hat{B}_K = 0.834(37) \quad \Leftrightarrow \quad B_K^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.604(27), \quad (8)$$

has the desired precision. Within errors it is compatible with results obtained by other groups using different lattice regularisations (see <sup>8</sup> for a recent review and further references). Hence, further progress will require the inclusion of dynamical quark flavours. The recent progress with simulation algorithms for Wilson-type quarks <sup>9</sup> will soon allow to compute  $B_K$  with a similar precision including the effect of light dynamical up and down quarks.

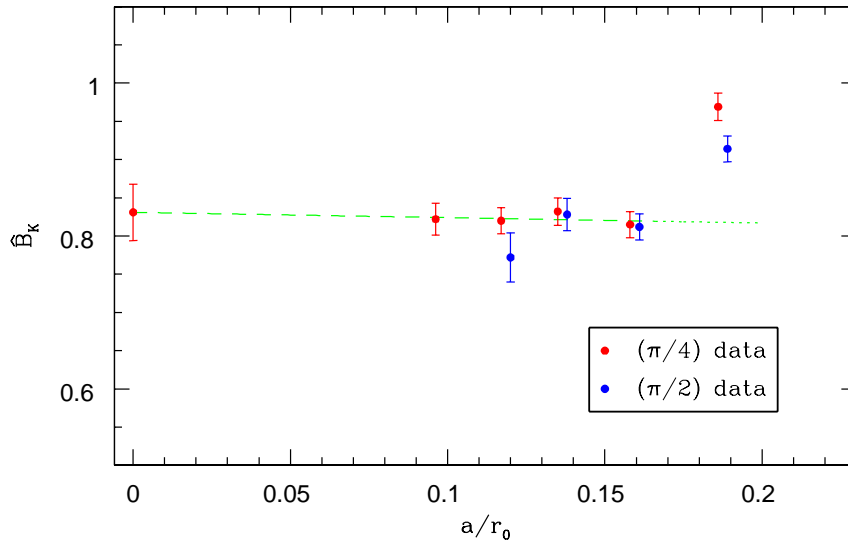
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Fig. 1.  $B_K$  data for both  $\pi/4$  and  $\pi/2$  scenarios. The  $\pi/2$  data is slightly shifted to the right to enhance readability. Also shown is the linear continuum extrapolation of  $\pi/4$  data excluding the data on the coarsest lattice.

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