

η_c - Glueball Mixing and Resonance X(1835)

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Abstract

The mixing of η_c and the lowest mass pseudoscalar glueball is estimated within the framework of the instanton liquid model. It is demonstrated that the mixing is large and may explain the difference between the observed mass of the glueball candidate X(1835) and the theoretical prediction of QCD sum rule analysis.

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Recently in [1] we presented our arguments to consider X(1835) resonance observed by BES Collaboration[2] in the reactions $J/\Psi \rightarrow \gamma p\bar{p}$ and $J/\Psi \rightarrow \gamma\eta'\pi^+\pi^-$ as the lowest mass pseudoscalar glueball³. Our interpretation is based on the appearance of the parity doublet structure for high mass hadronic excitations, which can be explained naturally within the instanton model for QCD vacuum. Thus, we have considered the doublet $[X(1835), f_0(1710)]$ as the parity doublet of lowest mass glueballs. Furthermore, the contribution of X(1835) to the flavor singlet axial vector coupling of proton and its influence to the proton spin problem with the large observed coupling of X(1835) to the $p\bar{p}$ channel were given there.

However, we left unexplained one doubt in interpreting the X(1835) as the lowest pseudoscalar glueball. That is the magnitude of its mass, which is lower than the predicted values of the quenched lattice approach, $2.1 \sim 2.5$ GeV [5], and QCD sum rules, 2.05 ± 0.19 GeV [6], 2.2 ± 0.2 GeV [7].

In this Letter we provide our conjecture that the η_c -glueball mixing can be a key factor of adjusting the mass of lowest pseudoscalar glueball to its experimental value.

The η_c has the same quantum numbers as the X(1835), ($I = 0, J^{PC} = 0^{-+}$) and the mass 2.98 GeV which is quite close to the lattice and QCD sum rule estimations of the pseudoscalar glueball mass. So the mixing of the η_c with glueball can be large due to the possible $c\bar{c}$ annihilation to two gluons. We will estimate this mixing by using the instanton model for QCD vacuum [8], [9]. The effective interaction responsible for the mixing follows from the gluon-gluon effective interaction induced by instantons [10, 8, 9]:

$$\mathcal{L}_{eff} = \int dU d\rho n(\rho) e^{-\frac{2\pi^2}{g_s} \rho^2 U_{ab} \bar{\eta}_{b\alpha\beta} G_{\alpha\beta}^a(x)} + (I \rightarrow \bar{I}), \quad (1)$$

where, $n(\rho)$ is the effective instanton density, ρ is the instanton size, g_s is a strong coupling constant, $\bar{\eta}_{b\alpha\beta}$ is the 'tHooft symbol, and U is the orientation matrix of the instanton in $SU(3)_c$ color space. The last term in Eq.(1) represents the contribution coming from anti-instantons. We are going to calculate the contribution of diagram illustrated in Fig.1 to the non-diagonal matrix element

$$\frac{\Delta M_{\eta_c G_P}^2}{2} = - \langle \eta_c | \mathcal{L}_{eff} | G_P \rangle . \quad (2)$$

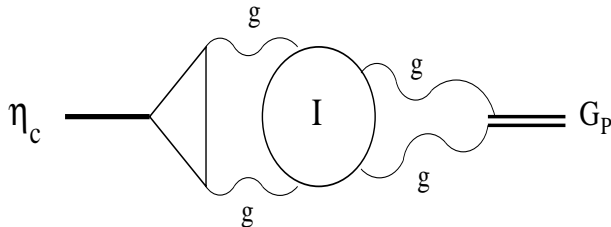


Figure 1: The η_c - pseudoscalar glueball G_P mixing induced by instanton. The symbol I denotes the instanton.

³The possibility of strong X(1835) coupling to gluons was pointed out by Rosner in [3]. The value of this coupling and X(1835) decay modes were discussed recently in [4] as well.

In the vacuum dominance approximation, which is very suitable in estimation of instanton contributions to various hadron decays (see [11] and [12]), the contribution corresponding the diagram in Fig.1 to the matrix element, Eq.(2), can be written in the following form

$$\frac{\Delta M_{\eta_c G_P}^2}{2} = \int d\rho n(\rho) \left(\frac{\pi^3 \rho^4}{8\alpha_s(\rho)} \right)^2 \langle \eta_c | G_{\alpha\beta}^a \tilde{G}_{\alpha\beta}^a | 0 \rangle \langle 0 | G_{\mu\nu}^b \tilde{G}_{\mu\nu}^b | G_P \rangle. \quad (3)$$

In framework of instanton model of QCD vacuum the matrix element $\langle \eta_c | G_{\alpha\beta}^a \tilde{G}_{\alpha\beta}^a | 0 \rangle$ has been calculated in [12] following the approach of [13]

$$\langle \eta_c | G_{\alpha\beta}^a \tilde{G}_{\alpha\beta}^a | 0 \rangle = \frac{64\alpha_s(2m_c)m_c^{3/2}|\Psi(0)|}{\pi^3\rho^4\sqrt{6}} I_{\eta_c}(\rho), \quad (4)$$

$$I_{\eta_c}(\rho) \simeq \frac{\pi^2 A_0 \rho^4 \log(1 + 1/(m_c \rho))}{1 + B_0 (m_c \rho)^4 \log(1 + 1/(m_c \rho))}, \quad (5)$$

where $A_0 = 0.213$, $B_0 = 0.124$. In Eq.(4) the $\Psi(0)$ is the 1S_0 wave function of charmonium at the origin. Our main result is

$$\frac{\Delta M_{\eta_c G_P}^2}{2} \simeq \int d\rho n(\rho) \frac{m_c^{3/2} \pi^3 \rho^4}{\sqrt{6} \alpha_s^2(\rho)} |\Psi(0)| I_{\eta_c}(\rho) \langle 0 | \alpha_s G_{\mu\nu}^b \tilde{G}_{\mu\nu}^b | G_P \rangle. \quad (6)$$

According to the instanton liquid model by Shuryak [14], instanton density is given by

$$n(\rho) = n_0 \delta(\rho - \rho_c), \quad (7)$$

where $n_0 \approx 0.5 \text{ fm}^{-4}$ and $\rho_c \approx 1/3 \text{ fm}$. We adopt our parameter values to fit the properties of charmonium, $m_c = 1.25 \text{ GeV}$, $|\Psi(0)| = 0.19 \text{ GeV}^{3/2}$ as in [12], and of the strong coupling constant at average instanton size $\alpha_s(\rho_c) = 0.52$ as in [9]. The coupling of X(1835) to gluons was obtained in our previous paper [1]

$$f_{G_P} = \langle 0 | \alpha_s G_{\mu\nu}^b \tilde{G}_{\mu\nu}^b | G_P \rangle \simeq 2.95 \text{ GeV}^3. \quad (8)$$

This value is consistent with the result of recent QCD sum rule analysis $f_{G_P} = 2.9 \pm 1.4 \text{ GeV}^3$ [7]. Our estimate of the mixing is

$$\Delta M_{\eta_c G_P}^2 \simeq 1.54 \text{ GeV}^2. \quad (9)$$

Now we are in the position to evaluate the effect of mixing on the mass of pseudoscalar glueball by using the following decomposition of physical charmonium and glueball states

$$\begin{aligned} |\eta_c \rangle &= |G_P^0 \rangle \sin\theta + |\eta_c^0 \rangle \cos\theta \\ |G_P \rangle &= |G_P^0 \rangle \cos\theta - |\eta_c^0 \rangle \sin\theta, \end{aligned} \quad (10)$$

where $|\eta_c^0 \rangle$ and $|G_P^0 \rangle$ are bare states. Let us assume that bare masses of glueball and η_c are following

$$M_{G_P}^0 = 2 \text{ GeV}, \quad M_{\eta_c}^0 = 2.9 \text{ GeV}. \quad (11)$$

These values lie inside the range of QCD sum rules expectation [6], [7], [15], [16] if one admits about 10% accuracy in predictions of this approach due to uncertainties in values

of various gluon condensates, mass of charm quark and α_s , high dimension operator contributions, etc. ⁴ As the result of the mixing Eq.(9), physical masses and mixing angle are

$$M_{G_P} \simeq 1.87 \text{ GeV}, \quad M_{\eta_c} \simeq 2.98 \text{ GeV}, \quad \theta \simeq 17^\circ. \quad (12)$$

Therefore, the mixing leads to the increasing of η_c mass to its experimental value and decreasing of the pseudoscalar glueball mass towards the mass of glueball candidate X(1835). The value of the mixing angle, Eq.(12), is rather large and should be taken into account in the calculation of different properties of η_c and pseudoscalar glueball with decay modes. In this connection we may point out that the mixing might be present behind of the observed large decay rates of η_c to $p\bar{p}$ and $\eta'\pi\pi$ final states [17] due to large coupling of glueball to these channels.

In principle, fine tuning of parameters allows us to bring the mass of glueball to observed one. However, we think that such procedure is beyond the accuracy of our approach based on the vacuum dominance approximation and definite instanton model for QCD vacuum. Furthermore, before the tuning process, some additional effects such as the glueball mixing with $\eta_c(2S)$, η' and others, which are beyond the scope of the present paper, should be taken into account.

In summary, we have shown that the instanton induced mixing of the charmonium and the pseudoscalar glueball is large and may explain the difference between the experimental mass of the glueball candidate X(1835) and the prediction of QCD sum rules. This observation provides the additional argument in favor of our suggestion in [1] to treat the X(1835) as the lowest mass pseudoscalar glueball.

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⁴Unfortunately, the available lattice results for the lowest mass pseudoscalar glueball have been performed in the quenched approximation [5]. It is rather hard to estimate the accuracy of such an approximation in the pseudoscalar channel where the light quark exchange between instantons plays an essential role (see discussion in [1]).

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