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GLUEBALLS AND THE POMERON – A LATTICE STUDY[a]

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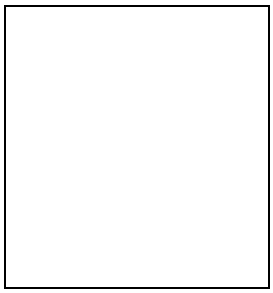

We perform lattice calculations of the lightest $J = 0, 2, 4, 6$ glueball masses in the D=3+1 SU(3) gauge theory and extrapolate to the continuum limit. Assuming that these masses lie on linear Regge trajectories we find a leading glueball trajectory $\alpha(t) = 0.93(24) + 0.28(2)\alpha'_R t$, where $\alpha'_R \simeq 0.9\,\mathrm{GeV}^{-2}$ is the slope of the usual mesonic Regge trajectories. This glueball trajectory has an intercept and slope similar to that of the Pomeron. We contrast this with the situation in D=2+1 where the leading glueball Regge trajectory is found to have too small an intercept to be important for high-energy cross-sections. We attempt to interpret the observed states and trajectories in terms of open and closed string models of glueballs.

1 Introduction and main results

The Pomeron trajectory is qualitatively different from other Regge trajectories in that it is much flatter (α' much smaller) and has a higher intercept[1], $\alpha_P(t = m^2) \simeq 1.08 + 0.25 m^2/\mathrm{GeV}^2$. There has been a long-standing speculation that the physical particles on the trajectory might be glueballs. If we consider the high-energy hadron scattering in a world deprived of the u, d and s quarks, it is difficult to imagine that the total cross-sections should behave very differently from those in the real world. For instance, in leading-logarithmic perturbative calculations[1], only the gluonic field contributes to the Pomeron. Thus it is reasonable to expect that the Pomeron phenomenon would also be observed in the absence of light quarks[6]. This constitutes the main motivation for the present investigation: we use numerical lattice techniques

[a] Work done in collaboration with Michael Teper.

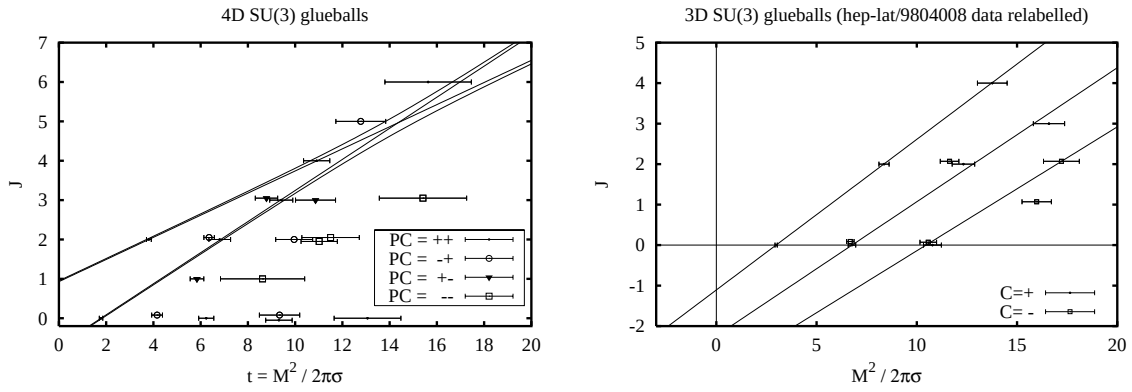

Figure 1: Chew-Frautschi plot of the continuum $SU(3)$ gauge theory, in $D = 3 + 1$ (left) and $D = 2 + 1$ (right).

to investigate whether the mass spectrum of the SU(3) pure gauge theory is consistent with approximately straight Regge trajectories, the leading one of which possesses the properties of the phenomenological Pomeron.

Our lattice calculations employ the standard Wilson action on a cubic lattice. We calculate ground and excited state masses, m, from Euclidean correlation functions using standard variational techniques. We calculate the string tension, σ, by calculating the mass of a flux loop that closes around a spatial torus. We perform calculations with a 2-level algorithm[4] at lattice spacings $a \simeq 0.10 - 0.05\,\mathrm{fm}$. The calculations are on lattices ranging from $16^3 36$ to $32^3 48$, corresponding to a spatial extent $L \simeq 1.5\,\mathrm{fm}$. We also perform a calculation on a lattice of size $2\,\mathrm{fm}$ so as to check that any finite volume corrections are small. We extrapolate the calculated values of the dimensionless ratio $m/\sqrt{\sigma}$ to $a = 0$ using an $a^2\sigma$ correction.

On the lattice, the problem with the identification of the lightest $J \geq 4$ states is that its cubic symmetry group is much smaller than the continuum rotation group and has just a few irreducible representations. Nonetheless, as $a \to 0$ an energy eigenstate belonging to one of these lattice representations will tend to some state that is labelled by spin J. So using continuity we can refer to a state at finite a as being of 'spin J' if a is small enough. At $a = 0$ a state of spin J is a multiplet of $2J + 1$ degenerate polarisations; if we now increase a from zero, these $2J+1$ polarisations will in general appear in different lattice representations, and the degeneracy will be broken at $O(a^2)$. So a particular polarisation of the ground state of spin $J = 4, 5, 6, ...$ will in general be a (highly) excited state in some lattice representation, thus complicating its identification. If we can perform this identification, then we can extrapolate the mass of the state to $a = 0$, so obtaining the mass of the lightest state of spin J. Our identification technique[2] is to perform a Fourier analysis of the rotational properties of any given lattice eigenstate. For this we may use as a 'probe' a set of fuzzy Wilson loops based on the propagator within one time-slice of a massive scalar field in the gauge field background. If we keep its mass fixed in physical units, it is guaranteed that the rotational invariance of the propagator is restored as $a \to 0$ (upon averaging over the gauge field). The Wilson loops, which have typically a size of 0.5fm, then have a rotational symmetry that is broken only by $O(a^2)$ effects, so that we can probe the rotational properties of the glueballs under rotations finer than $\pi/2$ to that accuracy. Subsequently we found more heuristic techniques to construct the probe operators, which however are computationally much cheaper and provided as good rotational properties as the propagator construction at the lattice spacings we were working at.

Having extrapolated our glueball masses to the continuum limit, we plot the squared masses against the spins in a Chew-Frautschi plot[2], as in Fig. 1 (left). We now assume that the states fall on approximately linear Regge trajectories. In that case the leading trajectory clearly passes through the lightest $J = 2$ and $J = 4$ glueballs. We note that there is no odd J state on this

trajectory: it is even signature. The parameters of the trajectory are $2\pi\sigma\alpha' = 0.281(22),\ \alpha_0 = 0.93(24)$, in agreement with the phenomenological Pomeron if we recall that the usual mesonic trajectories have slopes $\alpha'_{\mathcal{R}} \simeq \frac{1}{2\pi\sigma} \simeq 0.9\,\mathrm{GeV}^{-2}$. Of course, in comparing our leading pure-glue trajectory with the phenomenological Pomeron we should not ignore the fact that the latter will mix with the flavourless mesonic trajectory: this effect will presumably increase the intercept and the slope of the Pomeron.

We can identify the sub-leading glueball trajectory in Fig. 1 as well. It contains the lightest $J = 0$ glueball, the first-excited $J = 2$ glueball and the lightest $J = 3$ glueball. In striking contrast to what one finds for the usual mesonic trajectories, this secondary trajectory is clearly not parallel to the leading one. As we shall see, this is not unexpected in a string picture of glueballs. The trajectories, if linear, would cross somewhere near $J = 5$; because of unitarity the true trajectories will not cross but rather repel, as suggested on the figure.

The right plot shows the spectrum[9] in $D = 2 + 1$[b]. In contrast to $D = 3 + 1$, a linear trajectory between the lightest $J = 2$ and $J = 4$ states passes through the lightest $J = 0$ state. Between them the $J = 0, 2, 4$ states provide strong evidence for the approximate linearity of the trajectory. In contrast to $D = 3 + 1$ the secondary trajectory is approximately parallel to the leading one. The parameters of the leading trajectory are $2\pi\sigma\alpha' = 0.384(16),\ \alpha_0 = -1.144(71)$. Unlike the intercept, the slope of the trajectory is similar to what we found in $D = 3 + 1$.

2 Interpreting the glueball spectrum in terms of string models

A natural model for a high J meson is to see it as a rotating string with a q and $\bar{q}$ at its ends and at the classical level this leads to linear Regge trajectories, $J \overset{J\to\infty}{\sim} \frac{1}{2\pi\sigma}m^2$. If we now go to the pure gauge theory, this simple 'open string' model has an immediate analogue: two gluons joined by a string containing flux in the adjoint representation[7]. The rotating adjoint string produces a linear Regge trajectory $J \overset{J\to\infty}{\sim} \frac{1}{2\pi\sigma_A}m^2 \simeq \frac{2}{9\pi\sigma}m^2$ (assuming Casimir scaling). The trajectory is much flatter than the usual mesonic Regge trajectory, although not quite as flat as the phenomenological Pomeron or the leading glueball trajectory that we identified. Since the adjoint string comes back to itself under C, P or rotations of π, its spectrum contains $0^{++}, 2^{++}, 4^{++}, ...$ states, as expected for an even-signature Pomeron.

An equally natural model[8] pictures glueballs as composed of a closed loop of fundamental flux with no constituent gluons at all (one might expect some glueball states to be open strings and others to be closed strings, with mixing between the two). There are phonon-like excitations of this closed string which propagate around it and contribute to both its energy and its angular momentum. The whole loop can rotate around its diameter, obtaining angular momentum that way too. If one considers the set of states where the angular momentum is purely phononic one obtains an asymptotically linear Regge trajectory with slope[3] $J \overset{J\to\infty}{\sim} \frac{1}{8\pi\sigma}m^2$ while for a loop with purely (non-relativistic) orbital motion one obtains a linear trajectory with $J \overset{J\to\infty}{\sim} \frac{3\sqrt{3}}{32\pi\sigma}m^2$. In either case the slope $\alpha' \simeq 0.2 - 0.3\,\mathrm{GeV}^{-2}$ is in the right range for the Pomeron. The orbital trajectory leads to a trajectory of states with positive parity and $C = (-1)^J$, $J = 0, 1, 2\ldots$ For the leading phononic trajectory, the most striking feature is the absence of a $J = 1$ state: apart from a 0^{++} state, all PC combinations are then expected for $J \geq 2$.

The SU(3) gauge theory in $D = 2 + 1$ is linearly confining and therefore an effective string theory description is equally well motivated. Since the rotating open string lies in a plane, it provides a natural model for glueballs in two space dimensions. It will contribute states with J even and $C = +$. The closed string is also a possibility; the quantum numbers for the leading $C = +$ and $C = -$ phononic trajectories are $0^{++},\ 2^{\pm+},\ 3^{\pm+},\ 4^{\pm+}\ldots$ and $0^{--},\ 2^{\pm-},\ 3^{\pm-},\ 4^{\pm-}\ldots$ In the simplest form of the model, the two trajectories are degenerate.

[b] We do not refer to parity, because in two space dimensions one has automatic parity-doubling for $J \neq 0$.

Returning to the data, the $D = 2 + 1$ leading trajectory contains only even J states with $C = +$ and so is naturally interpreted as arising from a rotating open string. Since the intercept is sufficiently low, it can and does include a $J = 0$ state, in contrast to the case of 3 spatial dimensions. The first subleading trajectory has no $J = 1$ state, although it contains a $J = 3$ state, and possesses a $C = +/-$ degeneracy for the lower J where we have reliable calculations. All this strongly suggests a phononic trajectory of the closed string.

In $D = 3+1$, for $J \leq 4$ the leading trajectory contains only even spin states with $PC = ++$. This again suggests that the trajectory arises from a rotating open string carrying adjoint flux between the gluons at the end points. The subleading trajectory has no $J = 1$ state although it does appear to have a $J = 3$ state, again a feature of the closed string phononic spectrum.

The states one might expect to lie along the *odderon*[5] are the lightest odd J, $PC = --$ glueballs. From Fig. 1 we see that a trajectory defined by the lightest 1^{--} and 3^{--} will have a slope similar to the Pomeron and a very low, negative intercept. However, if the leading trajectory has an intercept around unity, as claimed phenomenologically, then the lightest 1^{--} glueball cannot lie on it, but will rather lie on a subleading trajectory. To test this possibility we need a good calculation of the lightest 5^{--} glueball, something we do not yet have.

3 Conclusions

Using novel lattice techniques, we have calculated the masses of higher spin glueballs in the continuum limit of the SU(3) gauge theory. We find a leading $PC = ++$ glueball trajectory $\alpha_P(t) = 0.93(24) + 0.25(2)t/\mathrm{GeV}^2$ (assuming linearity) which is entirely consistent with the phenomenological Pomeron. The sub-leading trajectory has a larger slope and eventually 'crosses' the Pomeron. We argue that such a rich Regge structure for glueballs occurs naturally within string models.

In contrast to this, we find that in 2+1 dimensions the intercept of the leading trajectory is negative so that it does not contribute significantly to scattering at high energies. We find evidence that the leading trajectory is an open string while the non-leading one is a closed string. In this case we have enough accurately calculated glueball states along the leading trajectory to demonstrate its approximate linearity.

In a world deprived of the u, d and s quarks, the mass gap would be given by the lightest glueball, $m_G \simeq 1.6\mathrm{GeV}$; the Froissart bound is then stronger by two orders of magnitude[10]. Experimentally, the high-energy pp cross-section lies only slightly below that bound. If the cross-section is found to exceed it at the LHC, then it will definitely be necessary to include the effects of light quarks in the description of the hadronic wave-functions at that energy.

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