

Strong decays of $D_{sJ}(2317)$ and $D_{sJ}(2460)$

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With the identification of $(D_{sJ}(2317), D_{sJ}(2460))$ as the $(0^+, 1^+)$ doublet in the heavy quark effective field theory, we derive the light cone QCD sum rule for the coupling of eta meson with $D_{sJ}(2317)D_s$ and $D_{sJ}(2460)D_s^*$. Through $\eta - \pi^0$ mixing we calculate their pionic decay widths, which are consistent with the experimental values (or upper limits). Combining the radiative decay widths derived by Colangelo, Fazio and Ozpineci in the same framework, we conclude that the decay patterns of $D_{sJ}(2317, 2460)$ strongly support their interpretation as ordinary $c\bar{s}$ mesons.

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I. INTRODUCTION

Since the discovery of $D_{sJ}(2317)$ [1] and $D_{sJ}(2460)$ [2], there have been lots of experimental investigations of these two narrow resonances [3, 4, 5, 6, 7, 8, 9, 10, 11]. They have the natural spin-parity assignment as the $0^+, 1^+$ charm-strange mesons from the observed final states. Their masses are about one hundred MeV lower than the quark model prediction [12], which are really unexpected. Many theoretical papers have been dedicated to the understanding of their underlying structure [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]. Proposed schemes include the $(0^+, 1^+)$ chiral partners of the D_s, D_s^* doublet in heavy quark effective field theory [13, 14, 15], DK molecules [16], four quark states [17, 18, 19, 20, 21, 22, 23, 24], $D\pi$ atom [25] and conventional $c\bar{s}$ states [13, 14, 26, 27, 28, 29, 30, 31, 32, 33].

These two states are lower than the DK and D^*K thresholds respectively. Their strong decays are isospin violating and occur through two steps: $D_{sJ}(2317) \rightarrow D_s + \eta \rightarrow D_s + \pi^0$ and $D_{sJ}(2460) \rightarrow D_s^* + \eta \rightarrow D_s^* + \pi^0$. The second step is induced by the $\eta - \pi^0$ mixing due to the mass difference between m_u and m_d [37]. There have been some discussions of their strong and radiative decays within the quark model [13, 17, 36, 38, 39, 40, 41]. The strong decay widths from various approaches differ significantly as can be seen from Table I. Their radiative decay widths were calculated using light cone QCD sum rule (LCQSR) not long ago [42]. Very recently, the branch ratios of the strong and radiative decays were measured quite accurately by Belle Collaboration [3, 4] and Babar Collaboration [8, 11]. In order to pin down the underlying quark content of these narrow states, a reliable calculation of their strong decay widths will be very helpful.

In this work, we assume $(D_{sJ}(2317), D_{sJ}(2460))$ as the $c\bar{s}$ states and study their strong decays in the LCQSR framework, which has been used extensively in extracting low-lying hadron masses and coupling constants in the past decade (see Ref. [43] for a review). This paper is organized as follows. We calculate the coupling constant $g_{D_{s0}D_s\eta}$ and the strong decay width of $D_{sJ}(2317) \rightarrow D_s\pi^0$ through $\eta - \pi^0$ mixing in Section II. $D_{sJ}(2460)$ decay is presented in Section III. We compare our results with experimental data and other theoretical approaches in literature and summarize our results in Section IV. We collect the light cone wave functions of the η meson in the appendix.

II. $D_{sJ}(2317) \rightarrow D_s + \eta \rightarrow D_s + \pi^0$

The amplitude of the strong decay $D_{sJ}(2317) \rightarrow D_s + \eta$ can be defined as

$$\langle \eta(q) D_s(p) | D_{s0}(p+q) \rangle = m_{D_{s0}} g_{D_{s0}D_s\eta} . \quad (1)$$

We calculate the coupling constant $g_{D_{s0}D_s\eta}$ through the following correlation function

$$F(p^2, (p+q)^2) = i \int d^4x e^{ip \cdot x} \langle \eta(q) | T [J_5^\dagger(x) J_0(0)] | 0 \rangle , \quad (2)$$

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where $J_0(x) = \bar{c}(x)s(x)$ and $J_5(x) = \bar{c}(x)i\gamma_5s(x)$ are the interpolating currents of \bar{D}_{s0} and \bar{D}_s respectively.

At the quark level, the correlation function can be expressed in terms of the eta meson light cone wave functions after the insertion of the charm quark propagator at the leading order

$$\langle 0|T\{c(x)\bar{c}(0)\}|0\rangle = i\hat{S}_c^0(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\not{k} + m_c}{m_c^2 - k^2 - i\epsilon}. \quad (3)$$

Now we have

$$F(p^2, (p+q)^2) = \int \frac{d^4k}{(2\pi)^4} \int d^4x \frac{e^{i(p-k)\cdot x}}{m_c^2 - k^2} [m_c \langle \eta(q) | \bar{s}(x) i\gamma_5 s(0) | 0 \rangle - ik^\alpha \langle \eta(q) | \bar{s}(x) \gamma_\alpha s(0) | 0 \rangle]. \quad (4)$$

In order to include the contribution from the twist-four eta meson light-cone wave functions, we need the three particle piece in the charm quark propagator:

$$\langle 0|T\{c(x)\bar{c}(0)\}|0\rangle = i\hat{S}_c^0(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[\frac{1}{2} \frac{\not{k} + m_c}{(m_c^2 - k^2)^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} + \frac{1}{m_c^2 - k^2} vx_\mu G^{\mu\nu}(vx) \gamma_\nu \right], \quad (5)$$

where $G_{\mu\nu} = G_{\mu\nu}^c \frac{\lambda^c}{2}$ with $\text{tr}(\lambda^a \lambda^b) = 2\delta^{ab}$, and g_s is the strong coupling constant. The complete expression of $F(p^2, (p+q)^2)$ up to twist-four reads:

$$\begin{aligned} F(p^2, (p+q)^2) &= \int_0^1 \frac{du}{m_c^2 - (p+uq)^2} \left\{ -F_\eta \varphi_\eta(u) p \cdot q + m_c F_\eta \mu_\eta \varphi_p(u) \right. \\ &\quad \left. - F_\eta m_\eta^2 \left(1 + \frac{m_c^2}{m_c^2 - (p+uq)^2} \right) G(u) + \frac{1}{4} F_\eta m_\eta^2 \frac{A(u) p \cdot q}{m_c^2 - (p+uq)^2} \left[1 + \frac{2m_c^2}{m_c^2 - (p+uq)^2} \right] \right\} \\ &\quad + F_\eta m_\eta^2 \int_0^1 dv \int \mathcal{D}\alpha_i \frac{p \cdot q}{m_c^2 - (p + (\alpha_1 + v\alpha_3)q)^2} \left\{ 2v(2\varphi_\perp - \varphi_\parallel) - (2\varphi_\perp - \varphi_\parallel + 2\tilde{\varphi}_\perp - \tilde{\varphi}_\parallel) \right\}, \quad (6) \end{aligned}$$

where

$$G(u) = - \int_0^u du' B(u'). \quad (7)$$

$\varphi_\eta(u)$, $\varphi_p(u)$, $B(u)$ etc are light cone amplitudes of η meson [44, 45], which are collected in the Appendix. $F_\eta = -\frac{2}{\sqrt{6}}f_\eta$, with f_η defined as

$$\langle 0 | \frac{1}{\sqrt{6}} (\bar{u}(0)\gamma_\mu\gamma_5u(0) + \bar{d}(0)\gamma_\mu\gamma_5d(0) - 2\bar{s}(0)\gamma_\mu\gamma_5s(0)) | \eta(q) \rangle = if_\eta q_\mu. \quad (8)$$

At the phenomenological level, $F(p^2, (p+q)^2)$ can be expressed as

$$F(p^2, (p+q)^2) = \frac{m_{D_{s0}}^2 m_{D_s} f_{D_{s0}} f_{D_s} g_{D_{s0} D_s \eta}}{(m_{D_s}^2 - p^2)(m_{D_{s0}}^2 - (p+q)^2)} + \dots. \quad (9)$$

The ellipse denotes the contribution from the continuum. The decay constants $f_{D_{s0}}$ and f_{D_s} are defined as

$$\langle 0 | J_5^\dagger | D_s \rangle = f_{D_s} m_{D_s}, \quad (10)$$

$$\langle D_{s0} | J_0 | 0 \rangle = f_{D_{s0}} m_{D_{s0}}. \quad (11)$$

Applying the double Borel transformation with respect to p^2 and $(p+q)^2$ to Eqs. (6) and (9) and invoking the quark-hadron duality, we get the following sum rule:

$$\begin{aligned} f_{D_{s0}} f_{D_s} g_{D_{s0} D_s \eta} &= \frac{1}{m_{D_{s0}}^2 m_{D_s}} e^{\frac{m_{D_{s0}}^2 + m_{D_s}^2}{2M^2}} \left\{ M^2 [e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}] \left[\frac{1}{2} M^2 F_\eta \varphi'_\eta(u_0) + m_c F_\eta \mu_\eta \varphi_p(u_0) \right. \right. \\ &\quad \left. \left. - F_\eta m_\eta^2 G(u_0) - \frac{1}{8} F_\eta m_\eta^2 A'(u_0) + \frac{1}{2} F_\eta m_\eta^2 (2I_1(2\varphi_\perp - \varphi_\parallel) - I_2(2\varphi_\perp - \varphi_\parallel + 2\tilde{\varphi}_\perp - \tilde{\varphi}_\parallel)) \right] \right. \\ &\quad \left. - e^{-\frac{m_c^2}{M^2}} F_\eta m_\eta^2 m_c^2 \left[G(u_0) + \frac{1}{8} A'(u_0) \right] \right\}_{u_0=1/2}. \quad (12) \end{aligned}$$

The functions I_1 and I_2 are defined as

$$I_1(\mathcal{F}) = \int_0^{u_0} d\alpha_1 \left[\frac{1}{u_0 - \alpha_1} \mathcal{F}(\alpha_1, 1 - u_0, u_0 - \alpha_1) - \int_{u_0 - \alpha_1}^{1 - \alpha_1} d\alpha_3 \frac{\mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)}{\alpha_3^2} \right], \quad (13)$$

$$I_2(\mathcal{F}) = - \int_0^{1 - u_0} d\alpha_3 \frac{\mathcal{F}(u_0, 1 - u_0 - \alpha_3, \alpha_3)}{\alpha_3} + \int_0^{u_0} d\alpha_1 \frac{\mathcal{F}(\alpha_1, 1 - u_0, u_0 - \alpha_1)}{u_0 - \alpha_1}, \quad (14)$$

where \mathcal{F} is one of the twist-four light cone amplitudes φ_{\parallel} , φ_{\perp} , $\tilde{\varphi}_{\parallel}$, $\tilde{\varphi}_{\perp}$.

In Eq.(12), $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$, $u_0 = \frac{M_1^2}{M_1^2 + M_2^2}$. Since there exists an overlapping working window for the two Borel parameters M_1^2, M_2^2 , it's convenient to let $M_1^2 = M_2^2$, i.e., $u_0 = \frac{1}{2}$. The eta meson light-cone wave functions are known quite well at $u_0 = \frac{1}{2}$. Such a choice allows the clean subtraction of the continuum contribution. We can simply introduce a threshold parameter s_0 and replace $M^2 e^{-\frac{m_c^2}{M^2}}$ with $M^2 \left(e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}} \right)$ to subtract the contribution from the continuum and excited states [46].

In the numerical analysis, the values of parameters in the above sum rule are: $m_{D_{s_0}} = 2.317$ GeV, $m_{D_s} = 1.968$ GeV, m_c (1 GeV)=1.35 GeV, $f_{D_s} = (266 \pm 32)$ MeV [47], $f_{D_{s_0}} = (225 \pm 25)$ MeV [43]. For f_{D_s} and $f_{D_{s_0}}$ we use the central values. Values of the other parameters are given in the Appendix. The variation of $g_{D_{s_0}D_s\eta}$ with M^2 for the different s_0 is shown in Fig. 1.

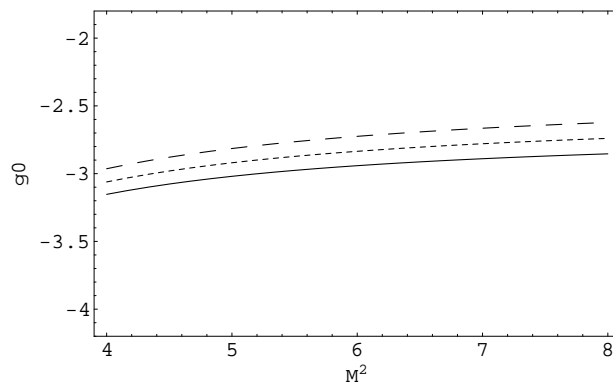


FIG. 1: The variation of the coupling constant $g_{D_{s_0}D_s\eta}$ with M^2 (in unit of GeV^2). The long-dashed, short-dashed and solid curves correspond to $s_0 = 6.0, 6.25, 6.5$ GeV^2 respectively.

In the working region of the Borel parameter $5 \text{ GeV}^2 < M^2 < 7 \text{ GeV}^2$, we get

$$-3.02 < g_{D_{s_0}D_s\eta} < -2.66, \quad (15)$$

where the uncertainty arises from the variation of M^2 and s_0 . Numerically, the twist-three term φ_p has the largest contribution to the sum rule.

The pionic decay of $D_{sJ}(2317)$ occurs through $\eta - \pi^0$ mixing, which is described by the isospin violating piece in the chiral lagrangian

$$\mathcal{L}_m = \frac{m_\pi^2 f^2}{4(m_u + m_d)} \text{Tr}(\xi m_q \xi + \xi^\dagger m_q \xi^\dagger), \quad (16)$$

where $\xi = \exp(i\tilde{\pi}/f_\pi)$, $\tilde{\pi}$ the light meson octet and m_q is the light quark mass matrix. The mass difference between up and down quarks induces the $\eta - \pi^0$ mixing with a suppression factor around $\frac{m_d - m_u}{m_s - \frac{m_u + m_d}{2}}$. Finally the strong decay width reads

$$\Gamma(D_{sJ}(2317) \rightarrow D_s \pi^0) = \frac{3}{144\pi} g_{D_{s_0}D_s\eta}^2 \left(\frac{m_d - m_u}{m_s - \frac{m_u + m_d}{2}} \right)^2 |\vec{p}_1|. \quad (17)$$

Numerically we have

$$\Gamma(D_{sJ}(2317) \rightarrow D_s \pi^0) = (34 - 44) \text{ keV}. \quad (18)$$

III. $D_{sJ}(2460) \rightarrow D_s^* + \eta \rightarrow D_s^* + \pi^0$

For $D_{sJ}(2460)$ decay, we define the following matrix element

$$\langle \eta(q) D_s^*(p) | D_{s1}'(p+q) \rangle = m_{D_{s1}'} g_{D_{s1}' D_s^* \eta} \eta^\mu \epsilon_\mu^*, \quad (19)$$

where η_μ and ϵ_μ are the polarization tensors for the 1^+ and 1^- states D_{s1}', D_s^* respectively. We start from the correlation function

$$F_{\mu\nu}(p^2, (p+q)^2) = i \int d^4x e^{ip \cdot x} \langle \eta(q) | T [J_\mu^\dagger(x) J_\nu^A(0)] | 0 \rangle \quad (20)$$

where $J_\mu(x) = \bar{c}(x) \gamma_\mu s(x)$ and $J_\nu^A(x) = \bar{c}(x) \gamma_\nu \gamma_5 s(x)$. At the hadron level, we have

$$F_{\mu\nu}(p^2, (p+q)^2) = \frac{m_{D_{s1}'}^2 m_{D_s^*} f_{D_{s1}'} f_{D_s^*} g_{D_{s1}' D_s^* \eta}}{(m_{D_{s1}'}^2 - p^2)(m_{D_{s0}}^2 - (p+q)^2)} (g_{\mu\nu} + \frac{m_{D_{s1}'}^2 - m_{D_s^*}^2}{2m_{D_{s1}'}^2 m_{D_s^*}^2} q_\mu p_\nu + \dots), \quad (21)$$

where we have kept the $g_{\mu\nu}$ and $q_\mu p_\nu$ structures. The decay constants $f_{D_{s1}'}$ and $f_{D_s^*}$ are defined as

$$\langle 0 | J_\mu^+ | D_s^* \rangle = f_{D_s^*} m_{D_s^*} \epsilon_\mu, \quad (22)$$

$$\langle D_{s1}' | J_\nu^A | 0 \rangle = f_{D_{s1}'} m_{D_{s1}'} \eta_\nu^*. \quad (23)$$

Following the same procedure as in Section II, we obtain a sum rule from the $g_{\mu\nu}$ structure

$$\begin{aligned} f_{D_{s1}'} f_{D_s^*} g_{D_{s1}' D_s^* \eta} &= \frac{1}{m_{D_{s1}'}^2 m_{D_s^*}^2} e^{\frac{m_{D_{s1}'}^2 + m_{D_s^*}^2}{2M^2}} \left\{ M^2 [e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}] \left[-\frac{1}{2} M^2 F_\eta \phi_\eta'(u_0) - m_c F_\eta \mu_\eta \varphi_p(u_0) \right. \right. \\ &+ \frac{1}{8} F_\eta m_c^2 A'(u_0) + \frac{1}{2} F_\eta m_\eta^2 (2I_1(\varphi_\parallel) - I_2(\varphi_\parallel + 2\tilde{\varphi}_\perp - \tilde{\varphi}_\parallel))] \\ &\left. \left. + e^{-\frac{m_c^2}{M^2}} \left[\frac{1}{8} F_\eta m_c^4 A'[u_0] + F_\eta m_\eta^2 m_c^2 G(u_0) \right] \right\}_{u_0=1/2}. \end{aligned} \quad (24)$$

Similarly we can get a second sum rule from the $q_\mu p_\nu$ structure

$$\begin{aligned} f_{D_{s1}'} f_{D_s^*} g_{D_{s1}' D_s^* \eta} &= \frac{2m_{D_s^*}}{m_{D_{s1}'}^2 - m_{D_s^*}^2} e^{\frac{m_{D_{s1}'}^2 + m_{D_s^*}^2}{2M^2}} \left\{ M^2 [e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}] [-F_\eta \varphi_\eta(u_0)] \right. \\ &+ e^{-\frac{m_c^2}{M^2}} \left[\frac{1}{3} m_c F_\eta \mu_\eta \varphi_\sigma(u_0) + \frac{1}{4} m_c^2 F_\eta (1 + \frac{m_c^2}{M^2}) A(u_0) - 2F_\eta m_\eta^2 u_0 G(u_0) \right. \\ &\left. \left. + F_\eta m_\eta^2 (I_3(\varphi_\parallel + 2\tilde{\varphi}_\perp - \tilde{\varphi}_\parallel) - 2I_4(2\varphi_\perp + \varphi_\parallel)) \right] \right\}_{u_0=1/2}. \end{aligned} \quad (25)$$

The functions I_3 and I_4 in Eqs. (24) and (25) are defined as

$$I_3(\mathcal{F}) = \int_0^{u_0} d\alpha_1 \int_{u_0 - \alpha_1}^{1 - \alpha_1} d\alpha_3 \frac{\mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)}{\alpha_3}, \quad (26)$$

$$I_4(\mathcal{F}) = \int_0^{u_0} d\alpha_1 \int_{u_0 - \alpha_1}^{1 - \alpha_1} d\alpha_3 \frac{u_0 - \alpha_1}{\alpha_3^2} \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3). \quad (27)$$

Unfortunately the sum rule Eq. (25) is very unstable. There is no working window for the Borel parameter M^2 . In the following we focus on the sum rule Eq. (24). We use $m_{D_{s1}'} = 2.460$ GeV, $m_{D_s^*} = 2.112$ GeV [47], $f_{D_{s1}'} \simeq f_{D_{s0}}$, $f_{D_s^*} \simeq f_{D_s}$ [42]. The variation of $g_{D_{s1}' D_s^* \eta}$ with M^2 is presented in Fig. 2.

In the working window of Borel parameter $4 \text{ GeV}^2 < M^2 < 6 \text{ GeV}^2$, we have

$$2.61 < g_{D_{s1}' D_s^* \eta} < 3.14. \quad (28)$$

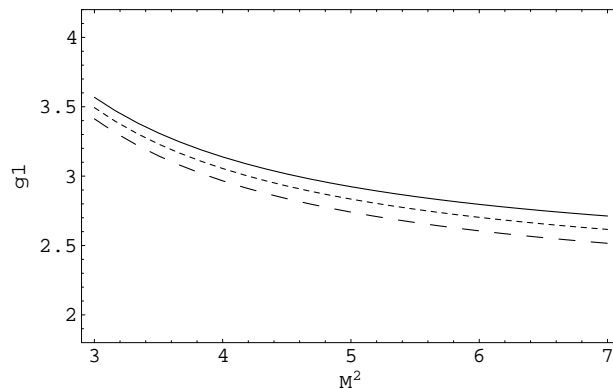


FIG. 2: The variation of the coupling constant $g_{D'_{s1}D_s^*\eta}$ with M^2 (in unit of GeV^2). The long-dashed, short-dashed and solid curves correspond to $s_0 = 6.25, 6.5, 6.75 \text{ GeV}^2$ respectively.

The contribution from φ_p term is also very important numerically. The pinonic decay width reads

$$\Gamma(D_{sJ}(2460) \rightarrow D_s^* + \pi^0) = \frac{g_{D'_{s1}D_s^*\eta}^2}{144\pi} \left(2 + \frac{(m_{D'_{s1}}^2 + m_{D_s^*}^2)^2}{4m_{D'_{s1}}^2 m_{D_s^*}^2} \right) \left(\frac{m_d - m_u}{m_s - \frac{m_u + m_d}{2}} \right)^2 |\vec{p}_1|. \quad (29)$$

Finally we have

$$\Gamma(D_{sJ}(2460) \rightarrow D_s^* + \pi^0) = (35 - 51) \text{ keV}. \quad (30)$$

IV. DISCUSSION

The strong decay widths of $D_{sJ}(2317) \rightarrow D_s + \pi^0$ and $D_{sJ}(2460) \rightarrow D_s^* + \pi^0$ have been calculated by several groups. Their results are collected in Table I together with ours. The first five calculations assume $c\bar{s}$ picture while the last two use composite non- $c\bar{s}$ pictures. The decay width of $D_{sJ}(2317)$ is roughly the same as that of $D_{sJ}(2460)$ from all approaches. The $1/m_c$ correction is expected to modify the small values in the second column from vector dominance model in the heavy quark limit [42].

TABLE I: Strong decay widths (in keV) of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ from various theoretical approaches.

	LCQSR	[41]	[13]	[38]	[39]	[17]	[48]
$D_{sJ}(2317) \rightarrow D_s\pi^0$	34-44	7 ± 1	$21.5 \simeq 10$	16	10-100	150 ± 70	
$D_{sJ}(2460) \rightarrow D_s^*\pi^0$	35-51	7 ± 1	$21.5 \simeq 10$	32		150 ± 70	

The radiative decay widths of $D_{sJ}(2317, 2460)$ were calculated using LCQSR in [42]: $\Gamma(D_{sJ}(2317) \rightarrow D_s^* + \gamma) = (4 - 6) \text{ keV}$, $\Gamma(D_{sJ}(2460) \rightarrow D_s\gamma) = (19 - 29) \text{ keV}$, $\Gamma(D_{sJ}(2460) \rightarrow D_s^* + \gamma) = (0.6 - 1.1) \text{ keV}$, $\Gamma(D_{sJ}(2460) \rightarrow D_{sJ}(2317) + \gamma) = (0.5 - 0.8) \text{ keV}$. Experimentally only $D_{sJ}(2460) \rightarrow D_s\gamma$ has been observed by Belle [3, 4] and Babar [8, 11]. We have collected the experimental ratio of radiative and strong decays of D_{sJ} mesons together with the central values of theoretical predictions from LCQSR based on Ref. [42] and present work in Table II. For $D_{sJ}(2460) \rightarrow D_s\gamma$, we get a range 0.37-0.83 for the ratio, consistent with both Belle and Babar's measurement.

In short summary, we have calculated the coupling constants $g_{D_{s0}D_s\eta}$ and $g_{D'_{s1}D_s^*\eta}$ in the framework of LCQSR. Through the $\eta - \pi^0$ mixing we obtain $\Gamma(D_{sJ}(2317) \rightarrow D_s\pi^0) = (34 - 44) \text{ keV}$ and $\Gamma(D_{sJ}(2460) \rightarrow D_s^*\pi^0) = (35 - 51) \text{ keV}$. These two widths are similar in magnitude, as expected from heavy quark symmetry. The ratio between the radiative widths and strong decay widths obtained in the same LCQSR framework is consistent with Belle and Babar's most recent data, which strongly indicates $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are conventional $c\bar{s}$ mesons. In the future, B decays into D_{sJ} mesons may also play an important role in exploring these charming states [49, 50, 51, 52, 53, 54, 55].

TABLE II: Comparison between experimental ratio of $D_{sJ}(2317, 2460)$ radiative and strong decay widths and theoretical predictions from LCQSR based on Ref. [42] and this work.

	Belle	Babar	CLEO [2]	LCQSR
$\frac{\Gamma(D_{sJ}^*(2317) \rightarrow D_s^* \gamma)}{\Gamma(D_{sJ}^*(2317) \rightarrow D_s \pi^0)}$	< 0.18 [4]	—	< 0.059	0.13
$\frac{\Gamma(D_{sJ}(2460) \rightarrow D_s \gamma)}{\Gamma(D_{sJ}(2460) \rightarrow D_s^* \pi^0)}$	$0.55 \pm 0.13 \pm 0.08$ [4]	$0.375 \pm 0.054 \pm 0.057$ [11]	< 0.49	0.56
$\frac{\Gamma(D_{sJ}(2460) \rightarrow D_s^* \gamma)}{\Gamma(D_{sJ}(2460) \rightarrow D_s^* \pi^0)}$	< 0.31 [4]	—	< 0.16	0.02
$\frac{\Gamma(D_{sJ}(2460) \rightarrow D_{sJ}^*(2317) \gamma)}{\Gamma(D_{sJ}(2460) \rightarrow D_s^* \pi^0)}$	—	< 0.23 [10]	< 0.58	0.015

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Appendix

We use $\bar{q}\Gamma q$ to denote $(\bar{u}\Gamma u + \bar{d}\Gamma d - 2\bar{s}\Gamma s)/\sqrt{6}$. Up to twist four, the two- and three-particle light-cone wave functions of eta meson can be written as [44, 45]:

$$\begin{aligned}
\langle \eta | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -i f_\eta q_\mu \int_0^1 du e^{iuqx} [\varphi_\eta(u) + \frac{1}{16} m_\eta^2 x^2 A(u)] \\
&\quad - \frac{i}{2} f_\eta m_\eta^2 \frac{q_\mu}{qx} \int_0^1 du e^{-iuqx} B(u), \\
\langle \eta | \bar{q}(x) i \gamma_5 q(0) | 0 \rangle &= f_\eta \mu_\eta \int_0^1 du e^{iuqx} \varphi_P(u), \\
\langle \eta | \bar{q}(x) \sigma_{\mu\nu} \gamma_5 q(0) | 0 \rangle &= \frac{i}{6} f_\eta \mu_\eta (q_\mu x_\nu - q_\nu x_\mu) \int_0^1 du e^{-iuqx} \varphi_\sigma(u), \\
\langle \eta | \bar{q}(x) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(ux) q(x) | 0 \rangle &= i f_\eta \mu_\eta \eta_3 [(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \\
&\quad \int \mathcal{D}\alpha_i \varphi_{3\eta}(\alpha_i) e^{-iqx(\alpha_1 + v\alpha_3)}, \\
\langle \eta | \bar{q}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) q(0) | 0 \rangle &= f_\eta m_\eta^2 \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{-iqx(\alpha_1 + v\alpha_3)} \\
&\quad + f_\eta m_\eta^2 \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{-iqx(\alpha_1 + v\alpha_3)}, \\
\langle \eta | \bar{q}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) q(x) | 0 \rangle &= i f_\eta m_\eta^2 \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{-iqx(\alpha_1 + v\alpha_3)} \\
&\quad + i f_\eta m_\eta^2 \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)},
\end{aligned}$$

where the operator $\tilde{G}_{\alpha\beta}$ is the dual of $G_{\alpha\beta}$: $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho}$; $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ and $f_\eta \simeq 1.2 f_\pi = 0.156$ GeV, $\eta_3 = 0.013$, $m_s(1 \text{ GeV}) = 0.125$ GeV, $m_\eta = 0.548$ GeV, $\mu_\eta = \frac{m_\eta^2}{m_s} = 2.4$ GeV [47].

The distribution amplitudes φ_η etc can be parameterized as

$$\begin{aligned}
\varphi_\eta(u) &= 6u\bar{u}(1 + a_2 C_2^{3/2}(\zeta) + a_4 C_4^{3/2}(\zeta)) , \\
\phi_p(u) &= 1 + (30\eta_3 - \frac{5}{2}\rho_\eta^2)C_2^{1/2}(\zeta) + (-3\eta_3\omega_3 - \frac{27}{20}\rho_\eta^2 - \frac{81}{10}\rho_\eta^2 a_2)C_4^{1/2}(\zeta) , \\
\phi_\sigma(u) &= 6u(1-u)\{1 + (5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{27}{20}\rho_\eta^2 - \frac{3}{5}\rho_\eta^2 a_2)\}C_2^{3/2}(\zeta) , \\
g_\eta(u) &= 1 + \{1 + \frac{18}{7}a_2 + 60\eta_3 + \frac{20}{3}\eta_4\}C_2^{1/2}(\zeta) + \{-\frac{9}{28}a_2 - 6\eta_3\omega_3\}C_4^{1/2}(\zeta) , \\
\mathbb{A}(u) &= 6u\bar{u}\{\frac{16}{15} + \frac{24}{35}a_2 + 20\eta_3 + \frac{20}{9}\eta_4 \\
&\quad + (-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3\omega_3 - \frac{10}{27}\eta_4)C_2^{3/2}(\xi) + (-\frac{11}{210}a_2 - \frac{4}{135}\eta_3\omega_3)C_4^{3/2}(\xi)\} \\
&\quad + (-\frac{18}{5}a_2 + 21\eta_4\omega_4)\{2u^3(10 - 15u + 6u^2) \ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \ln \bar{u} \\
&\quad + u\bar{u}(2 + 13u\bar{u})\} , \\
\varphi_{3\eta}(\alpha_1, \alpha_2, \alpha_3) &= 360\eta_3\alpha_1\alpha_2\alpha_3^2\{1 + \frac{1}{2}\omega_3(7\alpha_3 - 3)\} , \\
\varphi_{\parallel}(\alpha_1, \alpha_2, \alpha_3) &= 120\alpha_2\alpha_1\alpha_3(a_{10}(\alpha_1 - \alpha_2)) , \\
\varphi_{\perp}(\alpha_1, \alpha_2, \alpha_3) &= 30\alpha_3^2(\alpha_2 - \alpha_1)[h_{00} + h_{01}\alpha_3 + \frac{1}{2}h_{10}(5\alpha_3 - 3)] , \\
\tilde{\varphi}_{\parallel}(\alpha_1, \alpha_2, \alpha_3) &= 120\alpha_1\alpha_2\alpha_3(v_{00} + v_{10}(3\alpha_3 - 1)) , \\
\tilde{\varphi}_{\perp}(\alpha_1, \alpha_2, \alpha_3) &= -30\alpha_3^2\{h_{00}\bar{\alpha}_3 + h_{01}[\alpha_g\bar{\alpha}_3 - 6\alpha_1\alpha_2] + h_{10}[\alpha_3\bar{\alpha}_3 - \frac{3}{2}(\alpha_1^2 + \alpha_2^2)]\} ,
\end{aligned}$$

where $\bar{u} \equiv 1 - u$, $\zeta \equiv 2u - 1$, $\bar{\alpha} = 1 - \alpha$. $C_{2,4}^{3/2,1/2}(\zeta)$ are Gegenbauer polynomials. Here $g_\eta(u) = B(u) + \varphi_\eta(u)$. ρ_η^2 gives the mass correction and are defined as $\rho_\eta^2 = \frac{m^2}{m_\eta^2}$. a_{ij} , v_{ij} and h_{ij} are related to hadronic matrix elements η_4 , ω_4 and a_2 as

$$\begin{aligned}
a_{10} &= \frac{21}{8}\eta_4\omega_4 - \frac{9}{20}a_2 , & v_{10} &= \frac{21}{8}\eta_4\omega_4 , & v_{00} &= -\frac{1}{3}\eta_4 , \\
h_{01} &= \frac{7}{4}\eta_4\omega_4 - \frac{3}{20}a_2 , & h_{10} &= \frac{7}{2}\eta_4\omega_4 + \frac{3}{20}a_2 , & v_{00} &= -\frac{1}{3}\eta_4 .
\end{aligned}$$

The values of a_2 et al are: $a_2 = 0.115$, $a_4 = -0.015$, $\eta_3 = 0.013$, $\omega_3 = -3$, $\eta_4 = 0.5$, $\omega_4 = 0.2$. All of them are scaled at $\mu = 1$ GeV.

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