## Coulomb and hadronic scattering in elastic high-energy nucleon collisions

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The commonly used West and Yennie model approach to the description of the interference between Coulomb and hadronic scattering of nucleons is critically examined and its deficiencies are clarified. The preference of the more general eikonal model approach is summarized.

High-energy elastic scattering of nucleons is realized mostly due to the strong hadronic interactions, and in the case of charged hadrons also by the Coulomb interactions which are being commonly described with the help of the total elastic amplitude  $F^{C+N}(s,t)$ . This amplitude is usually written as the sum of hadronic amplitude  $F^N(s,t)$  and of Coulomb amplitude  $F^C(s,t)$  being mutually correlated by a relative phase  $\alpha \Phi(s,t)^{-1}$ :

$$F^{C+N}(s,t) = F^{C}(s,t) + F^{N}(s,t)e^{i\alpha\Phi(s,t)},$$
(1)

where  $\alpha = 1/137.036$  is the fine structure constant; here s is the square of CMS energy. For the phase function  $\Phi(s,t)$  West and Yennie<sup>2</sup> have derived the formula

$$\Phi(s,t) = \mp \left[ \ln \left( \frac{-t}{s} \right) - \int_{-4p^2}^0 \frac{dt'}{|t-t'|} \left( 1 - \frac{F^N(s,t')}{F^N(s,t)} \right) \right],\tag{2}$$

which contains an integral over all admissible t values and, therefore, has not been considered as an efficient tool for analysis of experimental data. In order to simplify it the following assumptions for hadronic amplitude  $F^N(s,t)$  have been accepted <sup>2</sup>: (i) the spins of all the particles involved can be neglected, (ii) the t dependence of the modulus  $|F^N(s,t)|$  is purely exponential in the whole kinematically allowed region of t and (iii) both the real and imaginary parts of the  $F^N(s,t)$  have the same t dependence for all admissible t values.

Then the total elastic scattering amplitude  $F^{C+N}(s,t)$  takes the form

$$F^{C+N}(s,t) = \pm \frac{\alpha s}{t} f_1(t) f_2(t) e^{i\alpha \Phi} + \frac{\sigma_{tot}}{4\pi} p \sqrt{s} (\rho+i) e^{Bt/2},$$
(3)

where the phase  $\alpha \Phi(s,t)$  equals <sup>2</sup>

$$\alpha \Phi(s,t) = \mp \alpha \left[ \ln \left( \frac{-Bt}{2} \right) + \gamma \right]. \tag{4}$$

Here  $\gamma = 0.577215$  is Euler's constant, p is the value of the momentum in CMS and  $\rho = \frac{ReF^N(s,t=0)}{\Im F^N(s,t=0)}$ . Together with the diffraction slope B the  $\rho$  is assumed to be independent of t. Both these quantities together with the total cross section  $\sigma_{tot}$  may be energy dependent only and may characterize the elastic hadron scattering at given energy. The two form factors  $f_1(t)$  and  $f_2(t)$  describe the space structure of colliding nucleons. The upper (lower) sign corresponds to the scattering of particles of the same (opposite) charges.

It has been shown recently <sup>3</sup> that among the mentioned three assumptions the last one is automatically involved in the requirement for the relative phase  $\alpha \Phi(s, t)$  given by integral formula (2) to be real. The reality of the phase requires immediately for the quantity  $\rho(s,t) \equiv \frac{\Re F^N(s,t)}{\Im F^N(s,t)}$  to be constant for all admissible t values.

All other assumptions having played the role in the derivation of Eqs. (3) and (4) have been studied in detail <sup>3,4</sup>. First, it has been shown that the existence of diffractive minimum is in a contradiction with the constant value of the quantity  $\rho$ . Second, it has been pointed out that the exponential t dependence of the modulus of hadronic amplitude, i.e.,  $|F^N(s,t)| \sim e^{Bt/2}$  can be considered as being approximately satisfied only in the region of t running from zero to the position of diffractive minimum in the differential cross section. However, this position moves to t = 0 when the energy increases. Thus the region of exponential behavior of the modulus  $|F^N(s,t)|$  becomes narrower with increasing energy. The deviations of the modulus  $|F^N(s,t)|$ from the exponential behavior can be exhibited by the t dependence of the diffractive slope B(s,t) defined as

$$B(s,t) = \frac{d}{dt} \left[ \ln \frac{d\sigma^N}{dt} \right] = \frac{2}{|F^N(s,t)|} \frac{d}{dt} |F^N(s,t)|.$$
(5)

It is evident that in the case of West and Yennie approach this quantity should be t independent while the experimental data exhibit its t dependence<sup>5</sup>. Thus, also the second assumption cannot be fulfilled at all kinematically allowed t values as required in derivation of Eq. (4). And we must conclude that both integral and simplified West and Yennie formulas contradict the elastic nucleon scattering differential cross section data.

The preference should be given to the eikonal model approach that is not burdened by similar limitations. The elastic scattering amplitude can be defined in this approach with the help of Fourier-Bessel transformation as

$$F(s,q^{2} = -t) = \frac{s}{4\pi i} \int_{\Omega_{b}} d^{2}b e^{i\vec{q}\vec{b}} \Big[ e^{2i\delta(s,b)} - 1 \Big],$$
(6)

where  $\delta(s, b)$  stands for the eikonal and  $\Omega_b$  represents the two-dimensional Euclidean space of the impact parameter  $\vec{b}$ . Mathematically consistent formulation of Fourier-Bessel transformation is guaranteed when the function F(s, t) in the region of unphysical t values is defined as analytical continuation from the region of physical t values in agreement with formula (6), comp. Adachi et al. <sup>7</sup> and Islam<sup>8</sup> who showed that it is then valid at any s and t.

The influence of both the Coulomb and strong interactions can be described with the help of the sum of Coulomb and hadronic eikonals  $^6$  and the total elastic scattering amplitude may be written as

$$F^{C+N}(s,t = -q^2) = \frac{s}{4\pi i} \int_{\Omega_b} d^2 b e^{i\vec{q}\vec{b}} \bigg[ e^{2i(\delta^C(s,b) + \delta^N(s,b))} - 1 \bigg].$$
(7)

Eq. (7) can be then transformed <sup>6</sup> into the form

$$F^{C+N}(s,t) = F^{C}(s,t) + F^{N}(s,t) + \frac{i}{\pi s} \int_{\Omega_{q'}} d^2 q' F^{C}(s,q'^2) F^{N}(s,[\vec{q}-\vec{q'}]^2), \tag{8}$$

where  $F^{C}(s,t)$  and  $F^{N}(s,t)$  are Coulomb and elastic hadronic amplitudes defined by expression (6) with the eikonals  $\delta^{C}(s,b)$  and  $\delta^{N}(s,b)$ . Eq. (8) includes the convolution integral of the two amplitudes defined over kinematically allowed region of momentum transfers  $\Omega_{q'}$ .

Eq. (8) can be rewriten as 9

$$F^{C+N}(s,t) = \pm \frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t) \bigg[ 1 \mp i \alpha G(s,t) \bigg],$$
(9)

where

$$G(s,t) = \int_{t_{min}}^{0} dt' \bigg\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'} \bigg[ f_1(t') f_2(t') \bigg] + \frac{1}{2\pi} \bigg[ \frac{F^N(s,t')}{F^N(s,t)} - 1 \bigg] I(t,t') \bigg\},$$
(10)

and

$$I(t,t') = \int_{0}^{2\pi} d\Phi'' \frac{f_1(t'')f_2(t'')}{t''};$$
(11)

here  $t'' = t + t' + 2\sqrt{tt'}\cos\Phi''$ . For the case of nucleon - nucleon scattering  $t_{min} = -s + 4m^2$ . Formulas (9) - (11) hold generally for any s and t up to the terms linear in  $\alpha$ . The expression in the last bracket of Eq. (9) may be regarded as the first term in the Taylor series expansion of the exponential  $e^{\pm i\alpha G}$ ; then one can write within the same precision

$$F^{N+C}(s,t) = F^{C}(s,t) + F^{N}(s,t)e^{\mp i\alpha G(s,t)},$$
(12)

the form being analog of original formula (1) of West and Yennie. The G(s,t) (being complex) cannot be interpreted as a mere relative phase. The reality of G(s,t) would be equivalent to require for the quantity  $\rho(s,t)$  to be constant and vice versa.

Formulas (9) - (11) can be used to determine the elastic hadronic amplitude from the differential cross section data provided its modulus and the phase are conveniently parameterized (for detail see analysis of pp and  $p\bar{p}$  scattering at ISR and SPS energies<sup>9</sup>) and for accurate prediction of the differential cross section at any t if the hadronic amplitude is given.

The analysis<sup>9</sup> of the mentioned experimental data showed that: (i) the quantites B(s,t) and  $\rho(s,t)$  are t dependent in the whole t region, (ii) the influence of Coulomb scattering cannot be neglected at higher |t| values, either, and (iii) the peripheral picture of elastic nucleon scattering seems to be slightly statistically preferred by experimental data.

The acurate determination of the elastic amplitude is also important when the luminosity  $\mathcal{L}$  should be calibrated on the basis of elastic process; it holds <sup>10</sup>

$$\frac{1}{\mathcal{L}}\frac{dN_{el}}{dt} = \frac{\pi}{sp^2} |F^{C+N}(s,t)|^2,$$
(13)

where  $\frac{dN_{el}}{dt}$  is the counting rate established at corresponding t. The so called Coulomb calibration in the region of smallest |t| where the Coulomb amplitude is dominant (reaching nearly 100 %) can be hardly realized at the LHC due to technical problems. The approach allowing to avoid corresponding difficulties may be based on Eq. (13), when the total elastic scattering amplitude  $F^{C+N}(s,t)$  with required accuracy at the smallest achievable |t| is established.

However, then it is very important which formula for the total elastic amplitude  $F^{C+N}(s,t)$  is made use of. In the following we will demonstrate possible difference at different t values. We have calculated the quantity

$$R(t) = \frac{|F_{eik}^{C+N}(s,t)|^2 - |F_{WY}^{C+N}(s,t)|^2}{|F_{eik}^{C+N}(s,t)|^2} * 100,$$
(14)

where  $F_{eik}^{C+N}(s,t)$  and  $F_{WY}^{C+N}(s,t)$  are the values of total elastic scattering amplitudes calculated in the framework of Bourelly-Soffer-Wu model <sup>11,12</sup> for pp elastic scattering at the LHC energy 14 TeV (the central behavior in the impact parameter space); the former quantity with the help of Eqs. (9) - (11) and the latter one with the help of Eqs. (2) - (3). The t dependence of the quantity R(t) is represented in Fig. 1. Its maximum value is 0.8 % at  $t = -0.006 \text{ GeV}^2$ showing that the resulting differential cross sections determined with the help of commonly used West and Yennie method and with the help of mathematically and physically consistent eikonal



Figure 1: t dependence of the quantity R(t) defined by Eq.(14) and calculated for the Bourrely-Soffer-Wu model Figure 2: t dependence of the same quantity R(t) defined by Eq. (14) and calculated for the model of Islam et al.

approach differ. This difference is a function of the t variable. Similar behavior of the quantity R(t) has been also obtained in the preliminary analysis <sup>13</sup> in the case of model of Islam et al.<sup>14</sup> where the maximum value of the quantity R(t) is much higher.

R(t) will also differ for the other types of the phase of elastic hadronic amplitude - see the analysis of pp and  $p\bar{p}$  scattering at ISR and SPS energies <sup>9</sup> corresponding to the central and peripheral distributions of elastic hadron scattering where this difference is yet larger in the peripheral case. It means that the luminosity determined for the central as well as peripheral distributions of elastic pp scattering at LHC energy of 14 TeV may be burdoned by a non-negligible systematic error.

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