Chiral Mass Splitting for $c\bar{s}$ and $c\bar{n}$ Mesons in the $\widetilde{U}(12)$-Classification Scheme of Hadrons

Kenji Yamada*, Tomohito Maeda* and Shin Ishida†

*Department of Engineering Science, Junior College Funabashi Campus, Nihon University, Funabashi 274-8501, Japan
†Research Institute of Science and Technology, College of Science and Technology, Nihon University, Tokyo 101-8308, Japan

Abstract. We investigate the chiral mass splitting of parity-doubled $J = 0,1$ states for $c\bar{s}$ and $c\bar{n}$ meson systems in the $\widetilde{U}(12)_{SF}$-classification scheme of hadrons, using the linear sigma model to describe the light-quark pseudoscalar and scalar mesons together with the spontaneous breaking of chiral symmetry, and consequently predict the masses of as-yet-unobserved $(0^+,1^+)$ $c\bar{n}$ mesons. We also mention some indications of their existence in the recent published data from the Belle and BABAR Collaborations.

INTRODUCTION

Recently, Ishida et al. have proposed the covariant $\widetilde{U}(12)_{SF}$-classification scheme of hadrons[1], which gives covariant quark representations for composite hadrons with definite Lorentz and chiral transformation properties. The $\widetilde{U}(12)_{SF}$-classification scheme has a unitary symmetry in the hadron rest frame, called “static $U(12)_{SF}$ symmetry”[2], embedded in the covariant $\widetilde{U}(12)_{SF}$-representation space, of which tensors can be decomposed into representations of $\widetilde{U}(4)_{DS} \times SU(3)_F$, $\widetilde{U}(4)_{DS}$ being the pseudounitary homogeneous Lorentz group for Dirac spinors. The static $U(12)_{SF}$ contains the Dirac spin group $U(4)_{DS}$ in its subgroups and $U(4)_{DS}$ contains two $SU(2)$ subgroups as $U(4)_{DS} \supset SU(2)_\rho \times SU(2)_\sigma$, where $SU(2)_\rho$ and $SU(2)_\sigma$ are the spin groups concerning the boosting and intrinsic-spin rotation, respectively, of constituent quarks, being connected with decomposition of Dirac γ-matrices, $\gamma \equiv \rho \otimes \sigma$. Thus the static $U(12)_{SF}$ symmetry includes the chiral $SU(3)_L \times SU(3)_R$ symmetry as $U(12)_{SF} \supset SU(3)_L \times SU(3)_R \times SU(2)_\sigma$. This implies that the $\widetilde{U}(12)_{SF}$-classification scheme is able to incorporate effectively the effects of chiral symmetry and its spontaneous breaking, essential for understanding of properties of the low-lying hadrons, into what is called a constituent quark model.

EXPERIMENTAL CANDIDATES FOR THE GROUND-STATE QUARK-ANTIQUARK MESONS

An essential feature of the $\widetilde{U}(12)_{SF}$-classification scheme is to have the static $U(4)_{DS}$ symmetry for light u,d,s quarks confined inside hadrons. The degree of freedom on the ρ-spin, being indispensable for covariant description of spin 1/2 particles, offers a basis to define the rule of chiral transformation for quark-composite hadrons. Since we have the ρ-spin degree of freedom, which is discriminated by the eigenvalues of ρ_3, $r = \pm$, in addition to the ordinary σ-spin, the ground states of light-quark $q\bar{q}$ mesons are composed of eight $SU(3)_F$ multiplets with respective J^{PC} quantum numbers, two pseudoscalars $(0^{-+}_{\rm N}, 0^{-+}_{\rm E})$, two scalars $(0^{++}_{\rm N}, 0^{+-}_{\rm E})$, two vectors $(1^{--}_{\rm N}, 1^{--}_{\rm E})$, and two axial-vectors $(1^{++}_{\rm N}, 1^{+-}_{\rm E})$ (N and E denoting "normal" and "extra"), where each N (E) even-parity multiplet is the chiral partner of the corresponding N (E) odd-parity multiplet and they form linear representations of the chiral symmetry.

Since the eigenstates only with the ρ_3-eigenvalue $r = +$ are taken for heavy quarks, we have for heavy-light meson systems two heavy-spin multiplets, $(0^-, 1^-)$ and $(0^+, 1^+)$, which are the chiral partner each other, while for heavy-heavy meson systems we have the same $(0^-, 1^-)$-spin multiplets as in the conventional nonrelativistic quark model.

The $\widetilde{U}(12)_{SF}$-scheme assignments for the observed mesons

We try to assign some of the observed mesons to the predicted $q\bar{q}$ multiplets, resorting to their J^{PC} quantum numbers and masses. The observed meson data are taken from the Particle Data Group 2004 edition[3], except for the following mesons:

- $\rho(1250)$. There are several experimental indications of the existence of the $\rho(1250)$ reported by the OBELIX[4] and LASS[5] Collaborations, and others.[1]
- $\omega(1200)$. The existence of $\omega(1200)$ is claimed in the analysis of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section by the SND Collaboration[6].

We accept the existence of these vector mesons as true[7]. The resulting assignments, though some of them are ambiguous, are shown in Table 1. Here we make some comments on these assignments.

(1) The light scalar mesons $\{a_0(980), \sigma, f_0(980), \kappa\}$ are assined to the $(0^{++}_{\rm N})$-nonet as a chiral partner of the π-meson $(0^{-+}_{\rm N})$-nonet.

(2) The low-mass vector mesons $\{\rho(1250), \omega(1200), K^*(1410)\}$ are assined to the $(1^{--}_{\rm E})$-nonet as a chiral partner of the $(1^{+-}_{\rm E})$-nonet $\{b_1(1235), h_1(1170), h_1(1380), K_1(1400)\}$.

(3) The axial-vector mesons $\{a_1(1260), f_1(1285), f_1(1420), K_1(1270)\}$ are assined to the $(1^{++}_{\rm N})$-nonet as a chiral partner of the $\rho(770)$-meson $(1^{--}_{\rm N})$-nonet.

(4) The recent observed mesons $\{D^*_{sJ}(2317), D_{sJ}(2460)\}$ are assined to the $(0^+, 1^+)$ multiplet as a chiral partner of the $(0^-, 1^-)$ multiplet $\{D_s, D^*_s\}$[8]. These newly observed mesons, together with the σ-meson nonet, are the best candidates for the hadronic states with $r = -$ whose existence is expected in the $\widetilde{U}(12)_{SF}$ scheme.

(5) It is noted that the normal (N) and extra (E) states with the same J^{PC} generally mix together due to the spontaneous as well as explicit breaking of chiral symmetry and some other mechanism.

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[1] See the $\rho(1450)$ Particle Listings and the "Note on the $\rho(1700)$" in [3].

TABLE 1. Experimental candidates for ground-state mesons in the $\widetilde{U}(12)_{SF}$-classification scheme.

$q\bar{q}$	P	S_A	$\tilde{P}$	$\tilde{S}_B$	V	A	$\tilde{V}$	$\tilde{B}$
	1^1S_0				1^3S_1			
	0^{-+}	0^{++}	0^{-+}	0^{+-}	1^{--}	1^{++}	1^{--}	1^{+-}
$n\bar{n}$ $s\bar{s}$	π	$a_0(980)$	$\pi(1300)$		$\rho(770)$	$a_1(1260)$	$\rho(1250)$	$b_1(1235)$
	η	σ	$\eta(1295)$		$\omega(782)$	$f_1(1285)$	$\omega(1200)$	$h_1(1170)$
	$\eta'(958)$	$f_0(980)$	$\eta(1475)$		$\phi(1020)$	$f_1(1420)$		$h_1(1380)$
$s\bar{n}$	K	κ	$K(1460)$		$K^*(892)$	$K_1(1270)$	$K^*(1410)$	$K_1(1400)$
$c\bar{n}$	D				D^*			
$c\bar{s}$	D_s	$D^*_{sJ}(2317)$			D^*_s	$D_{sJ}(2460)$		
$b\bar{n}$	B				B^*			
$b\bar{s}$	B_s				B^*_s			
$c\bar{c}$	$\eta_c(1S)$				$J/\psi(1S)$			
$b\bar{b}$	$\eta_b(1S)$				$Y(1S)$			

CHIRAL MASS SPLITTING FOR THE CHARMED AND CHARMED-STRANGE MESON SYSTEMS

In the $\widetilde{U}(12)_{SF}$-classification scheme heavy-light $(c\bar{q})$ meson fields, aside from the internal space-time wave functions, are given by

$$\Phi(v) = \frac{1}{2\sqrt{2}}(1 - iv\cdot\gamma)(i\gamma_5\mathbf{D} + i\tilde{\gamma}_\mu\mathbf{D}^*_\mu + \mathbf{D}_0 + i\gamma_5\tilde{\gamma}_\mu\mathbf{D}_{1\mu}) \tag{1}$$

with $v_\mu \equiv P_\mu/M,\ \tilde{\gamma}_\mu \equiv \gamma_\mu + v_\mu(v\cdot\gamma)$, where $(\mathbf{D},\mathbf{D}^*_\mu,\mathbf{D}_0,\mathbf{D}_{1\mu})$ represent the local fields for the $c\bar{q}$ mesons with $J^P = (0^-,1^-,0^+,1^+)$, P_μ (M) is the four-momentum (mass) of meson fields, and flavor indices are omitted for simplicity. To describe the light-quark pseudoscalar and scalar mesons together with the spontaneous breaking of chiral symmetry, we adopt the $SU(3)$ linear sigma model, introducing the chiral field Σ_5 defined by

$$\Sigma_5 = s - i\gamma_5\phi \tag{2}$$

with

$$s = \frac{1}{\sqrt{2}}s^a\lambda^a,\ \ \phi = \frac{1}{\sqrt{2}}\phi^a\lambda^a \quad (a = 0,\cdots,8),$$

where $\lambda^0 = \sqrt{2/3}\ \mathbf{1}$ and s^a (ϕ^a) are the scalar (pseudoscalar) fields. We now write a chiral-symmetric effective Lagrangian which gives the chiral mass splitting between the heavy-light $(0^-,1^-)$ and $(0^+,1^+)$ multiplets through the spontaneous breaking of chiral symmetry[11, 8]:

$$\mathscr{L}_{ND} = -g_{ND}\mathrm{Tr}[\Phi\Sigma_5\bar{\Phi}], \tag{3}$$

where g_{ND} is the dimensionless coupling constant of Yukawa interaction in the non-derivative form and the trace is taken over the spinor and flavor indices.

When the chiral symmetry is spontaneously broken, s has the vacuum expectation value, $\langle s \rangle_0 = \mathrm{diag}(a, a, b)$, where a and b are related to the pion and kaon decay constants by

$$a = \frac{1}{\sqrt{2}} f_\pi, \quad b = \frac{1}{\sqrt{2}} (2 f_k - f_\pi). \tag{4}$$

Then the mass splitting between the two multiplets is induced and the mass differences $\Delta M_\chi(c\bar{q})$ are given by $\Delta M_\chi(c\bar{n}) = 2 g_{ND} a$ and $\Delta M_\chi(c\bar{s}) = 2 g_{ND} b$, which leads to the relation

$$\Delta M_\chi(c\bar{n}) = \Delta M_\chi(c\bar{s}) \frac{a}{b} = \Delta M_\chi(c\bar{s}) \left(\frac{2 f_K}{f_\pi} - 1 \right)^{-1}. \tag{5}$$

From this relation with the experimental values[3], $\Delta M_\chi(c\bar{s}) = 348.0 \pm 0.8$ MeV and $f_{K^+}/f_{\pi^+} = 1.223 \pm 0.015$, we obtain $\Delta M_\chi(c\bar{n}) = 240.8 \pm 5.4$ MeV and consequently predict the masses

$$M(D_0^*) = 2.11 \pm 0.01 \text{ GeV}, \quad M(D_1) = 2.25 \pm 0.01 \text{ GeV} \tag{6}$$

for the $(0^+, 1^+)$ $c\bar{n}$ mesons, using the measured mass values[3] of the $D(0^-)$ and $D^*(1^-)$ mesons. We hereafter refer to these predicted mesons, respectively, as "$D_0^*(2110)$" and "$D_1(2250)$".

POSSIBLE INDICATIONS OF THE EXISTENCE OF LIGHT SCALAR AND AXIAL-VECTOR CHARMED MESONS

We could ask experimental data whether there was some evidence for the existence of $D_0^*(2110)$ and $D_1(2250)$. Here we check on the recent published data on the $D\pi$ and $D^*\pi$ mass distributions in $B \to (D\pi)\pi$, $(D^*\pi)\pi$ decays from the Belle[9] and BABAR[10] Collaborations.

- $D\pi$ **mass spectrum**: In the Belle data[2] we see an excess of events, a single data point of 20 MeV bin, at a mass of 2.13 GeV near the predicted mass of the $D_0^*(2110)$, and so might regard it as an indication of that resonance, though it is natural to think that its data point should be within a statistical error. On the other hand, it would seem to us that the BABAR data[3] around a mass of 2.1 GeV show a typical pattern of interference between two or more resonances.
- $D^*\pi$ **mass spectrum**: In the Belle data[4] there is also an excess of events, a single data point of 10 MeV bin, at a mass of 2.255 GeV near the predicted mass of the $D_1(2250)$, and so it might be an indication of the resonance. Although it is not clear, the BABAR data[5] around a mass of 2.26 GeV might show a typical pattern

[2] See the $D\pi$ mass distribution in Figure 3 of [9].
[3] See the $D\pi$ mass distribution in Figure 3 (right) of [10].
[4] See the $D^*\pi$ mass distribution in Figure 9 of [9].
[5] See the $D^*\pi$ mass distribution in Figure 3 (left) of [10].

of interference.

If the $D_0^*(2110)$ and $D_1(2250)$ resonances really exist, their widths have to be narrow, $\leq$ 20-30 MeV, judging from the data mentioned above. The dominant decay modes of these resonances are $D\pi$ and $D^*\pi$, respectively, and thus we examine their single pion transitions. To estimate the widths of $D_0^* \to D+\pi$ and $D_1 \to D^*+\pi$ decays, together with $D^* \to D+\pi$, we set up, in addition to the nonderivative interaction $\mathscr{L}_{ND}$ in Eq. (3), the chiral-invariant effective interaction with the derivative form:

$$\mathscr{L}_D = g_D \mathrm{Tr}[\Phi(\partial_\mu \Sigma_5)\gamma_\mu(F_U\bar{\Phi})], \tag{7}$$

where $F_U = \gamma\cdot\partial/\sqrt{\partial\cdot\partial}$ and g_D is the coupling constant with a dimension of $(mass)^{-1}$, which is related to the axial coupling constant g_A by $g_D = g_A/2a = g_A/\sqrt{2}f_\pi$. The pionic decay widths of the D^*, D_0^*, and D_1 states are derived from $\mathscr{L}_{ND}$ and $\mathscr{L}_D$, and the decay widths of D_0^* and D_1 are identical. Using the measured value of $\Gamma[D^{*+} \to D^0+\pi^+] =$ 65 keV[3] and $g_{ND} = 1.84$ from $\Delta M_\chi(c\bar{n}) = 241$ MeV, the coupling g_D is fixed to 3.96 GeV^{-1} (corresponding to $g_A = 0.521$), and then we obtain

$$\Gamma[D_0^*(2110) \to D+\pi] = \Gamma[D_1(2250) \to D^*+\pi] \approx 30 \text{ MeV}. \tag{8}$$

This value is consistent with the speculated widths of the $D_0^*(2110)$ and $D_1(2250)$.

CONCLUDING REMARKS

We have presented the possible assignments for some of the observed mesons in the covariant $\widetilde{U}(12)_{SF}$-classification scheme. It is necessary and important to examine the strong- and radiative-decay[12] properties of the assigned states in order to establish their assignments. On the basis of these assignments we have also predicted the existence of the low-mass $(0^+,1^+)$ $c\bar{n}$ mesons with narrow width, which might have been seen in the recent published data on the $D\pi$ and $D^*\pi$ mass distributions from the Belle and BABAR Collaborations.

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