

Effective $N = 1$ description of 5D conformal supergravity*

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Abstract

We construct an effective $N=1$ superfield description of five-dimensional conformal supergravity. By the use of this description, we reinterpret some physical systems such as Scherk-Schwarz supersymmetry breaking from the conformal supergravity viewpoint. We also show how to introduce a radion fluctuation mode in this framework.

The five-dimensional (5D) supergravity provides an interesting theoretical framework to the physics beyond the standard model (SM) in both bottom-up and top-down approach. In the former approach, the localized wavefunction can be the source of Planck and weak hierarchy [1] and/or hierarchical structures within SM, the supersymmetry (SUSY) breaking sector can be hidden in the extra dimension, AdS/CFT correspondence provides a way to analyze perturbatively the four-dimensional (4D) strongly coupled theories which can be the extension of Higgs sector in SM, and so on. In the latter approach, it is known that in some case the compactified eleven-dimensional supergravity (M-theory) can be described effectively by the 5D supergravity at certain energy scale [2]. Also because the 5D supergravity is the simplest model with extra dimension, we have complete off-shell formulations (e.g., Ref. [3]) which allow a systematic study. In this talk we show an effective $N = 1$ description of 5D conformal supergravity that will be useful in any approaches mentioned above.

First we briefly review the hypermultiplet compensator formulation of 5D conformal supergravity based on Ref. [3]. The 5D superconformal algebra consists of the Poincaré symmetry \mathbf{P} , \mathbf{M} , the dilatation symmetry \mathbf{D} , the $SU(2)$ symmetry \mathbf{U} , the special conformal boosts \mathbf{K} , $N = 2$ supersymmetry \mathbf{Q} , and the conformal supersymmetry \mathbf{S} . The gauge fields corresponding to these generators $\mathbf{X}_A = \mathbf{P}_m, \mathbf{M}_{mn}, \mathbf{D}, \mathbf{U}_{ij}, \mathbf{K}_m, \mathbf{Q}_i, \mathbf{S}_i$, are respectively represented by $h_\mu^A = e_\mu^m, \omega_\mu^{mn}, b_\mu, V_\mu^{ij}, f_\mu^m, \psi_\mu^i, \phi_\mu^i$. We use μ, ν, \dots as five-dimensional curved indices and m, n, \dots as the tangent flat indices. The i, j are $SU(2)_U$ index and ψ_μ^i and ϕ_μ^i are $SU(2)$ Majorana spinors. The relevant 5D superconformal multiplets to the following study are

$$\begin{array}{ll} \text{Weyl multiplet:} & (e_\mu^m, \psi_\mu^i, V_\mu^{ij}, b_\mu, v^{mn}, \chi^i, D), \\ \text{Vector multiplet:} & (M, W_\mu, \Omega^i, Y^{ij})^I, \\ \text{Hypermultiplet:} & (\mathcal{A}_i^\alpha, \zeta^\alpha, \mathcal{F}_i^\alpha), \end{array}$$

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where $I = 0, 1, 2, \dots, n_V$ and $\alpha = 1, 2, \dots, 2(p+q)$. The $n_V + 1$ is the number of vector multiplet and the p (q) stands for the number of compensator (physical) hypermultiplet. The superconformal gauge fixing for a reduction to 5D Poincaré supergravity is given by

$$\begin{aligned} \mathbf{D} & : \mathcal{N} = M_5^3 \equiv 1, \\ \mathbf{U} & : \mathcal{A}_i^\alpha \propto \delta_i^\alpha, \quad (p = 1) \\ \mathbf{S} & : \mathcal{N}_I \Omega^I = 0, \\ \mathbf{K} & : \mathcal{N}^{-1} \hat{\mathcal{D}}_m \mathcal{N} = 0, \end{aligned} \quad (1)$$

where $\mathcal{N} = C_{IJK} M^I M^J M^K$ is the norm function of 5D supergravity.

In the following we derive an effective $N = 1$ superspace description of 5D conformal supergravity by considering 4D Poincaré invariant background, $ds^2 = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$, $\psi_\mu^i = 0$ and so on, and neglecting the fluctuations of all the 5D gravitational fields including the graviphoton. The invariant action is written in the $N = 1$ superspace as [4, 5] $S = \int d^5x (\mathcal{L}_V + \mathcal{L}_H + \mathcal{L}_{N=1})$ where

$$\begin{aligned} \mathcal{L}_V & = \frac{3}{2} C_{IJK} \left[\int d^2\theta \left\{ i \Phi_S^I \mathcal{W}^J \mathcal{W}^K \right. \right. \\ & \quad \left. \left. + \frac{1}{12} \bar{D}^2 (V^I D^\alpha \partial_y V^J - D^\alpha V^I \partial_y V^J) \mathcal{W}_\alpha^K \right\} \right. \\ & \quad \left. + \text{h.c.} \right] - e^{2\sigma} \int d^4\theta V_T C_{IJK} \mathcal{V}_S^I \mathcal{V}_S^J \mathcal{V}_S^K, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_H & = -2e^{2\sigma} \int d^4\theta V_T d_\alpha^\beta \bar{\Phi}^\beta \left(e^{-2igV^I t_I} \right)_\gamma^\alpha \Phi^\gamma \\ & \quad - e^{3\sigma} \left[\int d^2\theta \Phi^\alpha d_\alpha^\beta \rho_{\beta\gamma} (\partial_y - 2g\Phi_S^I t_I)^\gamma_\delta \Phi^\delta \right. \\ & \quad \left. + \text{h.c.} \right], \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{N=1} = & \sum_{l=0,\pi} \Lambda_l \delta(y-lR) \left[\right. \\ & -\frac{3}{2} e^{2\sigma} \int d^4\theta \bar{\Sigma} \Sigma e^{-K^{(l)}(S,\bar{S})/3} \\ & + \left\{ \int d^2\theta \left(f_{I\bar{J}}^{(l)} \mathcal{W}^{\bar{I}} \mathcal{W}^{\bar{J}} + e^{3\sigma} \Sigma^3 W^{(l)}(S) \right) \right. \\ & \left. \left. + \text{h.c.} \right\} \right]. \end{aligned}$$

The $N=1$ vector and chiral superfields V^I and Φ_S^I come from the 5D vector multiplet and the $N=1$ chiral superfields Φ^α originate from the 5D hypermultiplets. We can find the relation between the original superconformal multiplets and these superfields in Ref. [5]. The compensator chiral multiplets are given by $\Sigma = (\Phi^{\alpha=2})^{2/3}$, $\Sigma^C = (\Phi^{\alpha=1})^{2/3}$, and we include spurious superfield, $V_T = \langle e_y^4 \rangle + i\theta^2 e^\sigma \langle V_y^{(1)} + iV_y^{(2)} \rangle - i\bar{\theta}^2 e^\sigma \langle V_y^{(1)} - iV_y^{(2)} \rangle$. The superfield S symbolically represents boundary (induced or own) chiral superfields. From V^I and Φ_S^I , we can construct gauge invariant superfields, $\mathcal{W}_\alpha^I = -\frac{1}{4} \bar{D}^2 D_\alpha V^I$, $\mathcal{V}_S^I = V_T^{-1} (-\partial_y V^I - i(\Phi_S^I - \bar{\Phi}_S^I))$. Note that the gauge groups are limited to the Abelian group in the above action for simplicity.

Next we derive the action after the superconformal gauge fixing (1). We assume the standard form of \mathcal{N} ,

$$\mathcal{N} = (M^{I=0})^3 - \frac{1}{2} M^{I=0} \sum_{x=1}^{n_V-1} (M^{I=x})^2,$$

and general hypermultiplet gaugings,

$$\begin{aligned} (T_{I=0}, T_{I=x}) \varphi^2 &= \left(-\frac{3}{2}k \epsilon(y), -r_x\right) i\sigma_3 \varphi^2, \\ (T_{I=0}, T_{I=x}) \varphi^{2v+2} &= (c \epsilon(y), q_x) i\sigma_3 \varphi^{2v+2}. \end{aligned}$$

Then at the leading order in an expansion in powers of $1/M_5$, the vector field in $V^{I=0}$ becomes the graviphoton that we neglect the fluctuation, and we find $V^{I=0} \simeq 0$, $\Phi_S^{I=0} \simeq \mathcal{T}$, $V_T \simeq \frac{\mathcal{T} + \bar{\mathcal{T}}}{2}$, $\Sigma \simeq 1 - \theta^2 \mathcal{F}_\Sigma$ and $\Sigma^C \simeq -\theta^2 \mathcal{F}_\Sigma^C$, where $\mathcal{T} = 1 - \theta^2 \mathcal{F}_T$ and $\mathcal{F}_T = -2i \langle V_y^{(1)} + iV_y^{(2)} \rangle$. The action after the gauge fixing [5] is found to be $S = \int d^5x (\mathcal{L}_{V'+H'} + \mathcal{L}_{N=1} + \mathcal{L}_{V_0+H_0} + \mathcal{L}_{\text{SB}})$ where

$$\begin{aligned} \mathcal{L}_{V'+H'} = & \left\{ \int d^2\theta \frac{1}{4} \mathcal{T} \mathcal{W}^x \mathcal{W}^x + \text{h.c.} \right\} \\ & + e^{2\sigma} \int d^4\theta \frac{2}{\mathcal{T} + \bar{\mathcal{T}}} \left(\partial_y V^x - \frac{\chi^x + \bar{\chi}^x}{\sqrt{2}} \right)^2 \\ & + e^{2\sigma} \int d^4\theta \frac{\mathcal{T} + \bar{\mathcal{T}}}{2} \left(\bar{H}^v e^{2q_x V^x} H^v \right. \\ & \left. + \bar{H}^{Cv} e^{-2q_x V^x} H^{Cv} \right) \\ & + e^{3\sigma} \left\{ \int d^2\theta H^{Cv} \left(\frac{1}{2} \overleftrightarrow{\partial}_y + c \mathcal{T} \right) \right. \\ & \left. + \sqrt{2} q_x \chi^x \right\} H^v + \text{h.c.} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{N=1} = & \sum_{l=0,\pi} \Lambda_l \delta(y-lR) \left[\right. \\ & -\frac{3}{2} e^{2\sigma} \int d^4\theta (\bar{\Sigma} e^{\frac{4}{3} r_x V^x} \Sigma e^{-\frac{1}{3} K^{(l)}(S,\bar{S})} \\ & + \left\{ \int d^2\theta \left(f_{\bar{x}\bar{y}}^{(l)}(S) \mathcal{W}^{\bar{x}} \mathcal{W}^{\bar{y}} \right. \right. \\ & \left. \left. + e^{3\sigma} \Sigma^3 W^{(l)}(S) \right) + \text{h.c.} \right\} \right], \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{V_0+H_0} = & -2e^{2\sigma} \int d^4\theta \frac{\mathcal{T} + \bar{\mathcal{T}}}{2} \left(\bar{\Sigma}^{\frac{3}{2}} e^{2r_x V^x} \Sigma^{\frac{3}{2}} \right. \\ & \left. + (\bar{\Sigma}^C)^{\frac{3}{2}} e^{-2r_x V^x} (\Sigma^C)^{\frac{3}{2}} \right) \\ & - 2e^{3\sigma} \left\{ \int d^2\theta (\Sigma^C)^{\frac{3}{2}} \left(\frac{1}{2} \overleftrightarrow{\partial}_y - \frac{3}{2} k \mathcal{T} \right. \right. \\ & \left. \left. + \sqrt{2} r_x \chi^x \right) \Sigma^{\frac{3}{2}} + \text{h.c.} \right\} \\ & - 8e^{2\sigma} \int d^4\theta \frac{\mathcal{T} + \bar{\mathcal{T}}}{2}, \end{aligned} \quad (2)$$

where $H^v = \sqrt{2} \Phi^{\alpha=2v+2}$, $H^{Cv} = \sqrt{2} \Phi^{\alpha=2v+1}$, $V^x = V^{I=x}/\sqrt{2}$ and $\chi^x = -i\Phi_S^{I=x}$. The last term in the action can not be written in terms of the $N=1$ superfields and is given by [5]

$$\begin{aligned} \mathcal{L}_{\text{SB}} = & e^{4\sigma} f_G \left\{ (\partial_y + 3\dot{\sigma} + 3k - f_G)(M^x)^2 \right. \\ & + \frac{3}{2} (\partial_y + \frac{5}{2}k + \frac{3}{2}\dot{\sigma})(|h|^2 + |h^C|^2) \\ & - (\sqrt{2}q_x M^x - c)(|h|^2 - |h^C|^2) \\ & \left. + \frac{\epsilon^{-2\sigma}}{4} (\chi_S^x \lambda^x + \text{h.c.}) \right\}, \end{aligned}$$

where $f_G = \dot{\sigma} - \frac{2}{3} \langle \mathcal{N}_I Y^{I(3)} \rangle$. For BPS background, f_G vanishes and then $\mathcal{L}_{\text{SB}} = 0$. Because f_G represents the deviation of the background geometry from the BPS one, we conclude that \mathcal{L}_{SB} describes the effect of the geometry mediated SUSY breaking.

Another immediate result from the action (2) is about the Scherk-Schwarz (SS) SUSY breaking [9]. SS SUSY breaking is the consequence of the twisted boundary condition $\Phi(x, y+2\pi R) = e^{-i\pi \vec{\omega} \cdot \vec{\sigma}} \Phi(x, y)$ where $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ is the twist vector and the Pauli matrices $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ acts on the $SU(2)_R$ index of the field Φ in the Poincaré supergravity. In the conformal supergravity point of view, the $SU(2)_R$ symmetry is the diagonal subgroup of $SU(2)_U \times SU(2)_\Sigma$ determined by the U -gauge fixing condition in (1), where $SU(2)_\Sigma$ is the rotation in terms of a index in the compensator hypermultiplet \mathcal{A}_i^a . Then the above SS twist is physically equivalent to the twisted U -gauge fixing $\mathcal{A}_i^a \propto \delta_i^a \rightarrow (e^{i\pi \vec{\omega} \cdot \vec{\sigma} \alpha(y)})_i^a$ where $\alpha(y)$ is a gauge fixing parameter which satisfies $\alpha(y+2\pi R) =$

$\alpha(y) + 2\pi$. The relevant part to this change in the action (2) is the y -derivative term of Σ , Σ^C and we find an additional term like $\partial_y \alpha(y) \left\{ \int d^2\theta \Sigma^3 W_0 + \text{h.c.} \right\}$, where $W_0 = \omega_1 + i\omega_2$. Here we can choose $\alpha(y)$ in two distinctive ways. For $\alpha(y) = y/R$, the additional term is the bulk constant superpotential, while $\alpha(y) = \frac{1}{2}\pi \sum_n (\text{sgn}(y - n\pi R) - \text{sgn}(-n\pi R))$ results in the constant superpotential at the boundaries because $\partial_y \alpha(y) = \pi \sum_n \delta(y - n\pi R)$. Then in the superconformal framework, we find directly that the SS SUSY breaking is equivalent to the constant superpotential. Note that the compensator must have vanishing gauge charges $k, r_x = 0$ for the case with the twisted U -gauge fixing, otherwise the corresponding gauge field acquires a nonvanishing mass without the Higgs mechanism. Namely the SS twist basically conflicts with AdS₅ geometry [7].

We finally consider how to include the radion fluctuation mode in the previous $N = 1$ superspace action. We start from the metric $ds^2 = e^{2F(b(x), y)} \eta_{\mu\nu} dx^\mu dx^\nu - G^2(b(x), y) dy^2$ with the radion fluctuation mode $b(x)$ by assuming a BPS radion stabilization mechanism (e.g., Ref. [8]) with a small backreaction. The background geometry of our system is AdS, and the function F and G should satisfy $F(\langle b \rangle, y) = \sigma(y) = -ky$ and $G(\langle b \rangle, y) = 1$. Because $b(x)$ is a modulus field, we also require $\partial_y F = -kG$. The embedding of $b(x)$ into the $N = 1$ superspace action is done by the replacement $\sigma(y) \rightarrow F(b(x), y)$ and $\langle e_y^A \rangle \rightarrow G(b(x), y)$ in the previous results. This yields a kinetic term for $b(x)$ after the superconformal gauge fixing which is compared to the corresponding one in the original supergravity action. The matching condition of these two as well as the above AdS and modulus conditions determines the $b(x)$ -dependence of F and G as $F = \frac{1}{2} \ln(e^{2\sigma(y)} + b(x))$, $G = (1 + e^{-2\sigma(y)} b(x))^{-1}$. The radion field itself should correspond to the proper length of the extra dimension, i.e., $r(x) = \frac{1}{\pi} \int_0^{\pi R} dy G$. Then the relation between $r(x)$ and $b(x)$ is given by $b(x) = e^{-k\pi R} \frac{\sinh \pi k(R-r(x))}{\sinh \pi k r(x)}$.

By promoting the radion field $r(x)$ to the superfield $T(x)$, we obtain the $N = 1$ superspace action with the dynamical radion superfield:

$$\begin{aligned} \mathcal{L}_{V'+H'} &= \left\{ \int d^2\theta \frac{1}{4} G(T) \mathcal{W}^x \mathcal{W}^x + \text{h.c.} \right\} \\ &+ e^{2\sigma} \int d^4\theta G_R^{-2}(T) \left(\partial_y V^x - \frac{\chi^x + \bar{\chi}^x}{\sqrt{2}} \right)^2 \\ &+ e^{2\sigma} \int d^4\theta G_R^{3/2}(T) \left(\bar{H}^v e^{2q_x V^x} H^v \right. \\ &\left. + \bar{H}^{Cv} e^{-2q_x V^x} H^{Cv} \right) \\ &+ e^{3\sigma} \left\{ \int d^2\theta H^{Cv} \left(\frac{1}{2} \overleftrightarrow{\partial}_y + c G(T) \right. \right. \\ &\left. \left. + \sqrt{2} q_x \chi^x \right) H^v + \text{h.c.} \right\}, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{N=1} &= \sum_{l=0, \pi} \Lambda_l \delta(y - lR) \left[\right. \\ &- \frac{3}{2} e^{2\sigma} \int d^4\theta G_R^{-1}(T) e^{-\frac{1}{3} K^{(l)}(S, \bar{S})} \\ &+ \left\{ \int d^2\theta \left(f_{\bar{x}y}^{(l)}(S) \mathcal{W}^{\bar{x}} \mathcal{W}^{\bar{y}} \right. \right. \\ &\left. \left. + e^{3\sigma} G^{-\frac{3}{2}}(T) W^{(l)}(S) \right) + \text{h.c.} \right\} \left. \right], \end{aligned}$$

$$\mathcal{L}_{V_0+H_0} = -3e^{2\sigma} \int d^4\theta \ln G_R(T),$$

where $G_R(T) = \frac{G(T)+G(\bar{T})}{2}$. Based on this action, we can calculate the radion mass for the radion stabilization mechanism proposed by Ref. [8]. We introduce a superpotential $W^{(l)}(S) = J_l S$ at the orbifold fixed points $y = lR$ ($l = 0, \pi$) where $S = G^{\frac{3}{4}}(T)H$ and H is a stabilizer hypermultiplet. We can easily obtain the 4D effective action in superspace for the zero mode $h_{(0)}$ of bulk hypermultiplet H and the radion T , and then find a BPS vacuum $\langle h_{(0)} \rangle = 0$ and $J_0 - J_\pi e^{-\left(\frac{3}{2}k+c\right)\pi R} = 0$ where $\langle T \rangle = R$. The radion mass on this BPS vacuum is derived up to $\mathcal{O}(|J_\pi|^2)$ as $m_{\text{rad}}^2 = \frac{k^2 |J_\pi|^2}{6} \left(1 - \frac{2c}{k}\right) \left(\frac{3}{2} - \frac{c}{k}\right)^2 e^{-2k\pi R} \frac{1 - e^{-2k\pi R}}{1 - e^{-(k-2c)\pi R}}$. We find that the radion mass is finite even in $k \rightarrow 0$ limit, namely the stabilization mechanism can work in the flat spacetime.

In summary, we have derived an effective $N = 1$ description of 5D conformal supergravity, and analyzed some SUSY breaking configurations such as geometry-mediated or Scherk-Schwarz breaking in this framework. We have also shown how to include dynamical radion mode in the $N = 1$ superspace action. This result will be useful for the phenomenological studies of 5D supergravity as well as the theoretical understanding [9]. An application to the 5D supergravity with parity odd couplings such as boundary FI terms [10] and the Green-Schwarz mechanism [11] would be fruitful.

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