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JOINT RESUMMATION FOR HEAVY QUARK PRODUCTION

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We present the joint threshold and recoil resummed transverse momentum distributions for heavy quark hadroproduction, at next-to-leading logarithmic accuracy. We exhibit their dependence on the production channel and the color configurations, and compare these distributions to eachother and to NLO.

Keywords: Resummation; heavy quark production.

1. Joint threshold and recoil resummation

The formalism $1,2,3$ $1,2,3$ $1,2,3$ $1,2,3$ $1,2,3$ of hadronic cross sections for the joint resummation of distributions singular at partonic threshold and at zero recoil has so far been applied to Z/W production $\overline{4}$, Higgs production $\overline{5}$, and prompt photon hadroproduction $\overline{6}$ $\overline{6}$ $\overline{6}$. For the latter $2 \rightarrow 2$ process, the formalism implements the notion that, in the presence of QCD radiation, the actual transverse momentum produced by the hard collision is not \vec{p}_T but rather $\vec{p}_T - \vec{Q}_T/2$, with \vec{Q}_T the total transverse momentum of unobserved soft recoiling partons. The joint-resummed partonic p_T spectrum has the form of a hard scattering cross section as a function of $p'_T \equiv |\vec{p}_T - \vec{Q}_T/2|$, convoluted with a *perturbative*, albeit resummed \vec{Q}_T distribution. We have extended ^{[7](#page-3-6)} the joint resummation formalism to the p_T distribution of heavy quarks produced in hadronic collisions. Key differences with the prompt-photon case are, first, the presence of the heavy quark mass m, preventing a singularity in the hard scattering function when $Q_T = 2p_T$ and, second, the possibility of multiple colored states for the produced heavy quark pair.

2. Resummed heavy quark transverse momentum spectra

We consider the inclusive p_T distribution of a heavy quark produced via the strong interaction in a hadron-hadron collision at center of mass (cm) energy \sqrt{S} . Exact

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higher order corrections to the differential cross sections for these partonic processes have been computed to NLO $8,9,10,11$ $8,9,10,11$ $8,9,10,11$ $8,9,10,11$ $8,9,10,11$ $8,9,10,11$. Up to corrections $\mathcal{O}(1/p_T^2)$, the observable may at any order 12 be written in the following factorized form

$$
\frac{d\sigma_{AB\to Q+X}}{dp_T} = \sum_{a,b} \int_0^1 d\xi_a d\xi_b \, \phi_{a/A}(\xi_a, \mu) \phi_{b/B}(\xi_b, \mu) \frac{d\hat{\sigma}_{ab\to Q+X}}{dp_T}(\xi_a, \xi_b, \alpha_s(\mu), p_T) ,
$$
\n(1)

with $d\hat{\sigma}_{ab\to Q+X}/dp_T$ the partonic differential cross-section, $\phi_{a/A}$ and $\phi_{b/B}$ parton densities, and μ the factorization/renormalization scale.

Threshold enhancements essentially involve the energy of soft gluons. In the context of the factorization [\(1\)](#page-1-0) we define hadronic and partonic threshold by the conditions $S = 4m_T^2$ and $\hat{s} = 4m_T^2$, respectively, with m_T the transverse mass $\sqrt{m^2 + p_T^2}$. It is convenient to define the scaling variables

$$
x_T^2 = \frac{4m_T^2}{S} \,, \qquad \hat{x}_T^2 = \frac{4m_T^2}{\xi_a \xi_b S} \,, \tag{2}
$$

so that hadronic (partonic) threshold is at $x_T^2 = 1$ ($\hat{x}_T^2 = 1$). The higher order corrections to the partonic cross section $d\hat{\sigma}_{ab}/dp_T$ contain distributions that are singular at partonic threshold. Threshold resummation organizes such distributions to all orders.

There are also recoil effects, resulting from radiation of soft gluons from initialstate partons. We wish to treat these effects in the context of joint threshold and recoil resummation. We identify a hard scattering with reduced cm energy squared Q^2 and at transverse momentum \vec{Q}_T with respect to the hadronic cm system. This hard scattering produces a heavy quark with transverse momentum

$$
\vec{p}'_T \equiv \vec{p}_T - \frac{\vec{Q}_T}{2} \,. \tag{3}
$$

The kinematically allowed range for the invariant mass Q of the heavy quark pair in this hard scattering is limited from below by $2m'_T = 2\sqrt{m^2 + p'^2_T}$ so that threshold in the context of joint resummation is defined by

$$
\tilde{x}_T^2 \equiv \frac{4m_T^{\prime 2}}{Q^2} = 1.
$$
\n(4)

A refactorization analysis 2 2 leads to the following expression for the observable in Eq. [\(1\)](#page-1-0)

$$
\frac{d\sigma_{AB\to Q+X}}{dp_T} = \int d^2Q_T \,\theta(\bar{\mu} - |\vec{Q}_T|) \frac{d\sigma_{AB\to Q+X}}{dp_T d^2 \vec{Q}_T} \,,\tag{5}
$$

Joint resummation for heavy quark production 3

where $\bar{\mu}$ is a cut-off and

$$
\frac{d\sigma_{AB \to Q+X}}{dp_T d^2 \vec{Q}_T} = \sum_{ab = q\bar{q}, gg} p_T \int \frac{d^2b}{(2\pi)^2} e^{i\vec{b}\cdot\vec{Q}_T} \int \frac{dN}{2\pi i} \phi_{a/A}(N,\mu) \phi_{b/B}(N,\mu) e^{E_{ab}(N,b)}
$$

$$
\frac{e^{-2\,Cr\,t(N)\,(\text{Re}L_\beta+1)}}{4\pi S^2} \left(\tilde{M}_1^2(N) + \tilde{M}_8^2(N)e^{C_A\,t(N)\left(\ln\frac{m_T^2}{m^2} + L_\beta\right)} \right)
$$

$$
\times \left(\frac{S}{4(m^2 + |\vec{p}_T - \vec{Q}_T/2|^2)} \right)^{N+1}.
$$
 (6)

Notice in particular the last factor, which provides a kinematic link between recoil and threshold effects. The exponential functions E_{ab} ^{[2](#page-3-1),[6](#page-3-5)} are to next-to-leading logarithmic (NLL) accuracy

$$
E_{ab}(N,b) = \int_{\chi(N,b)}^{Q} \frac{d\mu'}{\mu'} [A_a(\alpha_s(\mu')) + A_b(\alpha_s(\mu'))] 2 \ln \frac{\bar{N}\mu'}{Q} - gb^2, \qquad \bar{N} = N e^{\gamma_E},
$$
\n(7)

where the coefficients A_a and A_b can be found elsewhere ², and the function $\chi(N, b)$ is chosen to reproduce either NLL resummed recoil or threshold distributions in the appropiate limits 4 . We also added to the perturbative exponent the nonperturbative (NP) Gaussian smearing term $-gb^2$, in terms of the impact parameter b. We have introduced the variables

$$
t(N) = \int_{Q}^{Q/N} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')}{\pi}, \quad \text{Re} L_{\beta} = \frac{1+\beta^2}{2\beta} \left(\ln \frac{1-\beta}{1+\beta} \right), \quad \beta = \sqrt{1 - m^2/m_T^2}.
$$
 (8)

The functions $\tilde{M}^2_1(N), \tilde{M}^2_8(N)$ are the Mellin moments of the lowest order heavy quark production matrix elements for either the $q\bar{q}$ or $q\bar{q}$ channel, as appropiate, the index labeling the color-state of the heavy quark pair. Their explicit expressions can be found elsewhere⁷. The threshold-resummed result can now easily be derived, by substituting Eq. [\(6\)](#page-2-0) into [\(5\)](#page-1-1) and neglecting \vec{Q}_T in the last factor in Eq. (6). Then the \vec{Q}_T integral sets \vec{b} to zero everywhere, yielding the threshold-resummed result.

To illustrate these analytic results, we show for the case of top quark production at the Tevatron the p_T distribution for the dominant $q\bar{q}$ channel in Fig. [1.](#page-3-12) We observe that, while the resummed and NLO curves are close for small and moderate p_T (the inset provides a somewhat better view of the low p_T region), for large p_T values the resummed curves depart significantly from the NLO curve. Of course, cross sections for top quark production at such large p_T at the Tevatron are far too small to be measured, so that our plots at large p_T have only theoretical interest. For such large p_T values, the hadronic threshold, defined in Eq. [\(2\)](#page-1-2), approaches the partonic one, where larger N values dominate, a prerequisite for seeing significant effects for both resummations. The enhancements relative to the Born cross section are shown in the form of a K-factor. Threshold resummation produces an overall enhancement of the cross section that increases with increasing p_T , yielding e.g. an enhancement

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Fig. 1. Top quark p_T spectra and K-factors for the $q\bar{q}$ channel.

over NLO at $p_T = 800 \,\text{GeV}$. Joint resummation almost doubles that effect: the joint-resummed enhancement at large p_T effectively constitutes a smearing of the threshold-resummed p_T spectrum by a resummed recoil function.

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