Cosmological Supersymmetry Breaking and The Power of the Pentagon: A Model of Low Energy Particle Physics

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Abstract: I present a low energy Lagrangian implementing the idea of Cosmological SUSY breaking (CSB). The model predicts tan $\beta \sim 1$, and incorporates a new mechanism for breaking of $SU(2) \times U(1)$. The Higgs mass is determined by new physics and can evade the bounds of the MSSM. The model resolves the CP and flavor problems of SUSY. The up quark mass is non-vanishing. An axion-like particle, with TeV scale decay constant, appears, which could provide the solution of the strong CP problem. Such a particle is experimentally ruled out if it has conventional QCD axion couplings. The problem *may* be avoided by adding dimension 5 operators, which explicitly break the axial symmetry. However, it is likely that this reintroduces the strong CP problem.

KEYWORDS: Cosmological SUSY Breaking.

Contents

1. Introduction

In a previous paper[3], the author proposed a class of low energy models, which implemented the idea of Cosmological SUSY Breaking (CSB)[1][2]. The models contained an elementary singlet chiral superfield G , and a new strongly coupled gauge theory G , which contained matter multiplets with standard model quantum numbers. G was coupled to singlet bilinears in the new gauge theory, as well as $H_u H_d$. The model had an exact discrete R symmetry, with $R_G = 0$, which forbade a superpotential for G, as well as the existence of baryon and lepton number violating interactions of dimensions 4 and 5 (apart from the dimension 5 operator responsible for neutrino masses). The strong interaction scale, M , of the G theory had to be of order 1 TeV in order to obey constraints on chargino masses.

CSB was implemented by adding an R violating term

$$
\int d^2\theta \lambda^{1/4} M_P^2 f(G/M_P),
$$

to the Lagrangian, where λ is the cosmological constant. When G is small, this leads to an effective potential

$$
V \sim \lambda^{1/4} M_P(K^{G\bar{G}}(G/M, \bar{G}/M)|f_1|^2 - 3|f_0|^2),
$$

where f_0 and f_1 are the first two coefficients in the Taylor expansion of $f(x)$ around the origin. $K^{G\bar{G}}$ is the inverse Kahler metric generated for G by the strong dynamics of the G theory. If the potential has a minimum at $G \sim M$ then SUSY is broken. f_0 is fine tuned (its magnitude is of order one) to make the low energy effective cosmological constant equal to λ , which is a high energy input.

These models had many attractive features. They gave an acceptable pattern of SUSY breaking and solved the SUSY flavor and CP problems. They provided a μ term of the right order of magnitude. $SU(2) \times U(1)$ breaking was partly determined by strong interaction physics and the Higgs mass bounds of the MSSM were violated. Finally, if the G theory has automatic CP conservation, then the strong CP problem is solved, without axions or massless up quarks.

The only problem with this class of theories is that no one has found any members of the class. In order to keep G massless in the $\lambda = 0$ limit, the G theory had to preserve R symmetry (no spontaneous breaking) and was not allowed to have a low energy field of R charge 2. In all known examples, the $R = 2$ bilinear which couples to G in the microscopic theory, appears as an elementary field below M and ruins the SUSY breaking mechanism.

In this paper we examine a model with a specific choice of the $\mathcal G$ gauge theory. We then find that the addition of two other singlets, S, T , with R-charge 2, leads to a state with SUSY breaking. The low energy, non-gravitational, effective field theory, also contains SUSic minima. The rules of CSB tell us to add a constant term to the superpotential to tune the value of the potential at the SUSY violating minimum to λ. Given this tuning, SUSic minima have negative energy density of order $\lambda^{1/2} M_P^2$. We argue that transitions between the dS space and negative energy Big Crunches are so improbable that it is unlikely that any observer can survive long enough to experience them. The probability that an observer will experience a Big Crunch before it is destroyed in some other fashion is of order $e^{-c(RM_P)^2}$, where R is the radius of our dS horizon and c is a constant of order one.

In the next section we introduce an explicit and fairly unique candidate model, the Pentagon, for the low energy sector of the theory of Cosmological SUSY Breaking. We then explore some of its properties. In section 7 we show that by a simple change of quantum numbers we can convert the model into a model of dynamical breaking of SUSY and electroweak gauge symmetry. This section was motivated by a remark of M. Dine. The two interpretations of the model are phenomenologically similar, and differ mainly in their explanation of the absence of certain terms in the effective action. There is however a key difference with regard to a pseudo-Nambu-Goldstone boson, which plays the role of a QCD axions, and which is an inevitable consequence of this model. In the DSB interpretation of the model this PNGB gets mass only from the QCD anomaly. It is a low scale QCD axion, and is ruled out by experiment. It may be possible to solve this problem in the CSB interpretation of the model by breaking the symmetry through a dimension 5 operator. This can raise the mass of the axion, and might explain its absence in experiments done so far, and its lack of an effect on stellar evolution. At the moment, it appears that the requisite change in mass can be achieved only by setting the scale of the Pentagon gauge interactions at about 3 TeV, which seems to require a fine tuning of order one percent in dimensionless couplings in order to be consistent with the scale of electroweak symmetry breaking. This PNGB is either one of the worst phenomenological vulnerabilities of the Pentagon model, or a promising experimental signature.

The dual DSB/CSB interpretations point up the key new features of the model. The first is a new mechanism for electroweak symmetry breaking. The second is the use of *meta-stable* SUSY violating minima in a field theory which has super-symmetric solutions in the absence of gravity. The c.c. is chosen to be very small and positive at the meta-stable minimum, and this renders tunneling amplitudes to the negative energy density region of the potential unobservably small.

Some brief comments about experimental signatures are in the conclusions. The paper also contains two appendices. The first is devoted to an anthropic discussion of various coincidences in the physics and cosmology of this model. The second contains some calculations relevant to the structure of electroweak symmetry breaking.

I end this introduction with a caution to the reader. At a number of points in the exposition I will have to assume that certain couplings vanish. In assessing the plausibility of these assumptions, it is important to keep in mind the logic underlying the present paper. I am *assuming* that a quantum theory of dS space exists, compatible with the hypothesis of CSB. This paper is an attempt to construct a low energy model compatible with that assumption. Thus, if it is necessary to assume certain couplings vanish in order to find a SUSY violating minimum, then this will follow automatically from the rules of the underlying theory. We do not have to have symmetry explanations of every vanishing coupling. Nonetheless, like a good little effective field theorist, I have tried to find symmetry explanations. The reader will judge how well I succeeded.

Somewhat more disturbing are couplings that are required to be small only for phenomenological reasons. I have found symmetries which forbid all of these at vanishing cosmological constant. The most important of these is an R symmetry, which *must* be broken when the cosmological constant is non-vanishing. One must then worry about whether the dangerous couplings reappear. The mechanism of R symmetry breaking involves the still mysterious degrees of freedom on the horizon of dS space[2]. Thus, the answer to the question about phenomenologically dangerous R violating couplings (which don't disturb the SUSY breaking minimum) is not one which can be answered in low energy effective field theory.

2. The Beast Whose Number is 555

In order to generate gaugino masses consistent with experimental bounds, the $\mathcal G$ theory must contain chiral fields charged under $SU(1, 2, 3)$. To preserve coupling unification, it is best to add full $SU(5)$ multiplets, so $SU(5)$ must be an anomaly free subgroup of the flavor group of the theory. We will choose the $SU(5)$ supersymmetric gauge theory with 5 flavors, and will call this new gauge theory the Pentagon¹ and the matter fields P_i and \tilde{P}^i the *pentaquarks*² or p-quarks for short (The indices on pentaquark fields describe their transformation under the $SU(5) \times SU(5)$ flavor symmetry of the Pentagon model, and we hide indices referring to the strong Pentagon gauge group).

We will constantly use the seminal results of Seiberg[4] on the non-perturbative structure of SUSY QCD. In SUSY QCD with $N_F = N_C$ the quark and anti-quark superfields have R charge zero under the anomaly free $U(1)$ R symmetry. We choose the action on the pentaquarks of the exact Z_N R symmetry required by the rules of CSB, to be a subgroup of the anomaly free $U(1)_R$. That is, we assume that the full theory of the universe, in the limit $\lambda \to 0$, has an exact discrete symmetry, which acts on the fields of this model as a discrete subgroup of the anomaly free R symmetry. In order to conform to the literature, we will call the dynamical scale of the Pentagon Λ_5 rather than M

¹Quintessence is already in use, and Pentagram would be dangerous in the current funding climate.

²Although this term is already used for another concept, it is my impression that we won't have to worry about confusion much longer.

When the standard model couplings are turned off, the effective theory below Λ_5 has a chiral superfield $\mathcal{M}_i^j = P_i \tilde{P}^j$ in the $(5, \bar{5})$ of $SU(5)_L \times SU(5)_R[4]$. All components of this matrix have R charge 0. Under the diagonal $SU(5)$ it breaks up into the direct sum of a singlet and an adjoint. It is convenient to work with the two standard model singlets $\mathcal{M}_t = \text{tr } I_t \mathcal{M}$ with $t = 2, 3$. I_t are the two orthogonal $SU(1, 2, 3)$ invariant projectors in the 5 representation of $SU_V(5)$.

There are two more composite standard model singlets, which are massless when $\lambda = 0$. These are the P-baryon, B, and anti-P- baryon, B. They have P-baryon number ± 5 . P-baryon number is an exact symmetry of the low energy theory, which we presume broken via irrelevant terms of order $1/M_U$ or $1/M_P$. These are our dark matter candidates [6], and we will see that they obtain mass once λ is turned on.

Finally, we will introduce two chiral superfields, S and T , singlet under all gauge groups, and elementary at scales above Λ_5 . They have R charge 2, and Yukawa couplings

$$
\int d^2\theta \left\{ S[g_S(\mathcal{M}_2 + \mathcal{M}_3)\Lambda_5 + g_\mu H_u H_d] \right.
$$

$$
+ T[g_T(-\frac{3}{2}\mathcal{M}_2 + \mathcal{M}_3)\Lambda_5] \}.
$$

If the R symmetry group is Z_N with $N \geq 3$ and $N \neq 4$, then R symmetry does not allow any renormalizable terms involving only S and T in the Lagrangian. It would allow terms linear in S and T , multiplied by a function of G . However, there is a discrete symmetry group, F, which forbids these. The Pentagon gauge interactions leave a discrete subgroup of the axial $U_{AP}(1)$ of the penta-quarks unbroken. By giving S, T, H_u, H_d and the quarks and leptons of the standard model charge under an appropriate subgroup F of this discrete group, we can make it a symmetry of the whole Lagrangian. The terms involving G break this symmetry, and so must vanish when $\Lambda = 0.$

Note also that, since we have insisted that $H_u H_d$ have R charge 0, the action of the exact discrete R symmetry on ordinary quarks, must combine a discrete subgroup of the anomaly free $U_R(1)$ of SUSY QCD with six flavors, with a subgroup of the discrete Z_{12} subgroup of $U_A(1)$, which is left unbroken by QCD instantons.

We have omitted a possible coupling of the form TH_uH_d , which will turn out to be crucial to our discussion of CP violation. One possible excuse for this is that the couplings we have kept are compatible with S being a singlet, and T a component of an adjoint under the $SU_V(5)$ unified group, which is restored (probably in higher dimensions) at the scale M_U . Although $H_{u,d}$ nominally come from a [5] and [5], and can couple to both a singlet and an adjoint, there are also ways to prevent such a coupling at the unification scale. Non-renormalization theorems will prevent it from being generated as we scale down from M_U to Λ_5 .

On the subspace of moduli space where only \mathcal{M}_t are non-zero, the modified moduli space constraint yields

$$
\mathcal{M}_2^2 \mathcal{M}_3^3 = \Lambda_5^5.
$$

It is easy to see that there is a supersymmetric vacuum with $SU(2) \times U(1)$ broken at the scale Λ_5 . This follows from the moduli space constraint, and $F_S = 0$.

The five flavor $SU(5)$ gauge theory (the Pentagon), its implicit coupling to the SSM, and the Yukawa coupling of the singlets S and T constitute the new features of our model. The rest of the model is just the SSM, without a μ term or soft SUSY breaking terms, plus the Goldstino field G, which at the moment is a completely decoupled massless chiral superfield. The full gauge group of the model is $SU_P(5) \times SU_V(5)$, though only the $SU(1, 2, 3)$ subgroup of $SU_V(5)$ is visible at low energies. The full group is probably only realized in higher dimensions. The P_i are in the $[5, \bar{5}]$ of this product group, and the \tilde{P}^i are in the [5,5]. Quark/lepton fields are in three copies of the $[1, \bar{5} + 10]$. The Higgs fields are an incomplete multiplet, perhaps arising from a $[1, 5 + 5]$. The S and G fields are singlets of both groups and the T field may be a remnant of a [1, 24]. This model is supposed to be a complete description of low energy physics when $\lambda = 0$.

When we turn on λ , we add new R and F violating terms to the effective action. These terms are the *deus ex horizonae*, and we know very little about their nature, except that they scale to zero with λ , and they must give rise to a gravitino mass of order $\lambda^{1/4}$, and a c.c. of order λ . In [2] I gave a hand waving prescription for calculating these terms in terms of Feynman diagrams whose internal lines can interact with the horizon degrees of freedom. Such diagrams mix up infrared properties of the bulk, with Planck scale physics near the horizon. Each of the R violating terms will scale with a characteristic *critical exponent* $\left(\frac{\lambda^{1/4}}{M_E}\right)$ $\frac{\lambda^{1/4}}{M_P}$ as the c.c. goes to zero. It will also have some dependence on the IR scales of the theory, which will determine the number of powers of the Planck mass in the coefficient. Since the c.c. is so small, it is plausible that only one R violating term dominates most of the physics³.

For phenomenological reasons, and reasons of simplicity, I will make the assumption that this unique R violating term has the form

$$
\int d^2\theta \, \left[g_G G \Lambda_5 (\mathcal{M}_2 + \mathcal{M}_3) + \lambda^{1/4} M_P^2 f_0 \right].
$$

³I am not counting here the R violating constant in the superpotential, whose role is to tune the c.c. to λ .

We will see that this leads to SUSY breaking of the order of magnitude required by the hypothesis of CSB if $g_G \sim \frac{\Lambda^{1/4} M_F}{\Lambda_5^2}$ $\frac{\Lambda_{MP}^2}{\Lambda_5^2}$. The mechanism for SUSY breaking in the effective field theory is similar to that of Intriligator and Thomas[5]. One of the inverse powers of Λ_5 in this formula disappears when we go above the scale Λ_5 and write the coupling as a Yukawa coupling to penta-quarks. The other would be quite mysterious in conventional effective field theory. It is plausible because of the IR dependence of the diagrams of [2]. Note also that there is no way that we can take $\Lambda_5 \to 0$. The ratio of Λ_5 to the Planck scale is completely fixed in the $\lambda = 0$ limit⁴. λ is the only tunable parameter in the model.

It should be emphasized that we have gone beyond the rules of CSB at this point. We could have assumed that CSB was implemented by a superpotential term⁵ $\lambda^{1/4} M_P G$. G would be stabilized at the unification scale by invariant terms of the form $\delta W = \frac{T^p}{M_U^{p-3}} f(G/M_U)$, with f a bounded function (see below). This would give rise to a hidden sector model with the scale of SUSY breaking in the standard model far lower than the electroweak scale. Our decision to instead introduce SUSY breaking via an order 1 Yukawa coupling, g_G , is based frankly on phenomenology. We will have to understand a lot more than we do now about the underlying mechanism for CSB, before we can judge whether the present model can be derived, rather than postulated.

3. Baryon and lepton number

A central element in CSB is the discrete R symmetry that guarantees Poincare invariance in the the limiting model. This can be put to other uses. In [3] I showed that it can eliminate all unwanted dimension 4 and 5 baryon and lepton number violating operators in the supersymmetric standard model⁶. The interaction $\int d^2\theta \ H_u^2 L^2$, should not be forbidden by R. We will adopt the philosophy of a previous paper and insist that the texture of quark and lepton Yukawa couplings, as well as neutrino masses, are determined by physics at the unification scale.

We will choose the R charge of SSM fields to be independent of quark and lepton flavor, and denote it by the name of the corresponding field. All R charges are to be understood modulo N, where Z_N is the R symmetry group. Flavor dependent R charges

⁴This means that the phenomenological constraint $\Lambda_5 \sim 1$ TeV, is an implicit challenge to the underlying quantum gravity model. That model must predict the ratio of Λ_5 to the Planck scale.

⁵A term much less offensive to the effective field theorist, because it contains no inverse powers of the IR scale Λ_5 .

⁶Here we are invoking the hypothesis made above that R violating terms other than g_G scale with high powers of the c.c. and are negligible.

would require many important Yukawa couplings to vanish, and the corrections to the R symmetric limit are too small to account for the non-zero values of these couplings.

The condition that the standard Yukawa couplings are allowed by R symmetry is

$$
L + H_d + \bar{E} = Q + H_d + \bar{D} = Q + H_u + \bar{U} = 2.
$$
\n(3.1)

As noted above, the Yukawa couplings of S and T require

$$
H_u + H_d = 0.\t\t(3.2)
$$

Note that this condition forbids the standard μ term $\int d^2\theta$ $H_u H_d$. We will also impose $2L + 2H_u = 2$ to allow the dimension 5 superpotential responsible for neutrino masses. The renormalizable dynamics of the Pentagon gauge theory preserves all flavor symmetries of the standard model. This forbids the generation of the neutrino mass superpotential with coefficient $\frac{1}{\Lambda_5}$. As emphasized in [3], we imagine the neutrino mass superpotential, and the texture of the quark and lepton mass matrices, to be determined by physics at the scale M_U , probably via a Froggat Nielsen mechanism.

Dimension 4 baryon and lepton number violating operators in the superpotential will be forbidden in the limiting model by the inequalities

$$
2L + \bar{E} \neq 2\tag{3.3}
$$

$$
2\bar{D} + \bar{U} \neq 2,\tag{3.4}
$$

$$
L + Q + \bar{E} \neq 2. \tag{3.5}
$$

Absence of dimension 5 baryon number violating operators requires

$$
3Q + L \neq 2\tag{3.6}
$$

$$
3Q + H_d \neq 2\tag{3.7}
$$

$$
\bar{E} + 2\bar{U} + \bar{D} \neq 2,\tag{3.8}
$$

The condition that there be no baryon number violating dimension 5 D-terms is that none of $Q + \overline{U} - L$; or $U + E - D$, vanishes.

If we solve for L, \bar{D} , \bar{U} , \bar{E} in terms of Q and H_u , the inequalities become

$$
3Q - H_U \neq 1, 2, 4, 5 \tag{3.9}
$$

$$
Q + H_U \neq 0. \tag{3.10}
$$

These are all understood modulo N^7 . If $N = 3$, a possible solution is $H_u = 0$, $Q = 1, 2$. If $N \geq 4$, $H_u = 0$, $Q = 1$ is always a solution, and there are others. Thus, like the models of [3], the R symmetry of the Pentagon model can be chosen to forbid all dangerous baryon and lepton number violating terms.

4. CP Violation and Flavor

Except for the standard Yukawa couplings, the entire low energy effective Lagrangian is invariant under the $SU(3)_Q \times SU(3)_{\bar{U}} \times SU(3)_{\bar{D}}$ flavor group of the standard model. Thus, if we imagine that all higher energy physics comes in above the unification scale, then the Pentagon model has a GIM mechanism and predicts flavor changing neutral currents within experimental limits. Therefore, we adopt the philosophy of [3], according to which the origins of flavor and neutrino mass physics reside at the unification scale.

We begin our discussion of CP violation in the effective theory above the scale Λ_5 . We will continue to make the assumption that there is only one significant R violating coupling, g_G , in the low energy Lagrangian at this scale. This coupling can be made real by a phase rotation of the Goldstino superfield G . The problem of CP violation can then be addressed in the $\lambda = 0$ limit. The Lagrangian is that of an $SU(5) \times SU(1, 2, 3)$ gauge theory, coupled to two singlet chiral fields S, T with superpotential

$$
W = g_S S[P_i \tilde{P}^i] + g_\mu S H_u H_d + g_T T[Y_j^i P_i \tilde{P}^j]
$$

+
$$
\lambda_U^{mn} H_u Q_m \bar{U}_n + \lambda_D^{mn} H_d Q_m \bar{D}_n + \lambda_L^{mn} H_d L_m \bar{E}_n).
$$

The $SU(5)$ and $SU(1, 2, 3)$ gauge indices are implicit in these formulae. The matrix Y is defined by $Y \equiv I_3 - \frac{3}{2}$ $\frac{3}{2}I_2$. There is also a dimension five term which will generate neutrino masses when H_u gets a VEV, and a variety of other terms scaled by inverse powers of M_U or M_P . We will not discuss questions of flavor and CP violation in the lepton sector in this paper.

We begin our discussion of CP violation by performing $U_A(1)$ and $U_{AP}(1)$ transformations to eliminate the imaginary parts of the two strong gauge couplings θ_3 and θ_5 . From this point on we will only perform anomaly free transformations on chiral superfields, so these angles will remain zero.

We have noted that the only terms in the effective Lagrangian which violate the $SU(3) \times SU(3) \times U_B(1)$ flavor symmetry of the standard model, are the Yukawa couplings λ_U and λ_D . We can use the flavor group to eliminate all of the phases in $\lambda_{D,U}$ except the usual CKM angle and a single overall phase.

⁷ Actually, we have not written the most general solution. In solving the equation for \bar{E} we neglected a term that vanishes modulo $N/2$ but not modulo N.

Now we perform the following sequence of transformations:

- An anomaly free $U_R(1)$ transformation, with angle α .
- A rotation of S by phase angle β .
- A rotation of T by phase angle γ .
- A common phase rotation of H_u and H_d

It is easy to see that these rotations can be chosen to eliminate all phases in $g_{T,S,\mu}$ and arg det $[\lambda_u \lambda_d]$. The effective potential, which includes effects of the CP conserving Pentagon gauge interactions, is CP invariant. We will assume (this must be checked dynamically) that the minimum we find after SUSY breaking does not violate CP spontaneously. *If that is the case, the Pentagon model solves all CP problems of SUSY, as well as the strong CP problem.* The only CP violating phase in the renormalizable effective theory at scale $\Lambda_5 \sim 1$ TeV is the usual phase in the CKM matrix. However, we will see below that in effect what we have constructed is a supersymmetric version of the Peccei-Quinn axion model. The model has a pseudo-Nambu-Goldstone boson whose mass (taking into account only the renormalizable couplings in the Lagrangian) comes predominantly from QCD. This is a Peccei-Quinn-Weinberg-Wilczek axion, and is ruled out by a combination of laboratory experiments and stellar physics.

We can solve the laboratory problems by taking $\Lambda_5 \sim 3TeV$ (which may also alleviate possible problems with precision electroweak data, but may require fine tuning to get the electroweak scale right). The stellar problems can be resolved by raising the axion mass with a dimension 5 operator. Unfortunately, the phase of this new coupling, probably reintroduces the strong CP problem.

5. $SU(2) \times U(1)$ and SUSY breaking

We have noted above that $SU(2) \times U(1)$ is already broken when $\lambda = 0$. The nominal breaking scale is of order Λ_5 , and we see that Λ_5 is required to be 1 TeV or smaller. However, the uncertainties introduced by the strong Pentagon interactions, as well as by the new dimensionless couplings we have introduced, will prevent us from getting more than order of magnitude estimates of the masses of various particles. The electroweak scale is related to Λ_5 by a function that depends on the couplings g_S, g_μ , and the nonperturbative dynamics, which gives an expectation value to \mathcal{M}_t . We will see below that there are phenomenological reasons to want $\Lambda_5 \sim 3 \text{ TeV}$, in order to avoid experimental problems with an axion-like particle.

We demonstrate in Appendix 2, that once SUSY breaking is taken into account, there is a range of parameters for which $SU(2) \times U(1)$ is broken, and every physical component of the Higgs fields, $H_{u,d}$ gets a mass of order Λ_5 , independent of the standard model gauge couplings. Thus, the conventional SUSY upper bounds on the Higgs mass do not apply to this model. Precision electroweak fits put an upper bound of about 200 GeV on the Higgs mass (though this is in the absence of new physics, which is present in abundance in the Pentagon model).

A possible problem with a scale as low as 1 TeV is the detailed agreement of precision electroweak fits with the unadorned standard model. At the moment, I am too calculationally challenged to determine whether this is a problem for the Pentagon model, but increasing the scale to 3 TeV will help. On the positive side, it is worth pointing out that the pattern of electroweak symmetry breaking picked out by the Pentagon is precisely the same as that conventionally attributed to the effect of the standard model D terms.

In making the above analysis, I have used the fact that low energy physics in the Pentagon model is exactly CP invariant (up to the CKM angle), but have made the additional assumption that the potential chooses a CP conserving minimum for the fields. The latter assumption is certainly plausible, but needs further investigation.

5.1 SUSY breaking

Recall that the c.c. is a tunable parameter. Our best guess at the dynamical structure of this model is obtained in the limit that λ is much smaller than its observed value. In this regime, we can continue with the effective field theory analysis, which reveals the full qualitative structure. We emphasize that this analysis breaks down for the observed value of λ , where we must deal with the full dynamical complication of the strongly coupled Pentagon model. However, the small λ analysis gives a very attractive phenomenological picture, and we can hope that it survives a more rigorous treatment.

In the limit of small λ we deal with an effective field theory for the moduli, below the scale Λ_5 . This theory is determined by a Kahler potential, and a superpotential

$$
W = S[g_S(\mathcal{M}_2 + \mathcal{M}_3)\Lambda_5 + g_\mu H_u H_d] + g_T T(\frac{3}{2}\mathcal{M}_2 - \mathcal{M}_3)\Lambda_5
$$

$$
+ g_G G(\mathcal{M}_2 + \mathcal{M}_3)\Lambda_5 + H_u Q^T \lambda_u \bar{U} +
$$

$$
H_d(Q^T \lambda_d \bar{D} + L^T \lambda_l \bar{E})
$$

There is also a constraint on the moduli space

$$
\det \mathcal{M} - B\bar{B} = \Lambda_5^5.
$$

Apart from the $SU(1, 2, 3)$ gauge group of the standard model, only discrete subgroups of the R and flavor groups are exact symmetries of the world (when $\lambda = 0$), but we are neglecting terms inversely proportional to the unification scale. For constraining the dynamics of the strongly coupled Pentagon theory, we can consider its flavor group to be a continuous symmetry group.

It is convenient to write the first term in the superpotential as $tr(N\mathcal{M})$, where the spurion field N transforms in the $(5,5)$ representation of the flavor group. Later, we will set $N = (g_G G + g_S)I + g_T T Y$, where Y is weak hypercharge. The most general invariants we can make are $tr[(N\mathcal{M})^k, (\mathcal{M}\mathcal{M}^{\dagger})^k, (NN^{\dagger})^k, (\mathcal{M}^{\dagger}N^{\dagger})^k]$, with $1 \leq k \leq 5$.

We will restrict attention to the submanifold of moduli space where $\mathcal{M} = \mathcal{M}_3 I_3 +$ \mathcal{M}_2I_2 . We will verify later that the excitations of other moduli normal to this submanifold, are massive⁸. The same will be true for the baryonic components of the moduli space. On this submanifold, there are more invariants than variables, and the flavor symmetry of the Pentagon does not restrict the form of the Kahler potential. It is constrained only by CP invariance.

The F terms of the four independent chiral superfields, S, T, G, M_3 are

$$
F_S = g_S \Lambda_5 (\mathcal{M}_2 + \mathcal{M}_3) + g_\mu H_u H_d,
$$

$$
F_T = g_T \Lambda_5 (\mathcal{M}_3 - \frac{3}{2} \mathcal{M}_2),
$$

and

$$
F_G = g_G(\mathcal{M}_2 + \mathcal{M}_3)\Lambda_5,
$$

$$
F_3 = (g_S\Lambda_5 S + g_G\Lambda_5 G)(1 + \mathcal{M}'_2) + g_T T \Lambda_5 (1 - \frac{3}{2}\mathcal{M}'_2).
$$

In these equations, $\mathcal{M}_2 = \Lambda_5 \left(\frac{\Lambda_5}{\mathcal{M}_3} \right)$ $\frac{\Lambda_5}{\mathcal{M}_3}$, and \mathcal{M}'_2 is its derivative with respect to \mathcal{M}_3 . The F terms of $H_{u,d}$ are

$$
F_{u,d} = g_{\mu} S H_{d,u}
$$

(we set the squark and slepton VEVs to zero). It is clear that $F_S = F_T = F_G = 0$ is an inconsistent set of equations when $g_G \neq 0$, so SUSY is broken. When g_G is small, the energy will be minimized with all F terms $\leq F_G$. For the actual value of λ , this leads to energies of order Λ_5 or even greater (by a small factor). Thus, effective field theory is not a good tool in the regime of phenomenological interest. Nonetheless, since it is all we have available, we can hope that it gives the right qualitative physics.

In the low energy approximation in which we are working, the supergravity formula for the potential reduces to

⁸Though, to be honest, we will not verify the absence of tachyons.

$$
V = \mu^4 (K^{i\bar{j}} F_i \bar{F}_{\bar{j}} - |f_0|^2),
$$

where $\mu^2 \sim \lambda^{1/4} M_P$. The rescaled chiral fields X^i which appear in this formula are $g_G G/\Lambda_5, g_G^2(S-S^0)/\Lambda_5, g_G^2(T-T^0)/\Lambda_5, g_G^2(\mathcal{M}_3-\mathcal{M}_3^0)/\Lambda_5$, and $g_G^2(H_{u,d}-H_{u,d}^0)/\Lambda_5$. The symbols with zero superscripts refer to the value of the corresponding field at the supersymmetric minimum, when $\lambda = 0$. The linear, rather than quadratic dependence on g_G in the G dependence of the potential, is due to the fact that this field has a flat potential when $g_G = 0$. To leading order in electroweak gauge couplings, the Kahler metric depends on the Higgs fields only through the combination $g_{\mu}SH_uH_d$. When g_G is small, S is of order g_G^2 and the overall scale of the Higgs VEVS is fixed by the standard model D-terms. Some components of the Higgs will be lighter than Λ_5 by factors of standard model couplings. However, for g_G of order its experimental value, all physical Higgs fields have mass order Λ_5 , independent of the standard model couplings. We discuss this more extensively in Appendix 2.

For a range of parameters, we will have both $SU(2) \times U(1)$ and SUSY broken, with the electroweak scale of order Λ_5 , and the SUSY breaking scale of order $\sqrt{g_G}\Lambda_5$. The ratio of the two scales is of order one for the observed value of the c.c.⁹. The potential for the Higgs fields depends on their ratio, tan $\beta \equiv \frac{|H_u|}{|H_d|}$ $\frac{|H_u|}{|H_d|}$, only through $\sin 2\beta$ and is minimized when the ratio is one, throughout the parameter range in which electroweak symmetry is broken. Thus, up to radiative corrections, the model predicts $tan \beta = 1$.

One worrisome feature of the model is that, as $g_G \to 0$, the minimum of the potential for G wanders out to infinity. We may worry that SUSY will be exactly restored for large values of G , which would make the low energy model incompatible with the basic principle of CSB. In fact, this does not happen. The discrete symmetry which forbids renormalizable couplings of T and G, will allow terms of the form $W =$ $\frac{T^p}{M_U^{p-3}}f(G/M_U)$. When G is of order M_U the function f will be order 1. Recalling that $T \sim M$ we find an approximate form for the potential when $g_G \ll 1$ and $G \sim M_U$.

$$
V \sim \lambda^{1/2} M_P^2 |(g_G G)|^2 + \text{Re}[\lambda^{1/4} M_P \Lambda_5^2 f'(G/M_U)].
$$

If the second term dominates the first we find a minimum at $f'(G_0/M_U) = 0$, with a SUSY breaking F term of order $\lambda^{1/4} M_P$. The condition for this to happen is

$$
\lambda^{1/2} M_P^2 < \Lambda_5^4 \left(\frac{\Lambda_5}{M_U}\right)^p
$$

which will always be satisfied for small enough λ . That is, the system will stabilize, with the right scale of SUSY breaking, with a unification scale VEV for G , in the limit of asymptotically small λ .

 9 We discuss this coincidence in Appendix 1.

5.2 Massive moduli and light superpartners

When $\lambda = 0$, the Pentagon model has a plethora of exactly massless moduli. These all get mass for nonzero c.c. To estimate the magnitude of these masses, consider quartic terms in the Kahler potential

$$
\delta K \sim \frac{aG\bar{G}\bar{X}X}{M^2}.
$$

The scalar components of X will get mass of order

$$
m_X \sim \frac{F_G}{\Lambda_5^2}.
$$

Terms in K of the form

$$
b\bar{G}D_{\alpha}X D^{\alpha}X + h.c.,
$$

give similar masses to the fermionic components. For the observed value of λ , and $\Lambda_5 \sim 1$ TeV, these are all in the TeV range. The observation of these P-hadrons, with the standard model quantum numbers attributed to them by the model, would be a spectacular experimental signature. Among these particles are the components of the $P - baryon$. These are standard model singlets and make an excellent dark matter candidate^[6], since the P-baryon number is conserved up to interactions of order $\frac{1}{M_U^q}$ with $q \geq 1$. Although P-baryons cannot be thermal relics, a primordial asymmetry in this quantum number could account for the observed dark matter density. If the asymmetry were generated by the same physical mechanism as the baryon asymmetry, we might even hope to find an explanation of the dark matter to baryon ratio.

It is worth noting that pseudo-Nambu-Goldstone bosons associated with spontaneous breaking of the axial $SU(5)$ generators by the VEV of \mathcal{M}_3 are not in fact light. $SU_A(5)$ is explicitly broken by the combination of couplings g_S, g_T even in the SUSic limit, so these PNGBs will only be light if the Yukawa couplings are small. There are a variety of phenomenological reasons to assume that this is not the case. One would need a full understanding of UV boundary conditions at M_U and of the renormalization group equations in order to decide whether it is plausible that all of these couplings are relatively large. Note that they all involve fields which are coupled to the asymptotically free Pentagon gauge theory, which at least pushes things in the right direction. In the next subsection we will describe a pseudo-Nambu-Goldstone boson, which follows from an approximate continuous R symmetry of our low energy Lagrangian.

I have not been able to find an argument that all of the SUSY violating scalar masses squared are positive. This is another dynamical assumption (local stability of the SUSY violating minimum) for whose justification one must look to the solution of the strongly coupled Pentagon theory¹⁰.

The fact that when λ is much smaller than its observed value, the model predicts a lot of light neutral particles with couplings to the standard model of order $1/\Lambda_5$, as well as a lot of light charged particles, may be of interest for tightening the anthropic *lower bound* on λ . The change in the properties of dark matter¹¹ with λ will alter the nature of galaxies, which for sufficiently small λ will be purely baryonic. Light weakly coupled particles will alter the nature of stars, and new light charged states (in addition to the squarks, sleptons, and charginos) will change the nature of chemistry. We will discuss this briefly in Appendix 1.

It appears that, apart from the axion, all the states lighter than M, will be standard model particles, and their superpartners. Higgsinos, winos and zinos will have SUSic masses of order $g_{1,2}\Lambda_5$, and will also get SUSY violating masses of order $\frac{\alpha_{1,2}}{\pi}$ $\lambda^{1/4} M_{P}$ $\frac{M_P}{\Lambda_5}$. Gluinos and photinos will get SUSY violating masses given by analogous formulae, with appropriate standard model fine structure constants. This suggests that some charginos may not be far above their experimental lower bounds, and in the discovery range of LHC. Similarly, sleptons and squarks will have masses related to corresponding gaugino masses as they are in gauge mediated models. The suggestion that sleptons are relatively light is exciting, because in a low energy SUSY breaking model, like the Pentagon, they have spectacular decays.

5.3 A pseudo Goldstone boson

The following $U_F(1)$ R transformation is a symmetry of the classical Lagrangian of our model: let S, T, G have E charge 2 and the penta-squarks E charge 0. We have to assign the E charge $E_u = -E_d$ to the Higgs fields. Invariance of the usual quark Yukawa couplings requires that

$$
E_Q + E_{\bar{U}} + E_u = 2 = E_Q + E_{\bar{D}} - E_u,
$$

whence

$$
2E_Q + E_{\bar{U}} + E_{\bar{D}} = 4.
$$

 10 N. Seiberg suggested that one check stability in an extreme region of moduli space, where the Kahler potential is calculable. Preliminary investigation suggests that all squared masses are positive in this region, but the mass matrix in this region is not related in any simple way to that in the phenomenologically interesting region of moduli space.

¹¹The primordial asymmetry, and fluctuation spectrum are determined by high energy physics independent of λ .

The latter quantity is all that we need to evaluate the QCD anomaly of the E symmetry, which is non-vanishing. $U_E(1)$ acts on the penta-quarks like the anomaly free R symmetry and so has no Pentagon anomaly.

As a consequence, at the renormalizable level, the dominant source of E breaking is the QCD anomaly. We have also argued that the fields S, T get VEVs of order Λ_5 , so there is a pseudo-Goldstone boson, the axion, with decay constant of order 1 TeV, which gets mass from the QCD anomaly. It is well known that such a particle is problematic, both from the point of view of terrestrial experiments, and of stellar physics.

We can try to solve this problem by breaking the E symmetry explicitly using irrelevant operators from the unification scale. An operator of dimension d will give an axion mass of order

$$
m_{ax}^2 \sim \frac{Q_d}{\Lambda_5^2 M_U^{d-4}},
$$

which should be larger than the QCD induced mass

$$
m_{axQCD}^2 \sim \frac{\Lambda_{QCD}^4}{\Lambda_5^2}.
$$

With $\Lambda_5 \sim 1$ TeV, and $M_U \sim 10^{16}$ GeV, this can only be satisfied if $d = 5$ and the VEV is of order Λ_5^d . In that case the mass is about 10 times the value expected from QCD. There are no dimension 5 superpotentials constructed from the Pentagon degrees of freedom, S, T , and G , which violate the E symmetry, but preserve a discrete R symmetry when $g_G = 0$. However, if we assume the fundamental discrete R charge of G is zero, then we can write a dimension 5 Kahler potential term

$$
\int d^4\theta (P\bar{P}G/M_Uh_a+c.c.).
$$

In section 7 we will introduce an alternative interpretation of the low energy Lagrangian, in terms of Dynamical SUSY Breaking (DSB). In this version of the model the exact discrete R charge of G is 2, and the dimension 5 term would not be allowed. It would then appear that the model has a low scale QCD axion, and is ruled out experimentally. Thus, experiment mildly prefers the cosmological interpretation of the origin of g_G^{12} .

Of course, it is by no means certain that this irrelevant operator solves the problem. We are predicting a 100 keV particle with couplings to the standard model which are probably close to weak interaction strength. The QCD anomaly in $U_E(1)$ symmetry

¹²One could imagine, as originally suggested by Dine, replacing the mysterious cosmological breaking of R symmetry by conventional dynamical breaking. However, this always seems to produce $g_G \ll 1$, which is not phenomenologically viable.

gives the axion a mixing with the π^0 of order $c_{\Lambda}^{F_{\pi}}$ $\frac{F_{\pi}}{\Lambda_5}$, with c a number of order 1. This is enough to produce couplings to hadrons and photons which make the axion a problem both for terrestrial experiments and stellar evolution.

We can resolve the accelerator and reactor problems by postulating that $\Lambda_5/c > 3$ TeV. The stellar problems can be resolved by lowering the mass parameter in the dimension five operator by a factor of 10 (so that it has the same size as the parameter that appears in the dimension five operator, which generates neutrino masses), and taking $\Lambda_5 \sim 3TeV$. This gives an axion mass ~ $5MeV$, which means that axions will not be produced in most normal stars. On the other hand, they couple strongly enough to be trapped inside of supernovae.

This relatively high scale for the Pentagon gauge interactions is probably also good for protecting the agreement of our model with precision electroweak data. On the other hand (see Appendix 2) it seems to require fine tuning of dimensionless couplings with about one percent accuracy, in order to get the correct value for the electroweak scale. It is clear that even if we succeed in curing the problem of not seeing the axion in existing experiments, it should be discoverable by an extension of axion searches. It is perhaps one of the most definite experimental signatures of our model.

Finally, we note that the phase of the coupling h_a will determine the VEV of the axion field, and appears to reintroduce the strong CP problem. To be certain that this is the case, we should integrate out physics at the scale Λ_5 with care, and directly compute the neutron electric dipole moment in terms of arg h_a , but the result doesn't look promising.

5.4 Coupling constant unification

From the point of view of the standard model, the Pentagon gauge theory adds matter in the $[5]$ of the standard Grand Unified $SU(5)$. This means that, at one loop, coupling constant unification works the same way as it does in the Supersymmetric standard model, with the same unification scale of about 2×10^{16} GeV. This is the scale M_U that we have mentioned repeatedly in the text. The value of the unified couplings is different, and since there are 5 extra triplet-anti-triplet pairs of chiral superfields in the Pentagon, the unified coupling is on the edge of the weak coupling regime at the unification scale. Indeed, we need a full two loop calculation, taking into account the effect of the Yukawa couplings and threshold effects associated with the strongly interacting Pentagon theory at the TeV scale in order to assess whether perturbative unification truly occurs in this model. Note that the relatively strong unified coupling might bring dimension 6 proton decay amplitudes into experimental range.

Note that it is this property of coupling unification which specifies the Pentagon theory among all possible $N_F = N_C$ models, as the likely candidate for a description of

realistic particle physics in the framework of CSB. For $N_C < 5$ we cannot embed the standard model in a vector-like subgroup of the flavor group, while for $N_C > 5$ we lose perturbative unification.

A significant feature of the Pentagon model is that QCD is not asymptotically free above the TeV scale. In fact, the scale at which asymptotic freedom is lost scales like $\lambda^{1/4} M_P$ $\frac{M_{M_P}}{\Lambda_5}$. This is the TeV scale for the observed value of λ . Thus, the Pentagon model is a realization of old ideas for explaining the relative proximity (on a logarithmic scale) of the QCD and electro-weak scales[9].

6. Vacuum decay

We have explored a particular region of moduli space, and found a SUSY violating minimum of the potential. It may be meta-stable, and that is of course crucial to our enterprise. However, it is almost certain that there are SUSic points when other moduli are turned on, so the dS space we find is likely to suffer from Coleman-DeLucia vacuum decay. This decay is of no phenomenological significance. Part of our prescription for finding a model, was to tune the c.c. at the SUSY violating minimuum to λ . As a consequence, the energy density at SUSic minima of the potential will be of order $-\lambda^{1/2}M_P^2$ and there is likely to be a CDL instanton describing the "decay" of de Sitter space into a Big Crunch universe. A non-gravitational estimate of the action of this instanton is $\sim \frac{8\pi^2}{g^2}$ $\frac{2\pi^2}{g^2}$, with g a small coupling, and it could easily predict a lifetime longer than the age of the universe. In fact, in [13]I will argue that the gravitational corrections to this instanton are large and that its action is actually of order $I_{inst} \sim c \frac{M_P^4}{\lambda}$. If this is correct, then the lifetime is one of those times so long that the unit one measures it in is irrelevant. It is virtually the same number measured in current ages of the universe as it is in Planck times. It is much more probable, by factors of order e^{cR^2} for an observer in dS space to be destroyed by the nucleation of a black hole at the observer's position, than to experience CDL vacuum decay.

From a more fundamental point of view however, this decay is troubling, because we thought we were finding an effective description of a stable dS space. In a future paper[13] I will argue that the CDL instanton does not really represent a decay. Rather, it describes a highly improbable statistical fluctuation in which the system temporarily goes into a very low entropy state (like all the air in the room gathering in a corner).

7. The Pentagon as a model of dynamical SUSY breaking

In this section, I want to show how the Pentagon model can be turned into a model of

 ^{13}R is the radius of dS space.

dynamical SUSY breaking, abandoning its connection with de Sitter space and quantum gravity¹⁴. I have been reluctant to do this, because of a belief in the fundamental connection between SUSY breaking and the structure of space-time. However, the resulting model is in some ways simpler, and has fewer mysterious assumptions, than its CSB motivated progenitor.

The DSB model, begins with the supersymmetric Pentagon model, but changes the R charge assignment of G to $R_G = 2$, so that g_G can be non-zero from the outset. We view G and S as originating from $SU(5)$ singlets at the unification scale, while T comes from a [24]. Both S and G can couple to $P_i\tilde{P}^i$ as well as to H_uH_d , but we take can take linear combinations and define S to be the combination that couples to $H_u H_d$. We omit the coupling of T to $H_u H_d$ to avoid a CP violating phase. As in the CSB version of the Pentagon, we believe that this can be explained at the unification scale and propagated to low energies by the non-renormalization theorem. Recall that inclusion of this coupling does not affect the dynamics of electroweak or SUSY breaking, but only the strong CP problem.

Renormalizable polynomial interactions between T, S , and G are forbidden by a Z_N R symmetry with $4 \neq N \geq 3$. Terms of the form $\mu^2 X$, with $X = S, T, U$, are forbidden by the discrete $U_A P(1) \times U_A(1)$ symmetry that forbade terms of the form $S_f(G)$ in the previous sections. At the classical level, there is a supersymmetric vacuum where all fields have zero VEVs. Once we take into account the dynamical scale Λ_5 of the Pentagon theory, this vacuum is replaced by a SUSY violating one. The scale of SUSY breaking is determined by the smallest of the couplings g_G, g_T, g_S (in the low energy EFT approximation discussed above, g_G was always assumed smallest), multiplied by Λ_5 . $SU(2) \times U(1)$ is broken at a scale of order Λ_5 .

As in the CSB version, there are SUSic points in moduli space, in particular, a point where only the \mathcal{M}_t components of the meson field, as well as the product BB are non-zero. It is important to check that the Pentagon dynamics actually produces a locally stable SUSY violating minimum. If this is the case, and we tune the constant in the superpotential to make the potential of order λ at the SUSY violating minimum, then non-perturbative instabilities are *at least* phenomenologically irrelevant, and may not be instabilities at all when quantum gravity is properly understood[13].

The existence of this alternative interpretation of the Pentagon model has implications for the CSB interpretation as well. It highlights the existence of an accidental R symmetry of the low energy model under which G has R charge 2. From the low energy point of view, this is a $U(1)$ symmetry, broken to a discrete group only by

¹⁴The idea of severing the connection between the Pentagon model and CSB was suggested by M. Dine.

non-perturbative QCD effects and the dimension 5 operator we introduced to raise the axion mass. When g_G is small, we have used an effective field theory approach to estimate various scales of symmetry breaking and particle masses. In particular, the SUSY violating part of gaugino masses, is estimated by calculating the gauge coupling functions for the standard model gauge multiplets. This calculation is non-perturbative in the Pentagon interaction strength, and one loop in standard model couplings¹⁵. The $U(1)$ R symmetry implies that the gauge coupling functions can only depend on the moduli \mathcal{M}_t and not on S, T, G . It is therefore important that the F term of \mathcal{M}_3 be non-vanishing. When $g_G = 0$, F_3 can be made to vanish by choosing the otherwise free expectation values of S and T. When we turn on g_G , SUSY is broken. The potential has a complicated dependence on S, T and G, through the Kahler potential. There is no apparent reason for F_3 to vanish or be anomalously small.

One apparent advantage of the DSB interpretation of the Pentagon is that within the realm of effective field theory in flat space, it is consistent to assume that the discrete R symmetry is exact¹⁶, and only broken spontaneously at the scale Λ_5 . This leads to sufficient suppression of dangerous B and L violating interactions. However, this is probably not consistent with the size of f_0 , the constant in the superpotential which fine tunes the c.c. to λ . Thus, whichever way we look at it, the issues of SUSY breaking and the tuning of the c.c. are intimately related. In addition, this exact discrete R symmetry, implies the accidental continuous $U_F(1)$ symmetry, and forbids the dimension 5 Kahler potential term which could prevent the PNGB of this symmetry from being an experimentally challenged QCD axion. Thus, the conservative effective field theorists preference for the DSB interpretation of the model, leads in unpromising directions.

There have been several previous attempts, $[11][10]$ to break electroweak symmetry using supersymmetric dynamics. Our model most resembles that of [10] but differs in detail. In particular, we work on a part of the Pentagon moduli space which preserves $SU(2)\times U(1)$, so that the VEVs of the elementary Higgs fields are the primary source of electroweak breaking. The authors of [10] also considered, but rejected, the possibility of gauge mediated SUSY breaking for the gauginos and sparticles of the SSM. From their point of view we are insisting on a larger Yukawa coupling g_G than might be considered natural. Its value is motivated by CSB. If it should turn out that this coupling is unrealistically large, we could, from the CSB point of view, add a term $\lambda^{1/4} M_P G$ to the superpotential (this would not fit well in the dynamical context of

 15 We have not presented the calculation in this paper, but using the magic of holomorphy, it reduces to a one loop calculation in the Pentagon as well.

¹⁶This argument is somewhat naive. It seems clear that a consistent theory of dS space can have no exact symmetries for a local observer, because charge can flow out through the horizon.

the present section) to achieve the value of the gravitino mass required by CSB. This would spoil our solution of the strong CP problem. Another major difference between the present paper and previous work is the way in which coupling unification is achieved.

8. Conclusions

The ideas of CSB, when combined with the existence of the standard model, and of coupling unification, lead to a rather unique model for TeV scale physics, the Pentagon model. This model adds only a few new parameters to low energy physics. In the SUSic limit, these are just g_S , g_T , g_μ and the scale Λ_5 at which the Pentagon gauge interactions become strong. SUSY violation adds one new parameter g_G . The model automatically conserves CP, except for the usual CKM angle. In particular, the QCD vacuum angle vanishes. The model has a discrete R symmetry which forbids all baryon and lepton number violating operators of dimensions 4 and 5, apart from the operator that gives rise to neutrino masses.

SUSY is broken by an R violating superpotential, $(\lambda^{1/4} M_P^2 f_0 + g_G G P_i \tilde{P}^i)$, which is attributed to interactions with the horizon of de Sitter space (a *deus ex horizonae*). g_G is given by the formula $\frac{\lambda^{1/4} M_P}{\Lambda^2}$ $\frac{\mu_{MP}}{\Lambda_5^2}$, which seems designed to induce heart attacks in dedicated practitioners of effective field theory. f_0 must be tuned to guarantee that the cosmological constant in the low energy effective Lagrangian agrees with its fundamental input value. This is the usual tuning we do in effective field theory, but here motivated by the insight that λ is a high energy input which cannot be renormalized by low energy field theory effects. The other bizarre feature of the model is the unusual power law dependence of the superpotential on λ and Λ_5 . f_0 does not seem to have a dramatic effect on particle physics, at the level of approximation in which we have worked.

Our analysis is done in a limit where the cosmological constant is even smaller than its observed value. The effective composite field theory valid in that limit, predicts that SUSY is broken at a scale $\sqrt{\lambda^{1/4}M_P}$, while $SU(2) \times U(1)$ is broken at a scale Λ_{5} . The electroweak breaking mechanism is very different from that of the SSM, and conventional Higgs mass bounds do not apply. The ratio of Higgs VEVs, $tan\beta$ is predicted to be close to one. The flavor and CP problems of SUSY are completely solved, as well as the strong CP problem. In this model, the origin of flavor and of neutrino masses has to do with physics at very high energy, probably the unification scale, and is taken as input to be explained by a more ambitious theory.

The usual SUSic dark matter does not exist, because the gravitino is the LSP, and its longitudinal component is strongly coupled. The gravitino is relativistic even at today's temperature. However, the model contains a long lived penta-baryon, which could be the dark matter if an appropriate asymmetry in penta-baryon number is generated in the early universe[6] . One might even hope to explain the baryon to dark matter ratio of the universe if the ordinary and penta-baryon asymmetries were generated in the same physical process.

There is no reason to believe that raising λ to its observed value will make a qualitative change in the electroweak and SUSY physics. Even in the effective field theory approximation, we were unable to do precision calculations, because the physics depends on non-holomoprhic quantities in the strongly coupled Pentagon. For realistic values of the parameters one must truly solve the strongly coupled theory in order to compute precision observables.

The coincidence in scales $10\Lambda_5^2 \sim \lambda^{1/4} M_P$ may be "explained" by the anthropic considerations of the appendix, in which case it is related to the cosmic coincidence between dark energy and dark matter. In this context it is worth noting that the nature of Weinberg's galaxy bound changes in this model, because the properties of dark matter depend on λ .

It is worth concluding by comparing this model to the SSM. The Pentagon model introduces only four new dimensionless parameters, $g_{S,T}$, g_{μ} and g_G to the standard model parameter set, as compared to the 107 of the SSM. In the Pentagon model, quark and lepton flavor are automatically conserved, except for the usual quark Yukawa couplings and neutrino mass terms. Low energy CP violation resides only in the usual CKM phase, and the phase of h_a , rather than the myriad new phases of the SSM. On the face of it, the model has a strong CP problem, and some mechanism must be found to solve it.

The Pentagonal mechanism for electroweak symmetry breaking is quite different from that of the SSM, and may not suffer from the same fine tuning problems. The conventional Higgs mass bound is evaded. Dark matter is a penta-baryon rather than an LSP, and there is no conventional LSP. The experimental signatures of this kind of dark matter will be quite different from those of a LSP. Decays of superpartners of the standard model particles may proceed in a manner similar to gauge mediated models (though the details of this remain to be investigated). Note that the gravitino is extremely light, and causes no cosmological problems (in marked contrast to most gauge mediated models, and parts of SSM parameter space).

The price for all of these (mostly attractive) differences is an inability to do precision calculations. If the Pentagon really controls the world, high energy theorists and lattice gauge theorists have their work cut out for them. It will be important to find a way of calculating in this strongly coupled theory in order to make precise comparisons with experiment. *Also, we will have to verify three crucial dynamical assumptions: that CP is not violated spontaneously, that the SUSY violating minimum is locally stable, and* *that* F3*, the F term of the independent modulus of the Pentagon model, is of order* $\lambda^{1/4} M_P$.

In the last section, we showed that all of these attractive features were retained in a dynamical SUSY breaking model, obtained from the Pentagon model by a different choice of quantum numbers for the singlet field, G. This interpretation of the model was more attractive to the conservative effective field theorist than the original CSB version, because we did not have to rely on mysterious assumptions about the behavior of R violating terms induced by interactions with the cosmological horizon. Its phenomenological predictions are similar to those of the CSB model, apart from the resolution of the axion problem.

From the DSB point of view, key new dynamical features of the model are the new mechanism for electroweak symmetry breaking, and a different attitude towards metastability of SUSY violating vacua. In previous work on dynamical SUSY breaking, one always insisted that the flat space model have no supersymmetric solutions. Here we instead use the fact that if the c.c. is tuned close to zero at a meta-stable SUSY violating point, then the model contains no SUSic solutions. The low energy effective field theory has a stable SUSic AdS solution, but this is not part of the same quantum theory as the meta-stable dS minimum. The latter has tunneling amplitudes to a Big Crunch space-time, but they are so small as to be irrelevant. A local observer is super-exponentially¹⁷ more likely to be destroyed by thermally activated processes in dS space, than by tunneling to a Crunch. Those thermally activated processes themselves take place on a time scale super-exponentially longer than the current age of the universe. These instabilities are phenomenologically irrelevant. Furthermore, it is possible that in a true theory of quantum gravity, they are not instabilities at all, but merely improbable low entropy fluctuations of a finite system[13]. It may be that even if the idea of CSB does not survive, these new tricks for constructing models of dynamical SUSY breaking will come in handy.

The most pressing issue at this point is to work out distinctive experimental signatures of the Pentagon model. It certainly predicts superpartners in the LHC discovery range, and their decays should be sufficiently different from SSM predictions to distinguish the models at the LHC. In particular, the decays of sleptons into electron plus photon plus missing energy, characteristic of any low scale SUSY breaking model in which the gravitino is the LSP, should be a clear signature. These decays will certainly occur within the detector in the Pentagon model, because the coupling to the Goldstino component of the gravitino is quite strong. Although precise mass predictions are difficult in this model, it is clear that sleptons will be quite light.

¹⁷Super-exponentially small means an exponential of an inverse power of the c.c. in Planck units.

Finding a Higgs boson above the SSM bounds, in addition to gauginos at a few hundreds of GeV, would suggest additional degrees of freedom at the TeV scale, but could not single out the Pentagon as the culprit. The Pentagon model predicts a rich new strongly interacting sector, which may be just out of reach of the LHC. It is extremely important to search for arguments that some of the penta-hadrons are anomalously light, which would lead to really distinctive experimental signatures.

The axion, a very light particle predicted by the Pentagon, might already be ruled out experimentally. If not, it is likely to be one of the more accessible experimental signatures of our model. Finding it will require higher precision low energy experiments, rather than the LHC. It is important to estimate its production rate in accelerators and find its dominant decay modes. We have presented a mechanism for raising the axion mass and lowering its coupling, to make it compatible with both terrestrial experiments and stellar evolution. At the moment, we seem to require a 1% fine tuning of dimensionless couplings to make the model consistent with both these axion properties and the scale of electroweak symmetry breaking.

Finally, the Pentagon model predicts a distinctive new form of dark matter, with relatively large magnetic moments. This may lead to important observational constraints on the model^[12], or perhaps a way of discovering evidence for it^{18} .

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10. Appendix 1 - Anthropic Considerations

Let me begin by stating some opinions about the anthropic principle, which I view as an antidote to the somewhat thoughtless discussion of this topic in the high energy

¹⁸I would like to thank R. Caldwell for pointing out the strong constraints on the dipole moments of dark matter particles.

theory community and beyond it. Ever since the advent of inflationary cosmology, we have been faced with the possibility that theoretical models would describe multiple regions which remain causally disconnected into the indefinite future. If one accepts that possibility then one is immediately led to consider the possibility that some of the parameters in our low energy effective Lagrangian may not be derivable from first principles, but are instead properties of our immediate causal environment. Andrei Linde and I [14] pointed out independently that in this context an anthropic explanation for the puzzling value of the cosmological constant might be all we could ever hope for (Linde had previously discussed the anthropic principle in inflationary cosmology $\text{in}[15]$. Weinberg[16] did the crucial calculation which made these considerations quantitative: *other parameters remaining unchanged*, there can be no galaxies in a universe with positive cosmological constant greater than about 100 times its observed value. Refined versions of this argument[17] claim to find that the observed value is a "typical" value compatible with life. One does not need to believe these more precise arguments to realize that the observed value of the c.c. provides strong, if not compelling evidence for such a picture. An important part of the arguments discussed in this paragraph is the assumption that there is a distribution of values of the c.c., which is smooth and fairly dense near zero.

There are,in my opinion, two flaws in this general circle of ideas, the first philosophical, the second experimental. Part of the often expressed philosophy of these discussions is that "using anthropic arguments in *e.g.* eternal inflationary cosmology, is like using them to understand why we live on earth rather than on the surface of the sun". The fallacy here is that, while we can see the sun and all the other places where we don't live because the conditions are inhospitable, *we cannot, in principle, ever observe the myriad other universes that are required to exist in order to explain the c.c. in anthropic terms.* Therefore, discussions about them will remain forever in the realm of meta-physics, and perhaps one day (but certainly not at present) rigorous mathematics. Their existence is not subject to experimental test. I will refer to models based on such a metaphysical meta-universe as *metaphysical models*.

The second problem is that from a general point of view, and within the context of most explicit proposals for such a meta-verse, one would expect the Lagrangian we observe to be the most general one consistent with our existence, with otherwise random values of the couplings. *This proposal is ruled out experimentally. There are many parameters in the standard model, and its leading irrelevant corrections, which are much more finely tuned than the anthropic principle requires*[18]. *Anthusiasts*, wanting to have their cake and eat it too, propose to resolve this by a combination of anthropic reasoning and traditional symmetry arguments. But explicit models of a multiverse do not seem to give any special status to symmetries. There appear to be vastly more

anthropically allowed "vacua" without symmetries than with them, as one might expect from simple considerations of conditional probability¹⁹.

One proposal which avoids these experimental problems is the "Friendly Landscape" of [19]. These authors propose a toy model for a Landscape in which all but a few parameters have small fluctuations around a central value. Furthermore, the parameters with large fluctuations are precisely those for which we have unresolved fine tuning problems. Models like this would be highly predictive, and one could imagine that the probability distribution was peaked at symmetric points (since it is determined by mathematical, rather than anthropic criteria), avoiding the fine tuning problems for many parameters of a purely anthropic model. It remains to be seen if such friendly landscapes can be derived from a more fundamental theory²⁰, and whether they predict the correct central values.

I have gone into some detail in this discussion, in order to contrast general anthropic arguments with a similar line of argument which I will use in discussing CSB. The fundamental claim of CSB is that there is a countable set of theories of quantum gravity in an asymptotically de Sitter space with cosmological constant λ . The number of quantum states N in each of these theories is finite, and there is a one to one relation between λ and N, with λ going to zero as N becomes infinite. For $\lambda \to 0$, the theory becomes an $\mathcal{N} = 1$ Super Poincare invariant theory with a compact moduli space. The absence of know examples of such theories suggests that the limiting theory may be unique, or that the number of possibilities is much smaller than the number of string vacua that are claimed to exist in the Landscape. Another important property of these models is that for finite N they suffer from inherent quantum ambiguities in what the Hamiltonian and observables are, since the theory does not contain self consistent measuring devices which can measure quantum information with arbitrary precision. So one is led to consider the small λ limit in order to find precise predictions.

The simplest way to analyze the possible values of λ is to eschew the idea of an underlying meta-physical model, and simply declare that our theory of the world contains a free, discrete, dimensionless parameter (N) . One then views the Weinberg bound as simply a gross observational constraint on the value of this parameter: requiring the theory to have galaxies bounds λ by

$$
\lambda < c \ Q^3 \rho_0,
$$

¹⁹Like all such statements, this one should be taken with a grain of salt. Investigations of the Landscape of String Theory - the main proposal for a meta-verse - are at a quite primitive stage, and almost any claim about it might be wrong.

²⁰Much of the String Landscape appears to be unfriendly, and one would have to show that the correct probability distribution favored the friendly regions, if any.

where c is a constant of order one, Q is the amplitude of primordial density fluctuations, and ρ_0 is the dark matter density at the beginning of the matter dominated era. In the Pentagon model, ρ_0 scales like $\lambda^{1/4} \frac{M_P}{\Delta \epsilon}$ $\frac{M_P}{\Lambda_5} n_0$, where n_0 is the density of P-baryon number. The primordial ratio of the P-baryon number to entropy densities (which determines n_0 , as well as Q , are determined by physics well above the TeV scale and below the Planck scale. They are likely to be independent of λ for small λ , and completely fixed by the theory. This leads to a bound

$$
\lambda^{3/4} < Q^3 \frac{\Lambda_5}{M_P} n_0.
$$

The right hand side of this equation can be estimated in the limiting $\lambda = 0$ model. Of course, if we fix n_0 by the phenomenological requirement that the model reproduce the correct temperature at which matter domination begins, then this is just Weinberg's bound. The correct procedure is to assume that n_0 is independent of λ . If we assume that it takes on the right observational value for the observed value of λ , then we find that galaxy formation begins at a lower temperature for smaller values of λ , and a higher temperature for larger values. One might imagine that there are implications for anthropic arguments following from this observation. However, we should note that the temperature of matter radiation equality scales like $\lambda^{1/16}$ so the constraints are probably not very strong. We will see much more dramatic effects of changing λ in the microphysics of QCD.

If the c.c. is an input parameter, governing the number of states in the quantum theory, it is no longer safe to assume that the probability distribution determining it is flat near $\lambda = 0$. For example, a flat distribution in the number of states corresponds to a strong preference for very small λ . The argument that we observe a typical value for the c.c. that allows galaxies to exist is no longer so obvious. A meta-physical model, which introduces an *a priori* preference for large λ [20] could solve this problem. In this model, a meta-verse consists of a dense black hole fluid (whose coarse grained description is a flat FRW model with $p = \rho$) in which a distribution of asymptotically de Sitter bubbles of various sizes (each bubble consists of exactly one de Sitter horizon volume) match on to marginally trapped surfaces in the $p = \rho$ geometry (black holes in the black hole fluid!). The dS bubbles correspond to initial conditions of lower entropy than the generic initial condition which leads to the uniform dense black hole fluid. Thus, initial conditions which lead to a smaller de Sitter bubble, are more probable, and we have a preference for the largest cosmological constant consistent with anthropic bounds. We would need to know the functional form of the probability distribution and to decide on the relevance of refined anthropic considerations[17] in order to decide whether a cosmological constant of the order we observe is "natural".

On the other hand, there might be purely anthropic lower bounds on λ in the Pentagon model. It is extremely interesting that the qualitative low energy physics and cosmology of our model changes drastically as soon as $\sqrt{\lambda^{1/4}M_P} \sim 100~\text{GeV}$ rather than ∼ 3 TeV. In particular, QCD does not become asymptotically free until this rather low scale. If we assume that the value of the unified coupling, is independent of Λ , then the value of $\alpha_S(TeV)$ is the same as it is in the real world. The one loop renormalization group equations for five extra vectorlike fermions between 1 TeV and 100 GeV, then imply that Λ_{QCD} is lowered to a few MeV . There are also changes to the dimensionless electroweak couplings, but the electroweak scale is essentially unchanged, since electroweak breaking occurs for $\lambda = 0$. Note also that the standard model Yukawa couplings remain essentially unchanged so bare quark masses are the same. The up and down quark masses will now be of order the constituent quark mass. Isospin will be strongly broken.

The changes in the electromagnetic and strong interactions,with fixed weak interaction scale, and quark masses, will have a dramatic effect on nuclear and stellar physics. I have not worked out the details, but one can easily imagine that they would lead to an anthropic lower bound on λ in the Pentagon model.

It appears plausible then that the Pentagon model might be the low energy sector of a model of the world with a single dimensionless parameter, the cosmological constant in Planck units. Constraints on the existence of galaxies and more or less normal stars might bound this parameter (from both sides) within a few orders of magnitude of its observed value.

In such a situation, anthropic reasoning is much more attractive than it is in the context of a landscape. Much of the model, including the gauge group, is determined in a way which depends very weakly, or not at all, on λ . We do not have to worry about whether exotic forms of life could exist with different low energy gauge groups. The parameters which do vary, vary in a calculable manner, as a function of a single discrete variable. Very gross anthropic (really *galacto-* or stellar- thropic) considerations bound λ within a few orders of magnitude of its observed value, which is correlated with a variety of other observables.

11. Appendix 2

In the SUSic limit, assuming all Yukawa couplings of order one, S and T vanish and $\mathcal{M}_t \sim \Lambda_5$. $H_u H_d \sim \Lambda_5^2$, follows from the vanishing of F_S . $\tan\beta = 1$ follows from the vanishing of the electroweak D-terms. We can parametrize the Higgs VEVs by

$$
H_u = h_u \begin{pmatrix} 1 \\ 0 \end{pmatrix},
$$

$$
H_d = e^{i\phi} (A, B),
$$

where A, B and h_u are positive. If $A \neq 0$ electromagnetism is spontaneously broken. In this parametrization, $H_u H_d = e^{i\phi} h_u B$. The F term potential constrains ϕ and the product $h_u B$, but there are two components of the physical Higgs field whose masses are of order $g\Lambda_5$, where g is some combination of g_1 and g_2 in the standard model.

Once SUSY is broken, the situation changes. In general, we may expect the minimum to be at a place where all F terms are roughly comparable and of order $\lambda^{1/4} M_P$. The potential depends on S in a complicated way, through the Kahler potential (we still work in the low energy approximation valid when $\lambda^{1/4} M_P < M^2$). Thus, we expect S to get a non-zero VEV. If it does, the Higgs potential has a term

$$
|S|^2(|H_u|^2 + |H_d|^2) = |S|^2(A^2 + B^2 + h_u^2).
$$

If $\langle S \rangle \geq \Lambda_5$, as we may expect when we approach the observed value of the c.c., then the combination of this term and the $|F_S|^2$ term in the potential, gives mass $\sim \Lambda_5$ to all components of the Higgs field. Note also that this potential favors $A = 0$, preserving electromagnetism, just like the electroweak D term potential.

The potential for the Higgs fields h_u and $B = h_d$ will have the form

$$
a^2\Lambda_5^2(h_u^2+h_d^2) + (g_S\Lambda_5^2 - g_\mu h_u h_d)^2.
$$

a is a parameter giving the VEV of S in Λ_5 units. This is minimized at $h_u = h_d = \frac{1}{\sqrt{2\pi}}$ $\frac{1}{2}v$ where

$$
a^2\Lambda_5^2 + \frac{g_\mu}{2}(\frac{g_\mu}{2}v^2 - g_S\Lambda_5^2) = 0.
$$

The value of v^2 is given by the difference of two positive functions of the couplings and can be made smaller than Λ_5 by fine tuning. To get $v = 250 GeV$, with $\Lambda_5 = 3 TeV$ we require a fine tuning of about 1%.

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