QCD Correlators at Large N_C

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This talk explores the spin-1 correlators up to $\mathcal{O}(\alpha_s)$ through a large N_C resonance theory. The phenomenological analyses of this kind must take these corrections into account since they produce a larger impact than the first OPE condensates. It is also necessary to separate low and high energy regimes; fixing the parameters of the lightest multiplets through perturbative QCD arguments is unfair and introduces errors in the determination of the condensates and resonance parameters. This separation of regimes improves our understanding of the Minimal Hadronical Approximation. The study at $\mathcal{O}(\alpha_s)$ already allows discerning between different hadronical models, where the Regge-like mass spectrum shows the best agreement to phenomenology.

1. Introduction

This talk explores the spin-1 QCD correlators¹. The Operator Product Expansion (OPE) has resulted a very powerful and successful instrument to describe Quantum Chromodynamics (QCD) in the domain of deep euclidean momenta [2]; in addition to the purely perturbative QCD contributions (pQCD) one finds nontrivial operators of higher dimension. Likewise, in the large N_C limit -being N_C the number of colours—, QCD suffers large simplifications [3] (this limit of QCD will be denoted as QCD_{∞}). Assuming confinement, QCD_{∞} is dual to a theory with an infinite number of hadronic states where the processes are given by the tree-level topologies. The theory in terms of mesons will be denoted as Resonance Chiral Theory $(R\chi T)$ [4]. It must be built up chiral invariant in order to ensure the right low energy dynamics [5] even at the loop level [6].

The correlators are defined as

$$(q_{\mu}q_{\nu} - q^{2}g_{\mu\nu}) \Pi_{XY}(q^{2}) = i \int dx^{4} e^{iqx} \langle T\{J_{X}(x)_{\mu} J_{Y}(0)_{\nu}^{\dagger}\} \rangle, \quad (1)$$

denoting J_X^μ and J_Y^ν either vector or axial-vector current. We actually analyse the V-A and V+A

combinations ($\Pi_{LR} = \Pi_{VV} - \Pi_{AA}$ and $\Pi_{LL} = \Pi_{VV} + \Pi_{AA}$). Only the sector of light quarks u/d/s will be considered and within the chiral and large N_C limits.

Several large N_C resonance models have studied the V+A correlator at the free-quark level [7, 8,9]. The analysis is taken here with detail up to $\mathcal{O}(\alpha_s)$, profiting the relation between pQCD, the resonance couplings and the mass spectrum [1, 10]. One needs then to introduce a separation scale (here taken as $M_p^2 \sim 2 \text{ GeV}^2$) to split perturbative and non-perturbative dynamics [1].

Some models for the resonance mass spectrum are explored, getting a set of predictions for the $\rho(770)$ and $a_1(1260)$ parameters and the first OPE condensates. The Minimal Hadronical Approximation of $R\chi T$ (MHA) [4,15] arises as a well justified approach to QCD_{∞} that, however, introduces truncation errors that must be properly estimated.

2. pQCD and OPE

At $Q^2 \equiv -q^2 \gg \Lambda_{_{QCD}}^2$ the V-A correlator is well described by the OPE [2]:

$$\Pi_{LR}^{OPE} = \sum_{m=3}^{\infty} \frac{\langle \mathcal{O}_{(2m)}^{LR} \rangle}{Q^{2m}}.$$
 (2)

where the coefficients $\langle \mathcal{O}_{(2m)} \rangle$ are provided by the dimension–(2m) operator in the OPE. The

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V + A combination shows the structure [2]

$$\Pi_{LL}^{OPE} = \Pi_{LL}^{PQCD} + \sum_{m=2}^{\infty} \frac{\langle \mathcal{O}_{(2m)}^{LL} \rangle}{Q^{2m}}, \qquad (3)$$

with the OPE series starting by the pQCD contribution $\Pi_{LL}^{^{pQCD}}$.

3. Resonance theory at large N_C

At large N_C , the correlators are given by an infinite exchange of narrow-width resonances,

$$\Pi_{LR}^{N_C \to \infty} = -\frac{F^2}{Q^2} + \sum_{j=1}^{\infty} \frac{[-\pi_j] F_j^2}{M_j^2 + Q^2},
\Pi_{LL}^{N_C \to \infty} = \frac{F^2}{Q^2} + \sum_{j=1}^{\infty} \frac{F_j^2}{M_j^2 + Q^2},$$
(4)

where M_j and F_j are the mass and coupling constant of the j-th spin-1 multiplet at LO in $1/N_C$, and F is the pion decay constant. The factor π_j denotes the parity of the corresponding multiplet. We will consider an alternating parity spectrum $(\pi_j = (-1)^j)$ and $M_1 \leq M_2 \leq ...$

3.1. Separation of perturbative and nonperturbative QCD contributions

First of all, one has to impose a separation scale, taken in this work as $M_p^2 \sim 2 \text{ GeV}^2$, between the perturbative and non-perturbative regimes:

$$\Pi^{^{N_{C}\rightarrow\infty}}=\Delta\Pi_{^{pert.}}^{^{N_{C}\rightarrow\infty}}+\Delta\Pi_{^{n_{O}-p_{.}}}^{^{N_{C}\rightarrow\infty}}.$$
 (5

where the perturbative sub-series $\Delta\Pi_{pert.}^{N_C \to \infty}$ includes the resonances with $M_j^2 > M_p^2$, and the non-perturbative part $\Delta\Pi_{non-p.}^{N_C \to \infty}$ contains those with $M_i^2 \leq M_p^2$.

It is also necessary to make some assumption about the mass spectrum at high energies. A smooth M_j^2 dependence on j will be assumed for $M_j^2 > M_p^2$. If the couplings F_j also follow a smooth behaviour at high energies then, in order to recover $\Pi_{LL}^{p_{QCD}}$ at $Q^2 \gg \Lambda_{QCD}^2$, they must behave like [1,10]

$$F_j^2 = \delta M_j^2 \cdot \left[\frac{1}{\pi} \text{Im} \Pi_{LL} (M_j^2)^{p_{QCD}} + \mathcal{O} \left(\frac{1}{M_j^2} \right) \right], (6)$$

being $\delta M_j^2 \equiv M_j^2 - M_{j-1}^2$. The $\mathcal{O}\left(\frac{1}{M_j^2}\right)$ corrections were neglected in this work. Hence, at $Q^2 \gg M_p^2$, the perturbative sub-series results

$$\Delta\Pi_{pert.}^{N_{C}\to\infty} = \Pi^{pQCD} + \sum_{r=1}^{\infty} \frac{\Delta \langle \mathcal{O}_{(2m)} \rangle^{pert.}}{Q^{2m}}, \quad (7)$$

with $\Pi_{LR}^{pQCD}=0$. Remark that the $\Delta \langle \mathcal{O}_{(2m)}^{LL} \rangle^{pert.}$ cannot be recovered in general by trivially expanding Eq. (4) in powers of $\frac{M_R^2}{O^2}$.

On the other hand, the non-perturbative part of the series produces just $1/Q^{2m}$ power terms at $Q^2\gg M_p^2$, yielding the corresponding contributions to the condensates, e.g. $\Delta \langle \, \mathcal{O}_{_{(2m)}}^{^{LL}} \, \rangle^{^{non-p.}} = (-1)^{m-1} \, \sum_{j=1}^p F_j^2 \, (M_j^2)^{m-1} \, .$

Matching the sum $\langle \mathcal{O}_{(2m)} \rangle = \Delta \langle \mathcal{O}_{(2m)} \rangle^{pert.} + \Delta \langle \mathcal{O}_{(2m)} \rangle^{non-p.}$ to the OPE one gets the familiar

$$\langle \mathcal{O}_{(2)}^{LL} \rangle = \langle \mathcal{O}_{(2)}^{LR} \rangle = \langle \mathcal{O}_{(4)}^{LR} \rangle = 0.$$
 (8)

4. Mass spectrum models: phenomenology

Several models for the mass spectrum are studied in Ref. [1] with detail. The multiplets with $M_n^2 \gtrsim 2~{\rm GeV}^2~(n \ge 3)$ are included in $\Delta\Pi_{pert.}^{N_C \to \infty}$. An asymptotic M_n^2 dependence is taken for them and their masses are set such that $M_3^2 = M_{\rho'}^2 \simeq (1.45~{\rm GeV})^2$ and $M_4^2 = M_{a'_1}^2 \simeq (1.64~{\rm GeV})^2$ [11].

Given the asymptotic mass spectrum the couplings are fixed through Eq. (6) and the combination $\left(\Delta\Pi_{pert.}^{N_C\to\infty}-\Pi^{pQCD}\right)$ shows the structure of the st

ture
$$\sum_{m=1}^{\infty} \frac{\Delta \langle \mathcal{O}_{(2m)} \rangle^{pert.}}{Q^{2m}}$$
. The corresponding

 $\Delta \langle \mathcal{O}_{(2m)} \rangle^{pert.}$ are recovered through a numerical analysis for the range $Q^2 = 2-6 \text{ GeV}^2$ [1]. pQCD is only taken up to $\mathcal{O}(\alpha_s)$ so $\mathcal{O}(\alpha_s^2)$ uncertainties are expected in the derivation of the condensates.

The pion, $\rho(770)$ and $a_1(1260)$ are included into $\Delta\Pi_{non-p}^{N_C\to\infty}$. We take F=92.4 MeV and $M_{\rho}=0.77$ GeV as inputs. The remaining parameters, F_{ρ} , F_{a_1} and M_{a_1} , are recovered through the matching to OPE in Eq. (8). This fixes completely the correlators, producing a prediction for $\langle \mathcal{O}_{(4)}^{LL} \rangle$ and $\langle \mathcal{O}_{(6)}^{LR} \rangle$. The results for the two most

	$1+1 \ QCD_{\infty}$	5D-spectrum
$F_{\rho} \; (\mathrm{MeV})$	139 ± 3	174^{+5}_{-7}
$F_{a_1} (\mathrm{MeV})$	138 ± 3	169^{+6}_{-8}
$M_{a_1} \; ({ m MeV})$	1180^{+17}_{-19}	1029_{-13}^{+19}
$\langle \mathcal{O}_{(4)}^{^{LL}} \rangle (10^{-3} \text{ GeV}^4)$	-30^{+22}_{-30}	100^{+70}_{-60}
$\langle \mathcal{O}_{\scriptscriptstyle{(6)}}^{^{LR}} \rangle (10^{-3} \text{ GeV}^6)$	$-3.7^{+0.8}_{-0.3}$	$0.3^{+2.2}_{-1.5}$

Table 1

Predictions for the two first resonance multiplets within each mass spectrum model.

relevant models are shown in Table (1) and the corresponding Π_{LR} and $\mathcal{A}_{LL} = -Q^2 \frac{d\Pi_{LL}}{dQ^2}$ are compared in Fig. (1) to OPE [14].

4.1. QCD_{∞} in 1+1 dimensions

In several models, like QCD_{∞} in 1+1 dimensions and Regge theory, the high energy spectrum follows the dependence $M_n^2 = \Lambda^2 + n \delta M^2$ [3,7,8,10]. The analysis yields ρ and a_1 couplings of the same order as the asymptotic values $F_{\rho} \sim F_{a_1} \sim F_{j \geq 3}$. Both resonance couplings and the a_1 mass are in agreement with former analyses [4,11,12,13].

4.2. Five-dimensional spectrum

Another available scenario is provided by models in five dimensions [9], which produce a four dimensional effective spectrum with dependence $M_n = \Lambda + n \, \delta M$. One finds an acceptable value for M_{a_1} although F_{ρ} and F_{a_1} go high above the usual determinations [4,11,12,13]. Other sumrule analyses show that this spectrum produces a clearly distinctive pattern which differs from the Regge-like model and from the one found in the phenomenology [1].

4.3. Minimal Hadronical Approximation

The perturbative contributions to the condensates are found in general of the order of magnitude of the usual hadronical parameters – the pion decay constant and the light resonance masses–. For instance, for the Regge-like spec-

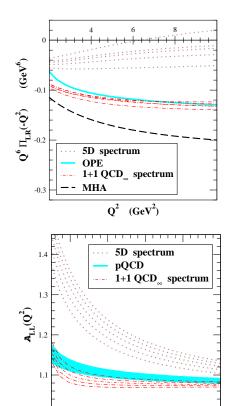


Figure 1. V-A correlator and V+A Adler function in $R\chi T$ up to $\mathcal{O}(\alpha_s)$ and within MHA $(M_{a_1}=M_\rho/\sqrt{2}, F_\rho/\sqrt{2}=F_{a_1}=F$ [12]). They are compared to the results from the OPE and pQCD at that order [14].

trum one gets

$$\Delta \langle \mathcal{O}_{(2m)}^{LL} \rangle^{pert} \simeq \frac{(-1)^m}{m} \frac{N_C}{12\pi^2} (M_{\rho'}^2)^m ,
\Delta \langle \mathcal{O}_{(2m)}^{LR} \rangle^{pert} \simeq (-1)^{m-1} \frac{N_C}{24\pi^2} \delta M^2 (M_{\rho'}^2)^{m-1} ,$$
(9)

up to corrections suppressed by $\frac{\delta M^2}{M_{\rho'}^2}$. Actually, from Eq. (9) one finds for the Regge spectrum that at euclidean momenta the perturbative subseries $\Delta \Pi_{LR,\,pert.}^{n_C \to \infty}$ is approximately equivalent to

the contribution from an effective ρ' resonance with coupling $\tilde{F}_{\rho'}^2 \equiv \frac{N_C}{24\pi^2} \delta M^2$, resulting of the order of F^2 . This exemplifies how some spectrum patterns provide a description equivalent to that from a theory with a finite number of multiplets.

In the MHA [15], $\Delta\Pi_{pert.}^{N_C\to\infty}$ is taken as zero, introducing a truncation error $\mathcal{O}\left(\delta M^2\left(M_{\rho'}^2\right)^{m-1}\right)$ in the value of the condensates. The short distance matching for Π_{LR} under MHA is then perfectly justified as long as one is aware of the inherent uncertainties:

$$\begin{array}{ll} F^2 - F_{\rho}^2 + F_{a_1}^2 &= \mathcal{O}\left(\frac{N_C}{24\pi^2}\delta M^2\right), \\ F_{a_1}^2 \, M_{a_1}^2 - F_{\rho}^2 \, M_{\rho}^2 &= \mathcal{O}\left(\frac{N_C}{24\pi^2}\delta M^2 \, M_{\rho'}^2\right). \end{array} \tag{10}$$

The resonance theory with a finite number of multiplets allows to build a proper quantum field theory for resonances with a well defined $1/N_C$ perturbative expansion and renormalisation scheme, being renormalisable order by order in $1/N_C$ [6]. The smooth variation of the resonance parameters from MHA to MHA+ ρ' [16] seems to hint the close relation between the truncated $R\chi T$ and the full QCD_{∞} .

5. Conclusions

It was shown how $R\chi T$ is able to recover the OPE up to $\mathcal{O}(\alpha_s)$. The analysis up to this order is essential for any V+A phenomenological analysis since the pQCD corrections are more important than those from the first OPE condensates.

In order to handle the infinite tower of hadronic states one needs to assume a structure M_j^2 for the spectrum at high energies and that the F_j^2 depend smoothly on j. The matching to Π_{LL}^{pQCD} in the deep euclidean imposes then strong constraints on the asymptotic structure of the couplings. some of the current models are tested. 1+1 QCD_{∞} $(M_n^2 \sim n)$ and the 5D theories $(M_n^2 \sim n^2)$ show the closest agreement to phenomenology, providing the first one the best description. Nevertheless, estimates on subleading corrections are still necessary.

The scale $M_p^2 \sim 2 \text{ GeV}^2$ splits the perturbative and the non-perturbative regimes, so the pion, ρ and a_1 are included in the non-perturbative sub-series. This splitting allows understanding how $R\chi T$ under MHA connects the full QCD_{∞} . Dropping the contribution from $\Delta\Pi_{pert.}^{NC\to\infty}$ is a well defined approximation as long as the truncation uncertainties are properly considered.

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