

Lagrangian perturbations at order $1/m_Q$ and the non-forward amplitude in Heavy Quark Effective Theory

F. Jugeau, A. Le Yaouanc, L. Oliver and J.-C. Raynal

*Laboratoire de Physique Théorique*¹

Université de Paris XI, Bâtiment 210, 91405 Orsay Cedex, France

Abstract

We pursue the program of the study of the non-forward amplitude in HQET. We obtain new sum rules involving the elastic subleading form factors $\chi_i(w)$ ($i = 1, 2, 3$) at order $1/m_Q$ that originate from the \mathcal{L}_{kin} and \mathcal{L}_{mag} perturbations of the Lagrangian. To obtain these sum rules we use two methods. On the one hand we start simply from the definition of these subleading form factors and, on the other hand, we use the Operator Product Expansion. To the sum rules contribute only the same intermediate states $(j^P, J^P) = (\frac{1}{2}^-, 1^-), (\frac{3}{2}^-, 1^-)$ that enter in the $1/m_Q^2$ corrections of the axial form factor $h_{A_1}(w)$ at zero recoil. This allows to obtain a lower bound on $-\delta_{1/m^2}^{(A_1)}$ in terms of the $\chi_i(w)$ and the shape of the elastic IW function $\xi(w)$. We find also lower bounds on the $1/m_Q^2$ correction to the form factors $h_+(w)$ and $h_1(w)$ at zero recoil. An important theoretical implication is that $\chi'_1(1)$, $\chi_2(1)$ and $\chi'_3(1)$ ($\chi_1(1) = \chi_3(1) = 0$ from Luke theorem) must vanish when the slope and the curvature attain their lowest values $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$. We discuss possible implications on the precise determination of $|V_{cb}|$.

LPT Orsay 05-43

July 2005

e-mails : frederic.jugeau@th.u-psud.fr, leyaouan@th.u-psud.fr, oliver@th.u-psud.fr

¹Unité Mixte de Recherche UMR 8627 - CNRS

1 Introduction.

The study of the non-forward amplitude, proposed first by Uraltsev [1]

$$T_{fi}(q) = i \int d^4x e^{-iq \cdot x} \langle B(v_f) | T[J_f(0)J_i(x)] | B(v_i) \rangle \quad (1)$$

where v_i is in general different from v_f and

$$J_f(0) = \bar{b}(0)\Gamma_f c(0) \quad J_i(x) = \bar{c}(x)\Gamma_i b(x) \quad (2)$$

(Γ_i, Γ_f are arbitrary Dirac matrices) has been very fruitful in Heavy Quark Effective Theory (HQET).

In the heavy quark limit, sum rules (SR) that generalize Bjorken [2] and Uraltsev [1] SR have been obtained within the Operator Product Expansion (OPE) that yield to bounds for all derivatives of the elastic Isgur-Wise (IW) function $\xi(w)$ [3] [4], in particular for the curvature [5]. The radiative corrections to these SR and bounds in the framework of HQET have been computed by Dorsten [6].

In a recent paper we have extended our formalism to the subleading order in $1/m_Q$ [7]. We did obtain the interesting relations, *valid for all w* :

$$\bar{\Lambda}\xi(w) = 2(w+1) \sum_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) + 2 \sum_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) \quad (3)$$

$$\xi_3(w) = (w+1) \sum_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) - 2 \sum_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) . \quad (4)$$

These remarkably simple relations were the basic results of ref. [7]. Both subleading quantities $\bar{\Lambda}\xi(w)$ and $\xi_3(w)$ can be expressed in terms of the leading quantities, namely IW functions $\tau_j^{(n)}(w)$ and level spacings $\Delta E_j^{(n)}$ ($j = \frac{1}{2}, \frac{3}{2}$). These equations give information on the $1/m_Q$ *Current* perturbations to the matrix elements. In the present paper we will deal with the *Lagrangian* perturbations.

The paper is organized as follows. Section 2 gives a simple derivation of the relevant SR, starting from the definition of the different subleading Lagrangian form factors. In Section 3 we summarize the basic results and comment on general theoretical features of the SR. In Section 4 we recall the contribution of 1^- intermediate states to the OPE sum rule at zero recoil at order $1/m_Q^2$ for the form factor $B \rightarrow D^*$. In Section 5, using Schwarz inequality, we obtain a bound on the correction δ_{1/m^2} to $F_{B \rightarrow D^*}(1)$ in terms of the Lagrangian elastic subleading form factors and the elastic

Isgur-Wise function. In Section 6 we also obtain lower bounds on the $1/m_Q^2$ corrections to the form factors $h_+(w)$ and $h_1(w)$ at $w = 1$. In Section 7 we summarize some theoretical features of the obtained bounds. In Section 8 we demonstrate that $\chi'_1(1)$, $\chi_2(1)$ and $\chi'_3(1)$ must vanish in the limit in which the slope ρ^2 and curvature σ^2 of the elastic IW function $\xi(w)$ attain their lowest values. In Section 9 we discuss phenomenological implications of our results for the exclusive determination of $|V_{cb}|$ and in Section 10 we conclude. In Appendix A we derive the same SR as in Section 2 using the Operator Product Expansion (OPE), following the same method developed for the derivation of the Current SR in [7]. In Appendix B we make a numerical analysis of the obtained bounds and in Appendix C we discuss the radiative corrections.

2 New sum rules on Lagrangian perturbations.

In this section, we will formulate new SR for the *Lagrangian* perturbations, parallel to the ones on the *Current* perturbations (3)-(4).

Instead of using the OPE, we will here simply use the definition of the subleading elastic $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ functions $\chi_i(w)$ ($i = 1, 2, 3$) [8]

$$\begin{aligned} & \langle D(v') | i \int dx T [J^{cb}(0), \mathcal{L}_v^{(b)}(x)] | B(v) \rangle = \\ & \frac{1}{2m_b} \left\{ -2\chi_1(w) Tr [\bar{D}(v') \Gamma B(v)] + \frac{1}{2} Tr [A_{\alpha\beta}(v, v') \bar{D}(v') \Gamma P_+ i \sigma^{\alpha\beta} B(v)] \right\} \end{aligned} \quad (5)$$

$$\begin{aligned} & \langle D(v') | i \int dx T [J^{cb}(0), \mathcal{L}_{v'}^{(c)}(x)] | B(v) \rangle = \\ & \frac{1}{2m_c} \left\{ -2\chi_1(w) Tr [\bar{D}(v') \bar{\Gamma} B(v)] - \frac{1}{2} Tr [\bar{A}_{\alpha\beta}(v', v) \bar{D}(v') i \sigma^{\alpha\beta} P_+ \bar{\Gamma} B(v)] \right\} \end{aligned} \quad (6)$$

with

$$\begin{aligned} A_{\alpha\beta}(v, v') &= -2\chi_2(w) (v'_\alpha \gamma_\beta - v'_\beta \gamma_\alpha) + 4\chi_3(w) i \sigma_{\alpha\beta} \\ \bar{A}_{\alpha\beta}(v', v) &= -2\chi_2(w) (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) - 4\chi_3(w) i \sigma_{\alpha\beta} \end{aligned} \quad (7)$$

where $\bar{A} = \gamma^0 A^+ \gamma^0$ denotes the Dirac conjugate matrix, the current $J^{cb}(0)$ denotes

$$J^{cb} = \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} \quad (8)$$

where Γ is any Dirac matrix, and $\mathcal{L}_v^{(Q)}(x)$ is given by

$$\mathcal{L}_v^{(Q)} = \frac{1}{2m_Q} [O_{kin,v}^{(Q)} + O_{mag,v}^{(Q)}] \quad (9)$$

with

$$O_{kin,v}^{(Q)} = \bar{h}_v^{(Q)} (iD)^2 h_v^{(Q)} \quad O_{mag,v}^{(Q)} = \frac{g_s}{2} \bar{h}_v^{(Q)} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(Q)}. \quad (10)$$

In relations (5)-(7), the $\chi_i(w)$ ($i = 1, 2, 3$) have dimensions of mass, and correspond to the definition given by Luke [9].

We will now insert intermediate states in the T -products (5). We can separately consider $\mathcal{L}_{kin}^{(b)}$ or $\mathcal{L}_{mag}^{(b)}$. The possible Z -diagrams involving heavy quarks contributing to the T -products are suppressed by the heavy quark mass since they are $b\bar{c}$ intermediate states.

Conveniently choosing the initial and final states, we find the following results (we use the normalization of the states as made explicit for example in formula (5.6) of ref. [10]) :

(1) With $\mathcal{L}_{kin,v}^{(b)}$, pseudoscalar initial state $B(v) = P_+(-\gamma_5)$ and pseudoscalar final state $\bar{D}(v') = \gamma_5 P'_+$, one finds, for any current (8)

$$\begin{aligned} & -2\chi_1(w) Tr [\bar{D}(v') \Gamma B(v)] \\ &= -Tr [\bar{D}(v') \Gamma B(v)] \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{(n)}(v) | O_{kin,v}^{(b)}(0) | B(v) \rangle}{\sqrt{4m_{B^{(n)}} m_B}} \end{aligned} \quad (11)$$

where

$$\langle D(v') | \bar{h}_{v'}^c(0) \Gamma h_v^b(0) | B^{(n)}(v) \rangle = -\xi^{(n)}(w) Tr [\bar{D}(v') \Gamma B(v)] \quad (12)$$

that yields

$$2\chi_1(w) = \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{(n)}(v) | O_{kin,v}^{(b)}(0) | B(v) \rangle}{\sqrt{4m_{B^{(n)}} m_B}}. \quad (13)$$

Likewise, we obtain, in the case of a vector initial state $B^*(v, \varepsilon) = P_+ \not{\varepsilon}$ and a vector final state $\bar{D}^*(v', \varepsilon') = \not{\varepsilon}' P'_+$

$$2\chi_1(w) = \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{*(n)}(v, \varepsilon) | O_{kin,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B^{*(n)}} m_{B^*}}} \quad (14)$$

since \mathcal{L}_{kin} is spin-independent. In the preceding expressions the energy denominators are

$$\Delta E_{1/2}^{(n)} = E_{1/2}^{(n)} - E_{1/2}^{(0)} \quad (n \neq 0). \quad (15)$$

(2) Consider $\mathcal{L}_{mag,v}^{(b)}$, pseudoscalar initial state $B(v) = P_+(-\gamma_5)$ and pseudoscalar final state $\overline{D}(v') = \gamma_5 P'_+$. Because of parity conservation by the strong interactions, the intermediate states $B^{(n)}$ must have the same parity than the initial state B . Moreover, $\mathcal{L}_{mag,v}^{(b)}$ being a scalar and producing transitions at zero recoil, the spin of B and $B^{(n)}$ must be the same. Therefore, only pseudoscalar intermediate states $B^{(n)}(0^-)$ can contribute, only states with $j = \frac{1}{2}^-$. One finds, for any current (8)

$$\begin{aligned} & 4(w-1)\chi_2(w)Tr[\overline{D}(v')\Gamma B(v)] - 12\chi_3(w)Tr[\overline{D}(v')\Gamma B(v)] \\ &= -Tr[\overline{D}(v')\Gamma B(v)] \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{(n)}(v) | O_{mag,v}^{(b)}(0) | B(v) \rangle}{\sqrt{4m_{B^{(n)}}m_B}} \end{aligned} \quad (16)$$

that gives

$$-4(w-1)\chi_2(w) + 12\chi_3(w) = \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{(n)}(v) | O_{mag,v}^{(b)}(0) | B(v) \rangle}{\sqrt{4m_{B^{(n)}}m_B}}. \quad (17)$$

It is remarkable that this linear combination depends only on $\frac{1}{2}^-$ intermediate states. We will comment on this feature below.

(3) Consider $\mathcal{L}_{mag,v}^{(b)}$ and a vector initial state $B^*(v, \varepsilon) = P_+\not{\varepsilon}$ and pseudoscalar final state $\overline{D}(v') = \gamma_5 P'_+$. Now we will have vector 1^- intermediate states, either $B^{*(n)}(\frac{1}{2}^-, 1^-)$ or $B^{*(n)}(\frac{3}{2}^-, 1^-)$. For the latter, we have to compute the current matrix element

$$\langle D(v') | J^{cb}(0) | B^{*(n)}(\frac{3}{2}^-, 1^-)(v, \varepsilon) \rangle = \tau_{3/2}^{(2)(n)}(w) Tr[\overline{D}(v')\Gamma F_v^\sigma v'_\sigma] \quad (18)$$

where the $(\frac{3}{2}^-, 1^-)$ operator is given by

$$F_v^\sigma = \sqrt{\frac{3}{2}} P_+ \varepsilon_\nu \left[g^{\sigma\nu} - \frac{1}{3} \gamma^\nu (\gamma^\sigma + v^\sigma) \right] \quad (19)$$

obtained from the $(\frac{3}{2}^+, 1^+)$ operator defined by Leibovich et al. (formula (2.5) of [10]), multiplying by $(-\gamma_5)$ on the right [11]. The Isgur-Wise functions $\tau_{3/2}^{(2)(n)}(w)$ correspond to $\frac{1}{2}^- \rightarrow \frac{3}{2}^-$ transitions, the superindex (2) meaning the orbital angular momentum [3] [4] [11]. As noticed by Leibovich et al., on general grounds the IW functions $\tau_{3/2}^{(2)(n)}(w)$ do not vanish at zero recoil.

One finds, for any current (8),

$$\begin{aligned} & \langle D(v') | J^{cb}(0) | B^{(n)}(\frac{3}{2}^-, 1^-)(v, \varepsilon) \rangle = \sqrt{\frac{3}{2}} \tau_{3/2}^{(2)(n)}(w) (\varepsilon \cdot v') Tr[\overline{D}(v')\Gamma P_+] \\ & - \frac{1}{\sqrt{6}} (w-1) \tau_{3/2}^{(2)(n)}(w) Tr[\overline{D}(v')\Gamma P_+\not{\varepsilon}] \end{aligned} \quad (20)$$

and finally

$$\begin{aligned}
& -4\chi_2(w)(\varepsilon \cdot v')Tr [\overline{D}(v')\Gamma P_+] + 4\chi_3(w)Tr [\overline{D}(v')\Gamma B^*(v, \varepsilon)] \\
& = -Tr [\overline{D}(v')\Gamma B^*(v, \varepsilon)] \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B^{*(n)}}m_{B^*}}} \\
& + \left\{ \sqrt{\frac{3}{2}}(\varepsilon \cdot v')Tr [\overline{D}(v')\Gamma P_+] - \frac{1}{\sqrt{6}}(w-1)Tr [\overline{D}(v')\Gamma B^*(v, \varepsilon)] \right\} \\
& \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle B_{3/2}^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B_{3/2}^{*(n)}}m_{B^*}}}. \tag{21}
\end{aligned}$$

The energy denominators $\Delta E_{1/2}^{(n)}$ and $\Delta E_{3/2}^{(n)}$

$$\begin{aligned}
\Delta E_{1/2}^{(n)} &= E_{1/2}^{(n)} - E_{1/2}^{(0)} \quad (n \neq 0) \\
\Delta E_{3/2}^{(n)} &= E_{3/2}^{(n)} - E_{1/2}^{(0)} \quad (n \geq 0). \tag{22}
\end{aligned}$$

To obtain other linearly independent relations, let us specify the final state and the current. We make explicit the pseudoscalar $\overline{D}(v') = \gamma_5 P'_+$ and take $\Gamma = \gamma_\mu \gamma_5$. This gives, from the preceding expression,

$$\begin{aligned}
& -4\chi_2(w)(\varepsilon \cdot v')(v'_\mu - v_\mu) + 4\chi_3(w)[(w-1)\varepsilon_\mu + (\varepsilon \cdot v')v_\mu] \\
& = -[(w-1)\varepsilon_\mu + (\varepsilon \cdot v')v_\mu] \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B^{*(n)}}m_{B^*}}} \\
& + \left\{ \sqrt{\frac{3}{2}}(\varepsilon \cdot v')(v'_\mu - v_\mu) - \frac{1}{\sqrt{6}}(w-1)[(w-1)\varepsilon_\mu + (\varepsilon \cdot v')v_\mu] \right\} \\
& \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle B_{3/2}^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B_{3/2}^{*(n)}}m_{B^*}}}. \tag{23}
\end{aligned}$$

Since the two four vectors $(v'_\mu - v_\mu)$ and $[(w-1)\varepsilon_\mu + (\varepsilon \cdot v')v_\mu]$ can be chosen to be independent, one obtains independent sum rules for $\chi_2(w)$ and $\chi_3(w)$, namely

$$-2\chi_2(w) = \frac{1}{2} \sqrt{\frac{3}{2}} \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle B_{3/2}^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B_{3/2}^{*(n)}}m_{B^*}}} \tag{24}$$

$$\begin{aligned}
4\chi_3(w) &= - \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle B^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B^{*(n)}}m_{B^*}}} \\
& - \frac{w-1}{\sqrt{6}} \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle B_{3/2}^{*(n)}(v, \varepsilon) | O_{mag,v}^{(b)}(0) | B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B_{3/2}^{*(n)}}m_{B^*}}} \tag{25}
\end{aligned}$$

As a final remark on this Section on the derivation of the sum rules, let us point out that if, instead of (5) that involves $\mathcal{L}^{(b)}$ we start from (6) with $\mathcal{L}^{(c)}$, we obtain the same SR as above, with the replacement $b \rightarrow c$ in the operators and in the states. The reason is that the IW functions and energy denominators are flavor-independent in the heavy quark limit.

3 Summary and comments on the Lagrangian sum rules.

To summarize, making explicit the c flavor, we have obtained the sum rules

$$\begin{aligned}\chi_1(w) &= \frac{1}{2} \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle D^{(n)}(v) | O_{kin,v}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}}m_D}} \\ &= \frac{1}{2} \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle D^{*(n)}(v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}}m_{D^*}}}\end{aligned}\quad (26)$$

$$\chi_2(w) = -\frac{3}{4\sqrt{6}} \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle D_{3/2}^{*(n)}(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}}m_{D^*}}}\quad (27)$$

$$\begin{aligned}\chi_3(w) &= -\frac{1}{4} \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle D^{*(n)}(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}}m_{D^*}}} \\ &\quad - \frac{w-1}{4\sqrt{6}} \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle D_{3/2}^{*(n)}(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}}m_{D^*}}}\end{aligned}\quad (28)$$

There are a number of striking features in relations (26)-(28).

(i) One should notice that *elastic subleading form factors of the Lagrangian type* are given in terms of *leading IW functions*, namely $\xi^{(n)}(w)$ and $\tau_{3/2}^{(2)(n)}(w)$, and *sub-leading form factors at zero recoil*.

(ii) $\chi_1(w)$ is given in terms of matrix elements of \mathcal{L}_{kin} , as expected from the definitions (5)-(6) and involve transitions $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$.

(iii) The *elastic subleading magnetic form factors* $\chi_2(w)$ and $\chi_3(w)$ involve $D^*(1^-) \rightarrow D^{*(n)}(1^-)$ transitions $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ and $\frac{1}{2}^- \rightarrow \frac{3}{2}^-$.

(iv) $\chi_1(w)$ and $\chi_3(w)$ satisfy, as they should, Luke theorem [9],

$$\chi_1(1) = \chi_3(1) = 0 \quad (29)$$

because the $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ IW functions at zero recoil satisfy

$$\xi^{(n)}(1) = \delta_{n,0} \quad (30)$$

(v) There is a linear combination of $\chi_2(w)$ and $\chi_3(w)$ that gets only contributions from $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ transitions, namely

$$-4(w-1)\chi_2(w) + 12\chi_3(w) = -3 \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle D^{*(n)}(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}}m_{D^*}}} \quad (31)$$

where the factor -3 is in consistency with (17), shifting from vector to pseudoscalar mesons.

This latter relation and (26) imply that the combination

$$L_1(w) = 2\chi_1(w) - 4(w-1)\chi_2(w) + 12\chi_3(w) \quad (32)$$

gets only contributions from $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ transitions. We will give an alternative demonstration of this feature using the OPE in Appendix A.

4 The OPE sum rule for $h_{A_1}(1)$.

It is well-known that the determination of $|V_{cb}|$ from the $\bar{B} \rightarrow D^* \ell \nu$ differential rate at zero recoil depends on the value of $h_{A_1}(1)$.

The interesting point is that precisely the subleading matrix elements of O_{kin} and O_{mag} at zero recoil, that enter in the SR (26)-(28), are related to the quantity $|h_{A_1}(1)|$, as we will see now.

The following SR follows from the OPE [12] [10],

$$\begin{aligned} |h_{A_1}(1)|^2 + \sum_n \frac{|\langle D^{*(n)}\left(\frac{1}{2}^-, \frac{3}{2}^-\right)(v, \varepsilon) | \vec{A} | B(v) \rangle|^2}{4m_{D^{*(n)}}m_B} \\ = \eta_A^2 - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \end{aligned} \quad (33)$$

where $D^{*(n)}$ are 1^- excited states, and

$$\begin{aligned} \mu_\pi^2 &= \frac{1}{2m_B} \langle B(v) | \bar{h}_v^{(b)} (iD)^2 h_v^{(b)} | B(v) \rangle \\ \mu_G^2 &= \frac{1}{2m_B} \langle B(v) | \bar{h}_v^{(b)} \frac{g_s}{2} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(b)} | B(v) \rangle \\ &= -\frac{3}{2m_B} \langle B^*(v, \varepsilon) | \bar{h}_v^{(b)} \frac{g_s}{2} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(b)} | B^*(v, \varepsilon) \rangle \end{aligned} \quad (34)$$

In relation (33) one assumes the states at rest $v = (1, \mathbf{0})$ and the axial current is space-like, orthogonal to v . The relation of (34) with the other common notation [8] [10] is $\mu_\pi^2 = -\lambda_1$ and $\mu_G^2 = 3\lambda_2$.

In the l.h.s. of relation (33),

$$h_{A_1}(1) = \eta_{A_1} + \delta_{1/m^2}^{(A_1)} \quad (35)$$

($\eta_{A_1} = 1 +$ radiative corrections) because there are no first order $1/m_Q$ corrections due to Luke theorem [9]. The sum over the squared matrix elements of $B \rightarrow D^{*(n)}(1^-)$ transitions contains two types of possible contributions, corresponding to $D^{*(n)}\left(\frac{1}{2}^-, 1^-\right)$ ($n \neq 0$), and $D^{*(n)}\left(\frac{3}{2}^-, 1^-\right)$ ($n \geq 0$). The r.h.s. of (33) exhibits the OPE at the desired order. From the decomposition between radiative corrections and $1/m_Q^2$ corrections (35) one gets, from (33), neglecting higher order terms,

$$\begin{aligned} -\delta_{1/m^2}^{(A_1)} &= \frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \\ &+ \frac{1}{2} \sum_n \frac{|\langle D^{*(n)}\left(\frac{1}{2}^-, \frac{3}{2}^-\right)(v, \varepsilon) | \vec{A} | B(v) \rangle|^2}{4m_{D^{*(n)}} m_B}. \end{aligned} \quad (36)$$

The correction $\delta_{1/m^2}^{(A_1)}$ is therefore negative, both terms being of the same sign.

The matrix elements $\langle D^{*(n)}\left(\frac{1}{2}^-, \frac{3}{2}^-\right)(v, \varepsilon) | \vec{A} | B \rangle$ have been expressed in terms of the matrix elements $\langle D^{*(n)}\left(\frac{1}{2}^-\right)(v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle$ and $\langle D^{*(n)}\left(\frac{1}{2}^-, \frac{3}{2}^-\right)(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle$ by Leibovich et al. [10], within the same normalization convention used in the preceding sections,

$$\begin{aligned} &\frac{\langle D^{*(n)}\left(\frac{1}{2}^-\right)(v, \varepsilon) | \vec{A} | B(v) \rangle}{\sqrt{4m_{D^{*(n)}} m_B}} \\ &= -\frac{\vec{\varepsilon}}{\Delta E_{1/2}^{(n)}} \left[\left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) \frac{\langle D^{*(n)}\left(\frac{1}{2}^-\right)(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right. \\ &\quad \left. + \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \frac{\langle D^{*(n)}\left(\frac{1}{2}^-\right)(v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right] \end{aligned} \quad (37)$$

$$\begin{aligned} &\frac{\langle D^{*(n)}\left(\frac{3}{2}^-\right)(v, \varepsilon) | \vec{A} | B(v) \rangle}{\sqrt{4m_{D^{*(n)}} m_B}} \\ &= -\frac{\vec{\varepsilon}}{\Delta E_{3/2}^{(n)}} \frac{1}{2m_c} \frac{\langle D^{*(n)}\left(\frac{3}{2}^-\right)(v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}} m_{D^*}}}. \end{aligned} \quad (38)$$

Therefore $-\delta_{1/m^2}^{(A_1)}$ (36) can be written as

$$\begin{aligned}
-\delta_{1/m^2}^{(A_1)} &= \frac{\mu_G^2}{6m_c^2} + \frac{1}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) (\mu_\pi^2 - \mu_G^2) \\
&+ \frac{1}{2} \sum_n \left[\left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) \frac{1}{\Delta E_{1/2}^{(n)}} \frac{\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right. \\
&+ \left. \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \frac{1}{\Delta E_{1/2}^{(n)}} \frac{\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right]^2 \\
&+ \frac{1}{2} \sum_n \left[\frac{1}{2m_c} \frac{1}{\Delta E_{3/2}^{(n)}} \frac{\langle D^{*(n)} \left(\frac{3}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}} m_{D^*}}} \right]^2. \tag{39}
\end{aligned}$$

The important point to emphasize here is that the matrix elements $\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle$ and $\langle D^{*(n)} \left(\frac{1}{2}^-, \frac{3}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle$ are precisely the same ones that enter in the SR (26)-(28). This allows to obtain an interesting lower bound on $-\delta_{1/m^2}^{(A_1)}$.

5 A lower bound on the inelastic contribution to the $-\delta_{1/m^2}^{(A_1)}$ correction of the $B \rightarrow D^*$ axial form factor at zero recoil.

We take now the relevant linear combinations of the matrix elements suggested by the r.h.s. of (39), and use (26), (27) and (31),

$$\begin{aligned}
&\sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \left\{ \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \frac{\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right. \\
&+ \left. \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) \frac{\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right\} \\
&= \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) 2\chi_1(w) - \frac{1}{3} \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) [-4(w-1)\chi_2(w) + 12\chi_3(w)] \tag{40}
\end{aligned}$$

$$\begin{aligned}
&\sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle D^{*(n)} \left(\frac{3}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}} m_{D^*}}} \\
&= -\frac{1}{2m_c} \frac{4\sqrt{6}}{3} \chi_2(w). \tag{41}
\end{aligned}$$

Using now Schwarz inequality

$$\left| \sum_n A_n B_n \right| \leq \sqrt{\left(\sum_n |A_n|^2 \right) \left(\sum_n |B_n|^2 \right)} \quad (42)$$

one finds

$$\begin{aligned} & \sum_{n \neq 0} \left[\xi^{(n)}(w) \right]^2 \sum_{n \neq 0} \left\{ \frac{1}{\Delta E_{1/2}^{(n)}} \left[\left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \frac{\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{kin,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right. \right. \\ & \quad \left. \left. + \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) \frac{\langle D^{*(n)} \left(\frac{1}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right] \right\}^2 \\ & \geq 4 \left\{ \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \chi_1(w) - \frac{1}{3} \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) [-2(w-1)\chi_2(w) + 6\chi_3(w)] \right\}^2 \end{aligned} \quad (43)$$

$$\begin{aligned} & \sum_n \left[\tau_{3/2}^{(2)(n)}(w) \right]^2 \sum_n \left\{ \frac{1}{\Delta E_{3/2}^{(n)}} \left[\frac{1}{2m_c} \frac{\langle D^{*(n)} \left(\frac{3}{2}^- \right) (v, \varepsilon) | O_{mag,v}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{*(n)}} m_{D^*}}} \right] \right\}^2 \\ & \geq \frac{32}{3} \left[\frac{1}{2m_c} \chi_2(w) \right]^2. \end{aligned} \quad (44)$$

These two last equations imply, from (39), the inequality

$$\begin{aligned} -\delta_{1/m^2}^{(A_1)} & \geq \frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \\ & \quad + 2 \frac{\left\{ \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \chi_1(w) - \frac{1}{3} \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) [-2(w-1)\chi_2(w) + 6\chi_3(w)] \right\}^2}{\sum_{n \neq 0} \left[\xi^{(n)}(w) \right]^2} \\ & \quad + \frac{16}{3} \frac{\left[\frac{1}{2m_c} \chi_2(w) \right]^2}{\sum_n \left[\tau_{3/2}^{(2)(n)}(w) \right]^2}. \end{aligned} \quad (45)$$

This inequality on $-\delta_{1/m^2}^{(A_1)}$ involves on the r.h.s. *elastic subleading* functions $\chi_i(w)$ ($i = 1, 2, 3$) in the numerator and sums over *inelastic leading IW functions* $\sum_{n \neq 0} \left[\xi^{(n)}(w) \right]^2$ and $\sum_n \left[\tau_{3/2}^{(2)(n)}(w) \right]^2$ in the denominator. We must emphasize that this inequality is valid for all values of w and constitutes a rigorous constraint between these functions and the correction $-\delta_{1/m^2}^{(A_1)}$. Let us point out that, near $w = 1$, since

$$\xi^{(n)}(w) \sim (w-1) \quad (n \neq 0) \quad (46)$$

and, due to Luke theorem

$$\chi_1(w), \chi_3(w) \sim (w - 1) \quad (47)$$

the second term on the r.h.s. of (45) is a constant in the limit $w \rightarrow 1$.

On the other hand, since $\chi_2(w)$ is not protected by Luke theorem,

$$\chi_2(1) \neq 0 \quad (48)$$

and in general, as pointed out by Leibovich et al. [10]

$$\tau_{3/2}^{(2)}(1) \neq 0 \quad (49)$$

the last term in the r.h.s. of (45) is also a constant for $w = 1$.

The inequality (45) is valid for all values of w , and in particular it holds in the $w \rightarrow 1$ limit. Let us consider this limit, that gives

$$\begin{aligned} -\delta_{1/m^2}^{(A_1)} &\geq \frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \\ &+ 2 \frac{\left\{ \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \chi_1'(1) - \frac{1}{3} \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) [-2\chi_2(1) + 6\chi_3'(1)] \right\}^2}{\sum_{n \neq 0} [\xi^{(n)'}(1)]^2} \\ &+ \frac{16}{3} \frac{\left[\frac{1}{2m_c} \chi_2(1) \right]^2}{\sum_n \left[\tau_{3/2}^{(2)}(1) \right]^2}. \end{aligned} \quad (50)$$

On the other hand, using the OPE in the heavy quark limit, we have demonstrated the following sum rules [5]

$$\sum_n \left[\tau_{3/2}^{(2)}(1) \right]^2 = \frac{4}{5} \sigma^2 - \rho^2 \quad (51)$$

$$\sum_{n \neq 0} \left[\xi^{(n)'}(1) \right]^2 = \frac{5}{3} \sigma^2 - \frac{4}{3} \rho^2 - (\rho^2)^2 \quad (52)$$

where ρ^2 and σ^2 are the slope and the curvature of the elastic Isgur-Wise function $\xi(w)$,

$$\xi(w) = 1 - \rho^2(w - 1) + \frac{\sigma^2}{2}(w - 1)^2 + \dots \quad (53)$$

The positivity of the l.h.s. of (51), (52) yield respectively the lower bounds on the curvature obtained in [4] [5],

$$\sigma^2 \geq \frac{5}{4} \rho^2 \quad (54)$$

$$\sigma^2 \geq \frac{1}{5} [4\rho^2 + 3(\rho^2)^2] . \quad (55)$$

On the other hand, Uraltsev [1] plus Bjorken [2] SR imply

$$\rho^2 \geq \frac{3}{4} \quad (56)$$

giving, from both (54), (55), the absolute bound for the curvature

$$\sigma^2 \geq \frac{15}{16} . \quad (57)$$

Relations (50)-(52) give finally the bound

$$\begin{aligned} -\delta_{1/m^2}^{(A_1)} &\geq \frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \\ &+ \frac{2}{3[5\sigma^2 - 4\rho^2 - 3(\rho^2)^2]} \left\{ \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) 3\chi_1'(1) - \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) [-2\chi_2(1) + 6\chi_3'(1)] \right\}^2 \\ &+ \frac{80}{3(4\sigma^2 - 5\rho^2)} \left[\frac{1}{2m_c} \chi_2(1) \right]^2 . \end{aligned} \quad (58)$$

We briefly discuss in Appendix B the radiative corrections to relations (51) and (52), computed in [6], and their impact on the bound (58).

6 Lower bound on the $1/m_Q^2$ corrections to $h_+(1)$ and $h_1(1)$.

Of theoretical interest are also the quantities at zero recoil $\ell_1(1)$, $\ell_2(1)$, that would correspond to the wave function overlaps in the non-relativistic quark model [8]. Using the notation of Falk and Neubert [8], these quantities are related to the matrix elements of the vector current at zero recoil,

$$\begin{aligned} \frac{\langle D(v)|V_\mu|B(v) \rangle}{\sqrt{4m_B m_D}} &= v_\mu h_+(1) \\ \frac{\langle D^*(v, \varepsilon)|V_\mu|B^*(v, \varepsilon) \rangle}{\sqrt{4m_{B^*} m_{D^*}}} &= v_\mu h_1(1) \end{aligned} \quad (59)$$

where

$$\begin{aligned} h_+(1) &= 1 + \delta_{1/m^2}^{(+)} + \dots \\ h_1(1) &= 1 + \delta_{1/m^2}^{(1)} + \dots \end{aligned} \quad (60)$$

with

$$\begin{aligned}\delta_{1/m^2}^{(+)} &= \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \ell_1(1) \\ \delta_{1/m^2}^{(1)} &= \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \ell_2(1) .\end{aligned}\tag{61}$$

On the other hand, using also the notations of [8], $\delta_{1/m^2}^{(A_1)}$ is given by the expression

$$\delta_{1/m^2}^{(A_1)} = \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \left[\frac{1}{2m_c} \ell_2(1) - \frac{1}{2m_b} \ell_1(1) \right] + \frac{1}{4m_c m_b} \Delta\tag{62}$$

where

$$\Delta = \ell_1(1) + \ell_2(1) + m_2(1) + m_9(1) .\tag{63}$$

We observe that

$$\begin{aligned}\delta_{1/m^2}^{(A_1)} &\rightarrow \frac{1}{4m_b^2} \ell_1(1) && \text{for } m_c \rightarrow \infty \\ \delta_{1/m^2}^{(A_1)} &\rightarrow \frac{1}{4m_c^2} \ell_2(1) && \text{for } m_b \rightarrow \infty .\end{aligned}\tag{64}$$

Therefore, since the lower bound (58) is valid for any value of m_c and m_b , we can obtain lower bounds on $-\ell_1(1)$ and $-\ell_2(1)$ by taking the limits (64). We find, in this way,

$$-\ell_1(1) \geq \frac{\mu_\pi^2 - \mu_G^2}{2} + \frac{6}{5\sigma^2 - 4\rho^2 - 3(\rho^2)^2} [-\chi_1'(1) + 2\chi_2(1) - 6\chi_3'(1)]^2\tag{65}$$

$$\begin{aligned}-\ell_2(1) &\geq \frac{3\mu_\pi^2 + \mu_G^2}{6} + \frac{2}{3[5\sigma^2 - 4\rho^2 - 3(\rho^2)^2]} [3\chi_1'(1) + 2\chi_2(1) - 6\chi_3'(1)]^2 \\ &\quad + \frac{80}{3(4\sigma^2 - 5\rho^2)} [\chi_2(1)]^2\end{aligned}\tag{66}$$

and from (61) we obtain lower bounds on $-\delta_{1/m^2}^{(+)}$ and $-\delta_{1/m^2}^{(1)}$.

7 General considerations on the bounds.

We have obtained lower bounds on the $-\delta_{1/m^2}$ corrections to some form factors, namely $h_{A_1}(w)$, $h_+(w)$ and $h_1(w)$, *that are protected by Luke theorem*. It is worth to summarize their expressions at zero recoil :

$$\begin{aligned}
-\delta_{1/m^2}^{(A_1)} &\geq \frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \\
&+ \frac{2}{3[5\sigma^2 - 4\rho^2 - 3(\rho^2)^2]} \left\{ \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) 3\chi_1'(1) \right. \\
&\left. - \left(\frac{1}{2m_c} + \frac{3}{2m_b} \right) [-2\chi_2(1) + 6\chi_3'(1)] \right\}^2 + \frac{80}{4\sigma^2 - 5\rho^2} \left[\frac{1}{2m_c} \chi_2(1) \right]^2
\end{aligned} \tag{67}$$

$$\begin{aligned}
-\delta_{1/m^2}^{(+)} &\geq \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \left\{ \frac{\mu_\pi^2 - \mu_G^2}{2} \right. \\
&\left. + \frac{6}{5\sigma^2 - 4\rho^2 - 3(\rho^2)^2} [-\chi_1'(1) + 2\chi_2(1) - 6\chi_3'(1)]^2 \right\}
\end{aligned} \tag{68}$$

$$\begin{aligned}
-\delta_{1/m^2}^{(1)} &\geq \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \left\{ \frac{3\mu_\pi^2 + \mu_G^2}{6} \right. \\
&\left. + \frac{2}{3[5\sigma^2 - 4\rho^2 - 3(\rho^2)^2]} [3\chi_1'(1) + 2\chi_2(1) - 6\chi_3'(1)]^2 + \frac{80}{3(4\sigma^2 - 5\rho^2)} [\chi_2(1)]^2 \right\}.
\end{aligned} \tag{69}$$

A number of remarks are worth to be made here :

(i) The bounds contain an OPE piece, dependent on μ_π^2 and μ_G^2 , and a piece that bounds the inelastic contributions, given in terms of the $1/m_Q$ elastic quantities $\chi_1'(1)$, $\chi_2(1)$, $\chi_3'(1)$ and on the slope ρ^2 and curvature σ^2 of the elastic IW function $\xi(w)$.

(ii) Taking roughly constant values for $\chi_1'(1)$, $\chi_2(1)$, $\chi_3'(1)$, as suggested by the QCD Sum Rules calculations (QCDSR) [13] [14] [15], the bounds for the inelastic contributions diverge in the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$, according to (57). This feature does not seem to us physical.

(iii) *Therefore, one should expect that $\chi_1'(1)$, $\chi_2(1)$ and $\chi_3'(1)$ vanish also in this limit.* We give a demonstration of this interesting feature in the next section.

(iv) Thus, the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$ seems related to the behaviour of $\chi_i(w)$ ($i = 1, 2, 3$) near zero recoil.

(v) The feature (iii) does not appear explicitly in the QCDSR approach, where one gets roughly $\rho_{ren}^2 \cong 0.7$, and where there is no dependence on ρ^2 of the functions $\chi_i(w)$ ($i = 1, 2, 3$).

(vi) In the nonrelativistic quark model the parameters $\ell_1(1)$ and $\ell_2(1)$ correspond

to the overlap of the wave functions at zero recoil [8]. The formulas (65) and (66) give a model-independent, rigorous bound for these quantities.

8 Behaviour of the subleading functions $\chi_i(\mathbf{w})$ ($i = 1, 2, 3$) in the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$.

In this Section we demonstrate that indeed $\chi'_1(1)$, $\chi_2(1)$ and $\chi'_3(1)$ vanish in the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$. Let us rewrite the relations (26), (27) and (31) in terms of pseudoscalar matrix elements

$$\chi_1(w) = \frac{1}{2} \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle D^{(n)}(v) | O_{kin}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \quad (70)$$

$$\chi_2(w) = \frac{1}{4\sqrt{6}} \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(w) \frac{\langle D_{3/2}^{*(n)}(v, \varepsilon) | O_{mag}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}} m_{D^*}}} \quad (71)$$

$$-4(w-1)\chi_2(w) + 12\chi_3(w) = \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)}(w) \frac{\langle D^{(n)}(v) | O_{mag}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \quad (72)$$

At zero recoil $w \rightarrow 1$ we have

$$\chi'_1(1) = \frac{1}{2} \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)'}(1) \frac{\langle D^{(n)}(v) | O_{kin}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \quad (73)$$

$$\chi_2(1) = \frac{1}{4\sqrt{6}} \sum_n \frac{1}{\Delta E_{3/2}^{(n)}} \tau_{3/2}^{(2)(n)}(1) \frac{\langle D_{3/2}^{*(n)}(v, \varepsilon) | O_{mag}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}} m_{D^*}}} \quad (74)$$

$$-4\chi_2(1) + 12\chi'_3(1) = \sum_{n \neq 0} \frac{1}{\Delta E_{1/2}^{(n)}} \xi^{(n)'}(1) \frac{\langle D^{(n)}(v) | O_{mag}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \quad (75)$$

Using again Schwarz inequality as in Section 5, we obtain

$$[\chi'_1(1)]^2 \leq \frac{1}{4} \sum_{n \neq 0} \left[\xi^{(n)'}(1) \right]^2 \sum_{n \neq 0} \left[\frac{1}{\Delta E_{1/2}^{(n)}} \frac{\langle D^{(n)}(v) | O_{kin}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \right]^2 \quad (76)$$

$$[\chi_2(1)]^2 \leq \frac{1}{96} \sum_n \left[\tau_{3/2}^{(2)(n)}(1) \right]^2 \sum_n \left[\frac{1}{\Delta E_{3/2}^{(n)}} \frac{\langle D_{3/2}^{*(n)}(v, \varepsilon) | O_{mag}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D^{(n)}} m_{D^*}}} \right]^2 \quad (77)$$

$$[-4\chi_2(1) + 12\chi_3'(1)]^2 \leq \sum_{n \neq 0} [\xi^{(n)'}(1)]^2 \sum_{n \neq 0} \left[\frac{1}{\Delta E_{1/2}^{(n)}} \frac{\langle D^{(n)}(v) | O_{mag}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \right]^2 \quad (78)$$

and from relations (51) and (52) we obtain

$$[\chi_1'(1)]^2 \leq \frac{1}{12} [5\sigma^2 - 4\rho^2 - 3(\rho^2)^2] \sum_{n \neq 0} \left[\frac{1}{\Delta E_{1/2}^{(n)}} \frac{\langle D^{(n)}(v) | O_{kin}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \right]^2 \quad (79)$$

$$[\chi_2(1)]^2 \leq \frac{1}{480} (4\sigma^2 - 5\rho^2) \sum_n \left[\frac{1}{\Delta E_{3/2}^{(n)}} \frac{\langle D_{3/2}^{*(n)}(v, \varepsilon) | O_{mag}^{(c)}(0) | D^*(v, \varepsilon) \rangle}{\sqrt{4m_{D_{3/2}^{*(n)}} m_{D^*}}} \right]^2 \quad (80)$$

$$[-4\chi_2(1) + 12\chi_3'(1)]^2 \leq \frac{1}{3} [5\sigma^2 - 4\rho^2 - 3(\rho^2)^2] \sum_{n \neq 0} \left[\frac{1}{\Delta E_{1/2}^{(n)}} \frac{\langle D^{(n)}(v) | O_{mag}^{(c)}(0) | D(v) \rangle}{\sqrt{4m_{D^{(n)}} m_D}} \right]^2 \quad (81)$$

Therefore, in the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$, one obtains

$$\chi_1'(1) = \chi_2(1) = \chi_3'(1) = 0 \quad (82)$$

as it has been expected from the inspection of relations (67)-(69).

This is a very strong correlation relating the behaviour of the elastic IW function $\xi(w)$ to the elastic subleading IW functions $\chi_i(w)$ ($i = 1, 2, 3$) near zero recoil.

9 Discussion and phenomenological implications on the determination of $|V_{cb}|$.

The bounds that relate second order subleading corrections δ_{1/m^2} , the first order $1/m_Q$ form factors $\chi_i(w)$ ($i = 1, 2, 3$) and the curvature and slope of the elastic Isgur-Wise function $\xi(w)$ should be taken into account in the exclusive determination of $|V_{cb}|$.

On the one hand, the usual present point of view is that the exclusive determination of $|V_{cb}|$ is not competitive with the inclusive determination, that looks much more precise. However, one must keep in mind that the hadronic uncertainties in

both methods are of different nature and that only a convergence of both can be satisfactory for a precise measurement of $|V_{cb}|$.

As an illustration of the most advanced measurements, let us quote the results of Babar [20]. To have a qualitative feeling, let us add the errors in quadrature,

$$|V_{cb}|_{inclusive} = 0.0414 \pm 0.0008 \quad (83)$$

$$|V_{cb}|_{exclusive} = 0.0370 \pm 0.0020 \quad (84)$$

where the exclusive determination comes from $\overline{B} \rightarrow D^* \ell \nu$ and uses the value

$$-\delta_{1/m^2}^{(A_1)} = 0.09 \pm 0.05 \quad (85)$$

discussed in Appendix B.

The slight disagreement between both determinations (83), (84) seems to suggest that $-\delta_{1/m^2}^{(A_1)}$ could be larger than (85).

On the other hand, although this is not the main object of our discussion, in obtaining $|V_{cb}|_{inclusive}$ one fits $\mu_G^2 = (0.27 \pm 0.07) \text{ GeV}^2$. This is roughly within 1σ in agreement with the experimental value obtained from the spectrum, namely $\mu_G^2 = 0.36 \text{ GeV}^2$. However, it seems to us that this parameter is a very well determined quantity that, in the fit, should be fixed at this latter value. This is just to emphasize that, even in the very efficient inclusive determination, there are presumably still hadronic uncertainties.

Coming back to the exclusive determination, it is well known that there is a great dispersion of the data in the different experiments using $\overline{B} \rightarrow D(D^*) \ell \nu$, as discussed in detail by Grinstein and Ligeti [16] (see also [20]).

Since in this determination, for example in $\overline{B} \rightarrow D^* \ell \nu$, enters $-\delta_{1/m^2}^{(A_1)}$ and also the subleading form factors $\chi_i(w)$ ($i = 1, 2, 3$), as well as the shape of the Isgur-Wise function $\xi(w)$, our bound (58) has to be taken into account, as well as the vanishing of $\chi'_1(1)$, $\chi_2(1)$, $\chi'_3(1)$ in the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$.

The functions $\chi_i(w)$ ($i = 1, 2, 3$) have been computed in the framework of the QCD Sum Rules approach [13] [14] [15], obtaining

$$\begin{aligned} \chi'_1(1) &= (0.15 \pm 0.10) \overline{\Lambda} \\ \chi_2(1) &= -(0.05 \pm 0.01) \overline{\Lambda} \\ \chi'_3(1) &= (0.009 \pm 0.004) \overline{\Lambda} . \end{aligned} \quad (86)$$

We have extracted these rough numbers from figures 5.5 of ref. [15], where the $\chi_i(w)$ ($i = 1, 2, 3$) are dimensionless, given in units of $\bar{\Lambda}$ and we have translated them in the definition of ref. [9], adopted in the present paper. On the other hand, one obtains, in the QCDSR approach

$$\rho_{ren}^2 \cong 0.7 \tag{87}$$

Therefore, the QCDSR approach does not make explicit the constraint that we have obtained, and our discussion cannot proceed further within this scheme.

In the case of $\bar{B} \rightarrow D\ell\nu$ the correction $-\delta_{1/m^2}^{(1)}$ is one of the pieces that constitute the $1/m_Q^2$ correction : besides ℓ_1 there is another correction ℓ_4 [8] not concerned by our bounds, and therefore the situation is less clear. Nevertheless, what we have said about $-\delta_{1/m^2}^{(A_1)}$ applies to $-\delta_{1/m^2}^{(1)}$.

By considering his BPS limit, Uraltsev [17] has obtained complementary results. We will discuss separately the relation of his approach with our above sum rules.

10 Conclusion.

To conclude, we have obtained bounds that relate $1/m_Q^2$ corrections of form factors protected by Luke theorem, namely $h_{A_1}(w)$, $h_+(w)$ and $h_1(w)$ to the $1/m_Q$ subleading form factors of the Lagrangian type $\chi_i(w)$ ($i = 1, 2, 3$) and to the shape of the elastic Isgur-Wise $\xi(w)$. These bounds should in principle be taken into account in the analysis of the exclusive determination of $|V_{cb}|$ in the channels $\bar{B} \rightarrow D(D^*)\ell\nu$. On the other hand, we have demonstrated an important constraint on the behavior of the subleading form factors $\chi_i(w)$ in the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$, since $\chi'_1(1)$, $\chi_2(1)$ and $\chi'_3(1)$ must vanish in this limit.

It would be very interesting to have a theoretical estimation of the functions $\chi_i(w)$ ($i = 1, 2, 3$) satisfying this constraint. Otherwise it seems questionable to try an exclusive determination of $|V_{cb}|$ by fitting the slope ρ^2 and considering uncorrelated subleading corrections, for example roughly constant values for $\chi'_1(1)$, $\chi_2(1)$ and $\chi'_3(1)$.

Appendix A. Derivation of the Lagrangian Sum Rules using the OPE.

In this Appendix we give an alternative derivation of the SR (26)-(28), following the same method used in ref. [7], based on the OPE, to obtain similar SR concerning the $1/m_Q$ perturbations of the heavy quark current.

To make easier the study of the subleading corrections we did consider the following limit

$$m_c \gg m_b \gg \Lambda_{QCD} . \quad (\text{A.1})$$

Then, as explained in [7], the difference between the two energy denominators in the T -product (1) is large

$$q^0 - E_f + E_{X_{\bar{c}bb}} - (q^0 + E_i - E_{X_c}) \sim 2m_c \quad (\text{A.2})$$

where X_c and $X_{\bar{c}bb}$ denote the intermediate states of the direct and Z orderings. Therefore, we can in this limit neglect the Z diagram, and consider the imaginary part of the direct diagram, the piece proportional to

$$\delta(q^0 + E_i - E_{X_c}) . \quad (\text{A.3})$$

Notice that one can choose q^0 such that there is a left-hand cut, even in the conditions (A.1). This means that q^0 is of the order of m_c and $m_c - q^0$ is fixed, of the order m_b . Our conditions are, in short, as follows :

$$\Lambda_{QCD} \ll m_b \sim m_c - q^0 \ll q^0 \sim m_c \rightarrow \infty . \quad (\text{A.4})$$

To summarize, we did consider the heavy quark limit for the c quark, but allowing for a large finite mass for the b quark.

The final result is the sum rule [7]

$$\begin{aligned} & \sum_{D_n} \langle B_f(v_f) | J_f(0) | D_n(v') \rangle \langle D_n(v') | J_i(0) | B_i(v_i) \rangle \\ &= \langle B(v_f) | \bar{b}(0) \Gamma_f \frac{1 + \not{v}'}{2v'^0} \Gamma_i b(0) | B(v_i) \rangle + O(1/m_c) \end{aligned} \quad (\text{A.5})$$

that is valid for *all powers* of an expansion in $1/m_b$, but only to leading order in $1/m_c$.

At leading order $m_b, m_c \rightarrow \infty$ one gets the SR formulated in [3]-[5]. In ref. [7] we considered the first order in $1/m_b$ to both the left and right hand sides of (A.5), using the formalisms of Falk and Neubert [8] for the $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ transitions and of Leibovich et al. [10] for the $\frac{1}{2}^- \rightarrow \frac{1}{2}^+$, $\frac{1}{2}^- \rightarrow \frac{3}{2}^+$ transitions. The formalism was extended to all possible transitions $\frac{1}{2}^- \rightarrow j^\pm$ [11].

We did consider only the $1/m_Q$ perturbations that are perturbations of the current, namely $L_4(w)$, $L_5(w)$ and $L_6(w)$, in the notation of [8]. To obtain the maximum information we did consider in [7] initial and final pseudoscalar $B(v_i) \rightarrow B(v_f)$ or vector states $B^*(v_i, \varepsilon_i) \rightarrow B^*(v_f, \varepsilon_f)$. This yielded to two interesting very simple sum rules. The reason is that we considered the SR at the frontier

$$(w_i, w_f, w_{if}) = (w, 1, w) \quad (\text{A.6})$$

of the domain of the variables $(w_i, w_f, w_{if}) = (v_i \cdot v', v_f \cdot v', v_i \cdot v_f)$ [3],

$$w_i \geq 1 \quad w_f \geq 1$$

$$w_i w_f - \sqrt{(w_i^2 - 1)(w_f^2 - 1)} \leq w_{if} \leq w_i w_f + \sqrt{(w_i^2 - 1)(w_f^2 - 1)}. \quad (\text{A.7})$$

In this Appendix we formulate new SR for the *Lagrangian* perturbations, parallel to the ones on the *Current* perturbations (3)-(4), using the OPE formalism of ref. [7]. We find the same results than with the simple method exposed in detail in Section 2.

In obtaining (3)-(4) we did use axial currents aligned along the initial and final velocities, $\Gamma_i = \not{v}_i \gamma_5$, $\Gamma_f = \not{v}_f \gamma_5$. Let us use now the *vector* heavy quark currents, aligned along the initial and final four-velocities,

$$\Gamma_i = \not{v}_i \quad \Gamma_f = \not{v}_f \quad (\text{A.8})$$

and proceed as in [7]. Using expression (A.5) we obtain two sum rules at order $1/m_b$ for initial and final pseudoscalar $B(v_i) \rightarrow B(v_f)$ or vector states $B^*(v_i, \varepsilon_i) \rightarrow B^*(v_f, \varepsilon_f)$.

To compute the SR we need the following matrix elements, including the $1/m_b$ order [8]

$$\begin{aligned} \langle D(v') | \overline{Q} \Gamma Q | B(v) \rangle &= -\xi(w) Tr[\overline{D}(v') \Gamma B(v)] \\ &- \frac{1}{2m_b} Tr \left\{ \overline{D}(v') \Gamma [P_+ L_+(v, v') + P_- L_-(v, v')] \right\} + O(1/m_c). \end{aligned} \quad (\text{A.9})$$

The 4×4 matrices write, for a pseudoscalar meson M (initial B or intermediate D)

$$M(v) = P_+(v)(-\gamma_5)$$

$$P_+(v)L_+(v, v') + P_-(v)L_-(v, v') = [L_1(w)P_+(v) + L_4(w)P_-(v)](-\gamma_5) \quad (\text{A.10})$$

where $w = v \cdot v'$, while for a vector meson M one has

$$M(v) = P_+(v)\not{e}$$

$$\begin{aligned} & P_+(v)L_+(v, v') + P_-(v)L_-(v, v') = \\ & P_+(v) [\not{e}L_2(w) + (\varepsilon_v \cdot v')L_3(w)] + P_-(v) [\not{e}L_5(w) + (\varepsilon \cdot v')L_6(w)] . \end{aligned} \quad (\text{A.11})$$

The matrix elements to excited states write [10]

$$\begin{aligned} & \langle D \left(\frac{3}{2}^+ \right) (v') | \bar{c} \Gamma b | B(v) \rangle = \sqrt{3} \tau_{3/2}(w) \text{Tr} [v_\sigma \bar{D}^\sigma(v') \Gamma B(v)] \\ & + \frac{1}{2m_b} \left\{ \text{Tr} [S_{\sigma\lambda}^{(b)} \bar{D}^\sigma(v') \Gamma \gamma^\lambda B(v)] + \eta_{ke}^{(b)} \text{Tr} [v_\sigma \bar{D}^\sigma(v') \Gamma B(v)] \right. \\ & \quad \left. + \text{Tr} [R_{\sigma\alpha\beta}^{(b)} \bar{D}^\sigma(v') \Gamma P_+(v) i\sigma^{\alpha\beta} B(v)] \right\} + O(1/m_c) \\ & \langle D \left(\frac{1}{2}^+ \right) (v') | \bar{c} \Gamma b | B(v) \rangle = 2\tau_{1/2}(w) \text{Tr} [\bar{D}(v') \Gamma B(v)] \\ & + \frac{1}{2m_b} \left\{ \text{Tr} [S_\lambda^{(b)} \bar{D}(v') \Gamma \gamma^\lambda B(v)] + \chi_{ke}^{(b)} \text{Tr} [\bar{D}(v') \Gamma B(v)] \right. \\ & \quad \left. + \text{Tr} [R_{\alpha\beta}^{(b)} \bar{D}(v') \Gamma P_+(v) i\sigma^{\alpha\beta} B(v)] \right\} + O(1/m_c) \end{aligned} \quad (\text{A.12})$$

where

$$\begin{aligned} & D_{2+}^\sigma(v') = P_+(v') \varepsilon_{v'}^{\sigma\nu} \gamma_\nu \\ & D_{1+}^\sigma(v') = -\sqrt{\frac{3}{2}} P_+(v') \varepsilon_{v'}^\nu \gamma_5 \left[g_\nu^\sigma - \frac{1}{3} \gamma_\nu (\gamma^\sigma - v'^\sigma) \right] \\ & D_{1+}(v') = P_+(v') \varepsilon_{v'}^\nu \gamma_5 \gamma_\nu \\ & D_{0+}(v') = P_+(v') . \end{aligned} \quad (\text{A.13})$$

The notation $S_{\sigma\lambda}^{(b)}$, $S_\lambda^{(b)}$ denote the perturbations to the current, and $\eta_{ke}^{(b)}$, $\chi_{ke}^{(b)}$ and $R_{\sigma\alpha\beta}^{(b)}$, $R_{\alpha\beta}^{(b)}$ denote respectively the kinetic and the magnetic Lagrangian perturbations.

Expanded in terms of Lorentz covariant factors and subleading IW functions, these tensor quantities read [10]

$$\begin{aligned}
S_{\sigma\lambda}^{(Q)} &= v_\sigma \left[\tau_1^{(Q)}(w)v_\lambda + \tau_2^{(Q)}(w)v'_\lambda + \tau_3^{(Q)}(w)\gamma_\lambda \right] + \tau_4^{(Q)}(w)g_{\sigma\lambda} \\
S_\lambda^{(Q)} &= \zeta_1^{(Q)}(w)v_\lambda + \zeta_2^{(Q)}(w)v'_\lambda + \zeta_3^{(Q)}(w)\gamma_\lambda \quad (Q = b, c)
\end{aligned} \tag{A.14}$$

for the Current perturbations, and

$$\begin{aligned}
R_{\sigma\alpha\beta}^{(b)} &= \eta_1^{(b)}(w)v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(b)}(w)v_\sigma v'_\alpha\gamma_\beta + \eta_3^{(b)}(w)g_{\sigma\alpha}v'_\beta \\
R_{\alpha\beta}^{(b)} &= \chi_1^{(b)}(w)\gamma_\alpha\gamma_\beta + \chi_2^{(b)}(w)v'_\alpha\gamma_\beta
\end{aligned} \tag{A.15}$$

for the Lagrangian magnetic perturbations.

We have also to consider the intermediate states $D\left(\frac{3}{2}^-, 1^-\right)$, $D\left(\frac{3}{2}^-, 2^-\right)$. The corresponding 4×4 matrices for the $\frac{3}{2}^-$ states will be given in terms of those of $\frac{3}{2}^+$ states (A.13) by [11]

$$D_{1^-}^\sigma(v') = D_{1^+}^\sigma(v')(-\gamma_5), \quad D_{2^-}^\sigma(v') = D_{2^+}^\sigma(v')(-\gamma_5) \tag{A.16}$$

and the current matrix elements, including $1/m_b$ corrections are

$$\begin{aligned}
\langle D\left(\frac{3}{2}^-\right)(v') | \bar{c}\Gamma b | B(v) \rangle &= \tau_{3/2}^{(2)}(w) \text{Tr} \left[v_\sigma \bar{D}^\sigma(v') \Gamma B(v) \right] \\
&+ \frac{1}{2m_b} \left\{ \text{Tr} \left[T_{\sigma\lambda}^{(b)} \bar{D}^\sigma(v') \Gamma \gamma^\lambda B(v) \right] + \rho_{ke}^{(b)} \text{Tr} \left[v_\sigma \bar{D}^\sigma(v') \Gamma B(v) \right] \right. \\
&\quad \left. + \text{Tr} \left[V_{\sigma\alpha\beta}^{(b)} \bar{D}^\sigma(v') \Gamma P_+(v) i\sigma^{\alpha\beta} B(v) \right] \right\} + O(1/m_c)
\end{aligned} \tag{A.17}$$

where

$$T_{\sigma\lambda}^{(b)} = v_\sigma \left[\sigma_1^{(b)}(w)v_\lambda + \sigma_2^{(b)}(w)v'_\lambda + \sigma_3^{(b)}(w)\gamma_\lambda \right] + \sigma_4^{(b)}(w)g_{\sigma\lambda} \tag{A.18}$$

denotes the Current perturbations, and

$$V_{\sigma\alpha\beta}^{(b)} = \rho_1^{(b)}(w)v_\sigma\gamma_\alpha\gamma_\beta + \rho_2^{(b)}(w)v_\sigma v'_\alpha\gamma_\beta + \rho_3^{(b)}(w)g_{\sigma\alpha}\gamma_\beta \tag{A.19}$$

the corresponding Lagrangian perturbations. In defining (A.19) we perform a different Lorentz decomposition as done in [10] for the $\frac{1}{2}^+$, $\frac{3}{2}^+$ states. The necessity of this alternative parametrization is explained in ref. [21].

Proceeding like in ref. [7], starting from the master formula (A.5), i.e. taking the formal limit $m_c \gg m_b$, and using now the vector currents (A.8), we find the following sum rules for

$$(w_i, w_f, w_{if}) = (w, 1, w) \quad \text{or} \quad (v_i, v_f, v) = (v, v', v'), \quad (\text{A.20})$$

respectively for the pseudoscalar $B(v) \rightarrow B(v')$ or $B^*(v, \varepsilon) \rightarrow B^*(v', \varepsilon')$ transitions,

$$L_1(w) = \sum_n \xi^{(n)}(w) L_1^{(n)}(1) \quad (\text{A.21})$$

$$L_2(w) + (w-1)L_3(w) = \sum_n \xi^{(n)}(w) L_2^{(n)}(1) - \frac{2}{3}(w-1) \sum_n \tau_{3/2}^{(2)(n)}(w) \rho_3^{(n)}(1) \quad (\text{A.22})$$

In (A.21) and (A.22) one has a relation between the elastic subleading form factors of Lagrangian type $L_1(w)$, $L_2(w)$ and $L_3(w)$ and excited leading IW functions $\xi^{(n)}(w)$, $\tau_{3/2}^{(2)(n)}(w)$ and excited subleading form factors of Lagrangian type at zero recoil, $L_1^{(n)}(1)$, $L_2^{(n)}(1)$ and $\rho_3^{(n)}(1)$. Notice that in the sums (A.21) and (A.22) the terms $\xi^{(0)}(w) L_1^{(0)}(1)$, $\xi^{(0)}(w) L_1^{(0)}(1)$ do not contribute due to Luke theorem [9]

$$L_1(1) = L_1(1) = 0. \quad (\text{A.23})$$

Therefore the SR (A.21), (A.22) actually reduce to

$$L_1(w) = \sum_{n \neq 0} \xi^{(n)}(w) L_1^{(n)}(1) \quad (\text{A.24})$$

$$L_2(w) + (w-1)L_3(w) = \sum_{n \neq 0} \xi^{(n)}(w) L_2^{(n)}(1) - \frac{2}{3}(w-1) \sum_n \tau_{3/2}^{(2)(n)}(w) \rho_3^{(n)}(1) \quad (\text{A.25})$$

We see that $L_1(w)$ and $L_2(w)$ satisfy Luke theorem at $w=1$, $L_1(1) = L_2(1) = 0$, due to $\xi^{(n)}(1) = \delta_{n,0}$.

A number of comments are worth here to be added.

(i) All current perturbation form factors, the elastic $L_4(w)$, $L_5(w)$ and $L_6(w)$ and the inelastic ones cancel in the sum rules. Only perturbations of the Lagrangian remain.

(ii) Only the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ intermediate states contribute at the frontier (A.20).

(iii) The SR (A.24)-(A.25) are reminiscent of the SR (3)-(4), that relate elastic subleading form factors of the current type to leading order excited IW functions and subleading excited form factors at zero recoil. In this case, however, these latter

form factors can be simply expressed, by the equations of motion, in terms of leading IW functions and level spacings.

(iv) It can be easily shown, following the same type of arguments as in [7] that higher excited states do not contribute to the SR (A.24)-(A.25) because we choose the frontier (A.20).

(v) Notice that for the SR concerning $1/m_Q$ perturbations to the *Current*, only $\frac{1}{2}^+$ and $\frac{3}{2}^+$ intermediate states survive. Similarly, in a symmetric way, for the $1/m_Q$ perturbations of the *Lagrangian*, only $\frac{1}{2}^-$ and $\frac{3}{2}^-$ intermediate states survive.

Writing the combinations $L_1(w)$ and $L_2(w) + (w - 1)L_3(w)$ in terms of the \mathcal{L}_{kin} and \mathcal{L}_{mag} matrix elements $\chi_i(w)$ [8] [9],

$$\begin{aligned} L_1(w) &= 2\chi_1(w) - 4(w - 1)\chi_2(w) + 12\chi_3(w) \\ L_2(w) &= 2\chi_1(w) - 4\chi_3(w) \\ L_3(w) &= 4\chi_2(w) \end{aligned} \tag{A.26}$$

We realize that, as obtained in Section 2, the combination $L_1(w)$ gets contributions only from $\frac{1}{2}^-$ intermediate states, while the combination $L_2(w) + (w - 1)L_3(w) = 2\chi_1(w) + 4(w - 1)\chi_2(w) - 4\chi_3(w)$ contains contributions from $\frac{1}{2}^-$ and $\frac{3}{2}^-$ intermediate states, as we have found in Section 3. In terms of the $\chi_i(w)$ ($i = 1, 2, 3$), the SR write

$$\begin{aligned} 2\chi_1(w) - 4(w - 1)\chi_2(w) + 12\chi_3(w) &= \sum_{n \neq 0} \xi^{(n)}(w) L_1^{(n)}(1) \\ 2\chi_1(w) + 4(w - 1)\chi_2(w) - 4\chi_3(w) &= \sum_{n \neq 0} \xi^{(n)}(w) L_2^{(n)}(1) - \frac{2}{3}(w - 1) \sum_n \tau_{3/2}^{(2)(n)}(w) \rho_3^{(n)}(1) \end{aligned} \tag{A.27}$$

On the other hand, since only \mathcal{L}_{kin} contributes to $\chi_1(w)$ and to $\chi_1^{(n)}(1)$, decomposing $L_1^{(n)}(1)$ and $L_2^{(n)}(1)$ in terms of $\chi_i^{(n)}(1)$ like in the first two relations (A.26), we can solve for $\chi_2(w)$ and $\chi_3(w)$ and find finally

$$\begin{aligned} \chi_1(w) &= \sum_{n \neq 0} \xi^{(n)}(w) \chi_1^{(n)}(1) \\ \chi_2(w) &= -\frac{1}{4} \sum_n \tau_{3/2}^{(2)(n)}(w) \rho_3^{(n)}(1) \end{aligned}$$

$$\chi_3(w) = \sum_{n \neq 0} \xi^{(n)}(w) \chi_3^{(n)}(1) - \frac{1}{12}(w-1) \sum_n \tau_{3/2}^{(2)(n)}(w) \rho_3^{(n)}(1) \quad (\text{A.28})$$

From the definition of $\chi_1^{(n)}(1)$, $\chi_3^{(n)}(1)$ and $\rho_3^{(n)}(1)$ from the T -products as in (5) and (6) for $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ transitions, but allowing for $n \neq 0$, and the corresponding one for $\frac{1}{2}^- \rightarrow \frac{3}{2}^-$ transitions, we see that these relations are identical with (26)-(28), obtained in Section 2 just from the definition of the form factors $\chi_i(w)$. The inelastic form factors at zero recoil $\chi_1^{(n)}(1)$ ($n \neq 0$) are given by the matrix elements $\frac{1}{2}^-$ ($n=0$) $\rightarrow \frac{1}{2}^-$ ($n \neq 0$) of \mathcal{L}_{kin} ponderated by the corresponding energy denominators. Similarly, $\chi_3^{(n)}(1)$ and $\rho_3^{(n)}(1)$ ($n \geq 0$) are given by the matrix elements $\frac{1}{2}^-$ ($n=0$) $\rightarrow \frac{1}{2}^-$ ($n \neq 0$) and $\frac{1}{2}^-$ ($n=0$) $\rightarrow \frac{3}{2}^-$ ($n \geq 0$) coming from \mathcal{L}_{mag} .

Appendix B.

Although the QCDSR results (86) do not explicitly satisfy the constraints (82), it could be of some interest to use these results to estimate the r.h.s. of (67)-(69) varying the input for the slope ρ^2 and the curvature σ^2 . The aim would be to see how these bounds evolve as one approaches the limit $\rho^2 \rightarrow \frac{3}{4}$, $\sigma^2 \rightarrow \frac{15}{16}$.

We denote the bounds under the form of the contribution of the OPE term of the matrix elements (34), plus the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ inelastic contributions,

$$-\delta_{1/m^2}^{(A_1)} \geq \text{OPE} + \frac{1}{2}^- + \frac{3}{2}^- . \quad (\text{B.1})$$

In view of the theoretical comments on the bounds made in the preceding Section, we can only provide some qualitative numerical illustrations that will show the general trend of the results. We give some numerical results in Tables 1, 2 and 3 using the parameters

$$\begin{aligned} m_c &= 1.25 \text{ GeV} & m_b &= 4.75 \text{ GeV} & \bar{\Lambda} &= 0.50 \text{ GeV} \\ \mu_\pi^2 &= 0.50 \text{ GeV}^2 & \mu_G^2 &= 0.36 \text{ GeV}^2 \end{aligned} \quad (\text{B.2})$$

and for the curvature σ^2 of the Isgur-Wise function we use its value in terms of the slope ρ^2 given by the ‘‘dipole’’ Ansatz [18]

$$\xi(w) = \left(\frac{2}{w+1} \right)^{2\rho^2} \quad (\text{B.3})$$

namely

$$\sigma^2 = \frac{\rho^2}{2} + (\rho^2)^2. \quad (\text{B.4})$$

We use this relation between the curvature and the slope because, as shown in [5], (B.3) satisfies all the bounds that we have obtained for the derivatives of the elastic IW function [3] [4] [5].

Parameters	$-\delta_{1/m^2}^{(A_1)} \geq \text{OPE} + \frac{1}{2}^- + \frac{3}{2}^-$
(i) $\rho^2 = 1$ $\sigma^2 = 1.5$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\delta_{1/m^2}^{(A_1)}$ $\geq 0.052 + 0.000 + 0.003$ $= 0.055$
(ii) $\rho^2 = 1$ $\sigma^2 = 1.5$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.15 \bar{\Lambda}$	$-\delta_{1/m^2}^{(A_1)}$ $\geq 0.052 + 0.113 + 0.003$ $= 0.168$
(iii) $\rho^2 = 0.9$ $\sigma^2 = 1.26$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\delta_{1/m^2}^{(A_1)}$ $\geq 0.052 + 0.000 + 0.005$ $= 0.057$
(iv) $\rho^2 = 0.8$ $\sigma^2 = 1.04$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\delta_{1/m^2}^{(A_1)}$ $\geq 0.001 + 0.001 + 0.017$ $= 0.070$
(v) $\rho^2 = 0.76$ $\sigma^2 = 0.96$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\delta_{1/m^2}^{(A_1)}$ $\geq 0.052 + 0.004 + 0.088$ $= 0.14$

Table 1. The lower bound (58) for $-\delta_{1/m^2}^{(A_1)}$ for different values of the parameters. OPE denotes the contribution depending on μ_π^2 , μ_G^2 and $\frac{1}{2}^-$, $\frac{3}{2}^-$ the inelastic contributions of the corresponding 1^- excited states. We fix the values $m_c = 1.25$ GeV, $m_b = 4.75$ GeV, $\bar{\Lambda} = 0.5$ GeV, $\mu_\pi^2 = 0.50$ GeV², $\mu_G^2 = 0.36$ GeV².

An order of magnitude estimation of the r.h.s. of relation (36) assumes that the inelastic term is roughly a factor χ of the OPE result [19], i.e.

$$-\delta_{1/m^2}^{(A_1)} = (1 + \chi) \left[\frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \right]. \quad (\text{B.5})$$

Taking $\chi = 0.5 \pm 0.5$ [19], one gets, for $\mu_\pi^2 = 0.50$, $\mu_G^2 = 0.36$,

$$-\delta_{1/m^2}^{(A_1)} \cong 0.09 \pm 0.05. \quad (\text{B.6})$$

We observe from the results of Table 1 that the lower bound grows rapidly as one approaches the lower bounds $\rho^2 = \frac{3}{4}$, $\sigma^2 = \frac{15}{16}$. However, for the values chosen for ρ^2 , σ^2 , the guess (B.6) can be accomodated with the QCDSR estimations for $\chi_i(w)$ ($i = 1, 2, 3$).

Let us comment on the different entries of Table 1. Our results can only pretend to give the qualitative trend of the bounds. In the choice of parameters (i) we have used the central values (86) and $\rho^2 = 1$. The lower bound on $-\delta_{1/m^2}^{(A_1)}$ is dominated by the OPE contribution, and specially the $\frac{1}{2}^-$ contributions are very small because of a strong cancellation between two terms in (58). In the second row (ii), just as an illustration, we have taken the central values of (86) except for $\chi'_3(1)$, for which we have taken the large value suggested by Grinstein and Ligeti [16] to fit the different experiments on $\overline{B} \rightarrow D(D^*)\ell\nu$, keeping however $\rho^2 = 1$. We observe that now the $\frac{1}{2}^-$ contribution becomes very large. In choices (iii), (iv) and (v) we take still the central values of (86) and we decrease the value of $\rho^2 = 0.9, 0.8, 0.76$, and consequently the curvature. For (v) the inelastic contributions become sizeable, specially for the $\frac{3}{2}^-$ contributions. Of course, the bounds diverge for $\rho^2 = \frac{3}{4}$, $\sigma^2 = \frac{15}{16}$. This value for ρ^2 is not far away from the QCDSR value $\rho_{ren}^2 = 0.7$. However, strictly speaking, we cannot make a comparison because we do not have computed the radiative corrections to our bound. The same comment applies to the functions $\chi_i(w)$ computed in the QCDSR approach. Therefore, our numerical results can only be indicative of what can be expected.

In Tables 2 and 3 we give the lower bounds on $-\ell_1(1)$ and $-\ell_2(1)$ and the corresponding lower bounds on $-\delta_{1/m^2}^{(+)}$, $-\delta_{1/m^2}^{(1)}$, using the same sets of parameters as in Table 1.

Parameters	$-\ell_1(1) \geq \text{OPE} + \frac{1}{2}^- + \frac{3}{2}^-$	$-\delta_{1/m^2}^{(+)} \geq \text{OPE} + \frac{1}{2}^- + \frac{3}{2}^-$
(i) $\rho^2 = 1$ $\sigma^2 = 1.5$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\ell_1(1) \geq$ $(0.075 + 0.288 + 0) \text{ GeV}^2$ $= 0.363 \text{ GeV}^2$	$-\delta_{1/m^2}^{(+)}$ $\geq 0.007 + 0.025 + 0$ $= 0.032$
(ii) $\rho^2 = 1$ $\sigma^2 = 1.5$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.15 \bar{\Lambda}$	$-\ell_1(1) \geq$ $(0.075 + 3.967 + 0) \text{ GeV}^2$ $= 4.042 \text{ GeV}^2$	$-\delta_{1/m^2}^{(+)}$ $\geq 0.007 + 0.345 + 0$ $= 0.351$
(iii) $\rho^2 = 0.9$ $\sigma^2 = 1.26$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\ell_1(1) \geq$ $(0.075 + 0.534 + 0) \text{ GeV}^2$ $= 0.609 \text{ GeV}^2$	$-\delta_{1/m^2}^{(+)}$ $\geq 0.007 + 0.046 + 0$ $= 0.053$
(iv) $\rho^2 = 0.8$ $\sigma^2 = 1.04$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\ell_1(1) \geq$ $(0.075 + 1.802 + 0) \text{ GeV}^2$ $= 1.877 \text{ GeV}^2$	$-\delta_{1/m^2}^{(+)}$ $\geq 0.007 + 0.156 + 0$ $= 0.163$
(v) $\rho^2 = 0.76$ $\sigma^2 = 0.96$ $\chi'_1(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi'_3(1) = 0.01 \bar{\Lambda}$	$-\ell_1(1) \geq$ $(0.075 + 9.484 + 0) \text{ GeV}^2$ $= 9.559 \text{ GeV}^2$	$-\delta_{1/m^2}^{(+)}$ $\geq 0.007 + 0.824 + 0$ $= 0.831$

Table 2. The lower bounds for $-\ell_1(1)$ (65) and $-\delta_{1/m^2}^{(+)}$ (68) for the same set of parameters and notations of Table 1.

Parameters	$-\ell_2(1) \geq \text{OPE} + \frac{1}{2}^- + \frac{3}{2}^-$	$-\delta_{1/m^2}^{(1)} \geq \text{OPE} + \frac{1}{2}^- + \frac{3}{2}^-$
(i) $\rho^2 = 1$ $\sigma^2 = 1.5$ $\chi_1'(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi_3'(1) = 0.01 \bar{\Lambda}$	$-\ell_2(1) \geq$ $(0.308 + 0.028 + 0.017) \text{ GeV}^2$ $= 0.353 \text{ GeV}^2$	$-\delta_{1/m^2}^{(1)}$ $\geq 0.027 + 0.002 + 0.001$ $= 0.030$
(ii) $\rho^2 = 1$ $\sigma^2 = 1.5$ $\chi_1'(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi_3'(1) = 0.15 \bar{\Lambda}$	$-\ell_2(1) \geq$ $(0.308 + 0.101 + 0.017) \text{ GeV}^2$ $= 0.426 \text{ GeV}^2$	$-\delta_{1/m^2}^{(1)}$ $\geq 0.027 + 0.009 + 0.001$ $= 0.037$
(iii) $\rho^2 = 0.9$ $\sigma^2 = 1.26$ $\chi_1'(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi_3'(1) = 0.01 \bar{\Lambda}$	$-\ell_2(1) \geq$ $(0.308 + 0.052 + 0.031) \text{ GeV}^2$ $= 0.391 \text{ GeV}^2$	$-\delta_{1/m^2}^{(1)}$ $\geq 0.027 + 0.004 + 0.003$ $= 0.034$
(iv) $\rho^2 = 0.8$ $\sigma^2 = 1.04$ $\chi_1'(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi_3'(1) = 0.01 \bar{\Lambda}$	$-\ell_2(1) \geq$ $(0.308 + 0.175 + 0.104) \text{ GeV}^2$ $= 0.587 \text{ GeV}^2$	$-\delta_{1/m^2}^{(1)}$ $\geq 0.027 + 0.015 + 0.009$ $= 0.051$
(v) $\rho^2 = 0.76$ $\sigma^2 = 0.96$ $\chi_1'(1) = 0.15 \bar{\Lambda}$ $\chi_2(1) = -0.05 \bar{\Lambda}$ $\chi_3'(1) = 0.01 \bar{\Lambda}$	$-\ell_2(1) \geq$ $(0.308 + 0.922 + 0.548) \text{ GeV}^2$ $= 1.778 \text{ GeV}^2$	$-\delta_{1/m^2}^{(1)}$ $\geq 0.027 + 0.080 + 0.048$ $= 0.155$

Table 3. The lower bounds for $-\ell_2(1)$ (66) and for $-\delta_{1/m^2}^{(1)}$ (69) for the same set of parameters and notations of Table 1.

Concerning Table 2, we observe that the bounds on $-\ell_1(1)$ and on $-\delta_{1/m^2}^{(+)}$ are now dominated by the $\frac{1}{2}^-$ contribution, and that the OPE contribution is small, contrarily to the bounds on $-\delta_{1/m^2}^{(A_1)}$, $-\ell_2(1)$ and $-\delta_{1/m^2}^{(1)}$. On the other hand, the sets of parameters (i), (iii) and (iv) are unphysical, since the lower bound on $-\ell_1(1)$ is very large. In Table 3 we observe that the bounds on $-\ell_2(1)$ and on $-\delta_{1/m^2}^{(1)}$ are always dominated by the OPE contribution, except when ρ^2 and σ^2 approach $\frac{3}{4}$ and $\frac{15}{16}$, like in the set of parameters (iii) and (iv).

Appendix C. Radiative corrections

The radiative corrections to the relations (51), (52) have been computed by Dorsten within HQET [6]. In this approach there are two parameters, namely the subtraction point μ and the cut-off Δ on the sums. To avoid large logarithms, one should take $2\Delta \cong \mu$.

Our relations (51), (52) are modified in the following way (formulas (34), (35) and (18) of [6]), adopting $2\Delta = \mu$ to simplify :

$$\sum_{n=0}^{n(\mu/2)} \left[\tau_{3/2}^{(2)(n)}(1) \right]^2 = \frac{4}{5} \sigma^2(\mu) - \rho^2(\mu) \left(1 + \frac{32\alpha_s}{27\pi} \right) + \frac{4}{5} \frac{193\alpha_s}{675\pi} \quad (\text{C.1})$$

$$\sum_{n>0}^{n(\mu/2)} \left[\xi^{(n)'}(1) \right]^2 = \frac{5}{3} \sigma^2(\mu) - \frac{4}{3} \rho^2(\mu) \left(1 + \frac{20\alpha_s}{27\pi} \right) - [\rho^2(\mu)]^2 + \frac{5}{3} \frac{148\alpha_s}{675\pi} \quad (\text{C.2})$$

Taking $\alpha_s = 0.3$ for $\mu = 2$ GeV, we obtain, keeping the algebraic factors as in (51), (52)

$$\sum_{n=0}^{n(\mu/2)} \left[\tau_{3/2}^{(2)(n)}(1) \right]^2 = \frac{4}{5} \sigma^2(\mu) - 1.11 \rho^2(\mu) + 0.02 \quad (\text{C.3})$$

$$\sum_{n>0}^{n(\mu/2)} \left[\xi^{(n)'}(1) \right]^2 = \frac{5}{3} \sigma^2(\mu) - \frac{4}{3} 1.07 \rho^2(\mu) - [\rho^2(\mu)]^2 + 0.03 \quad (\text{C.4})$$

We observe that the radiative corrections do not modify in a significant way our results, since the corrections are small. However, we must emphasize that, using (B.4) as a model of a relation between slope and curvature, the divergences of the denominators of the bounds are shifted away from $\rho^2 = \frac{3}{4}$ to slightly higher values.

References

- [1] N. Uraltsev, Phys. Lett. **B501**, 86 (2001) ; N. Uraltsev, hep-ph/0012336, J. Phys. **G27**, 1081 (2001).
- [2] J. D. Bjorken, invited talk at Les Rencontres de la Vallée d'Aoste, La Thuile, SLAC-PUB-5278, 1990 ; N. Isgur and M. Wise, Phys. Rev. **D43**, 819 (1991).
- [3] A. Le Yaouanc, L. Oliver and J.-C. Raynal, hep-ph/0210233, Phys. Rev. **D67**, 114009 (2003).

- [4] A. Le Yaouanc, L. Oliver and J.-C. Raynal, hep-ph/0210231, Phys. Lett. **B557**, 207 (2003).
- [5] A. Le Yaouanc, L. Oliver and J.-C. Raynal, hep-ph/0307197, Phys. Rev. **D69**, 094022 (2004).
- [6] M. Dorsten, hep-ph/0310025, Phys. Rev. **D70**, 096013 (2004).
- [7] F. Jugeau, A. Le Yaouanc, L. Oliver and J.-C. Raynal, hep-ph/0407176, Phys. Rev. **D71**, 054031 (2005).
- [8] A. F. Falk and M. Neubert, Phys. Rev. **D47**, 2965 (1993).
- [9] M. E. Luke, Phys. Lett. **B252**, 447 (1990).
- [10] A. Leibovich, Z. Ligeti, I. Steward and M. Wise, Phys. Rev. **D57**, 308 (1998).
- [11] A. Falk, Nucl. Phys. **B378**, 79 (1992).
- [12] I. I. Bigi et al., Phys. Rev. **D52**, 196 (1995).
- [13] M. Neubert, Phys. Rev. **D46**, 3914 (1992).
- [14] M. Neubert, Z. Ligeti and Y. Nir, Phys. Lett. **B301**, 101 (1993) ; Phys. Rev. **D47**, 5060 (1993).
- [15] M. Neubert, Phys. Reports **245**, 259 (1994).
- [16] B. Grinstein and Z. Ligeti, hep-ph/0111392, Phys. Lett. **B526**, 345 (2002).
- [17] N. Uraltsev, hep-ph/0312001, Phys. Lett. **B585**, 253 (2004).
- [18] V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal, Phys. Rev. **D56**, 5668 (1997).
- [19] I. Bigi, M. Shifman and N. Uraltsev, Ann. Rev. Nucl. Part. Sci. **47**, 591 (1997).
- [20] V. G. Lüth, Contribution to the 32nd International Conference on High Energy Physics (ICHEP 04), Beijing, August 2004, hep-ex/0411047, SLAC-PUB-10843, BABAR-PROC-04-117.
- [21] F. Jugeau, A. Le Yaouanc, L. Oliver and J.-C. Raynal, LPT Orsay 05-44.

Acknowledgement.

We are indebted to the EC contract HPRN-CT-2002-00311 (EURIDICE).