The tensor force in HQET and the semileptonic $\overline{\mathbf{B}}$ decay to excited vector mesons $\mathbf{D}\left(\frac{3}{2}^-, 1^-\right)$

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Abstract

We extend the formalism of Leibovich, Ligeti, Stewart and Wise in the $1/m_Q$ expansion of Heavy Quark Effective Theory for the $\overline{B}$ semileptonic decays into excited $D\left(\frac{3}{2}^+\right)$ mesons to the opposite parity states $D\left(\frac{3}{2}^-\right)$. For $D\left(\frac{3}{2}^+\right)$ the $1/m_Q$ current perturbation dominates over the leading term at zero recoil, while for $D\left(\frac{3}{2}^-\right)$ the $1/m_Q$ perturbation due to $\mathcal{L}_{mag}$ dominates also at zero recoil. We show that the corresponding $1/m_Q$ magnetic coupling is proportional to the mixing between the states $D\left(\frac{3}{2}^-, 1^-\right)$ and $D\left(\frac{1}{2}^-, 1^-\right)$ induced by the tensor force. We point out some subtleties that appear in this respect in HQET.

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In this note we deal with the relation between the tensor force and weak transitions at zero recoil in Heavy Quark Effective Theory (HQET), and some subtleties related to this question.

In the quark model, the tensor force between two quarks Q and $\overline{q}$ of unequal masses is given by the expression [1]

$$H_{tensor}^{Qq} = \frac{1}{m_Q m_q} U(r_{Qq}) \left[\frac{3(\mathbf{S}_Q \cdot \mathbf{r}_{Qq})(\mathbf{S}_q \cdot \mathbf{r}_{Qq})}{r_{Qq}^2} - (\mathbf{S}_Q \cdot \mathbf{S}_q) \right] \tag{1}$$

where, for one-gluon exchange, $U(r_{Qq})$ is positive and proportional to $\frac{\alpha_S}{r_{Qq}^3}$.

Expression (1) shows that in the case of heavy-light mesons ($m_Q \gg m_q$), H_{tensor}^{Qq} is proportional to $1/m_Q$. Therefore, the tensor force should appear in HQET at the first order in the $1/m_Q$ expansion.

In mesons, this force is responsible for mixing between the vector states $D\left(\frac{3}{2}^-, 1^-\right)$ and $D\left(\frac{1}{2}^-, 1^-\right)$. In the quark model, a straighforward calculation using (1) gives

$$< D\left(\tfrac{3}{2}^-, 1^-\right) | H_{tensor}^{Qq} | D\left(\tfrac{1}{2}^-, 1^-\right) > \sim \frac{< \psi_1 | U(r_{Qq}) | \psi_0 >}{m_Q m_q} \tag{2}$$

where $\psi_L(r_{Qq})$ $(L = 0, 1)$ are the radial wave functions of the ground state and of the first orbitally excited state.

In HQET, the mixing between $D\left(\frac{3}{2}^-, 1^-\right)$ and $D\left(\frac{1}{2}^-, 1^-\right)$, equivalent to (2), will be given by the matrix element

$$< D\left(\tfrac{3}{2}^-, 1^-\right)(v, \varepsilon) | \mathcal{L}_{mag,v}^{(c)}(0) | D\left(\tfrac{1}{2}^-, 1^-\right)(v, \varepsilon) > \tag{3}$$

with

$$\begin{aligned} \mathcal{L}_{mag,v}^{(Q)} &= \frac{1}{2m_Q} O_{mag,v}^{(Q)} \\ O_{mag,v}^{(Q)} &= \frac{g_s}{2}\, \overline{h}_v^{(Q)} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(Q)} \end{aligned} \tag{4}$$

The $\frac{3}{2}^-$ and the $\frac{1}{2}^-$ fields of spin 1 are given by [2] [3]

$$\begin{aligned} H_v\left(\tfrac{1}{2}^-, 1^-\right) &= P_+ \not{\varepsilon}_v \\ F_v^\sigma\left(\tfrac{3}{2}^-, 1^-\right) &= \sqrt{\frac{3}{2}} P_+ \varepsilon_v^\rho \left[g_\rho^\sigma - \frac{1}{3}\gamma_\rho(\gamma^\sigma + v^\sigma) \right] \end{aligned} \tag{5}$$

where $P_+ = \dfrac{1 + \not{v}}{2}$ and the last expression follows from

$$F_v^\sigma\left(\tfrac{3}{2}^+, 1^+\right) = -\sqrt{\frac{3}{2}} P_+ \varepsilon_v^\rho \gamma_5 \left[g_\rho^\sigma - \frac{1}{3}\gamma_\rho(\gamma^\sigma - v^\sigma) \right] \tag{6}$$

multiplying by $(-\gamma_5)$ on the right.

The mixing will then be given by the matrix element

$$\frac{< D\left(\frac{3}{2}^-,1^-\right)(v,\varepsilon)|\mathcal{L}^{(c)}_{mag,v}(0)|D\left(\frac{1}{2}^-,1^-\right)(v,\varepsilon)>}{\sqrt{m_{D_{3/2}}\ m_{D_{1/2}}}} = \frac{1}{2m_c}Tr\left[M^{(c)}_{\sigma\alpha\beta}\overline{F}^{\sigma}_{v}i\sigma^{\alpha\beta}H_v\right] \quad (7)$$

where

$$\overline{F}^{\sigma}_{v} = \gamma^0 F^{\sigma +}_{v}\gamma^0 = \sqrt{\frac{3}{2}}\left[g^{\sigma}_{\rho} - \frac{1}{3}\left(\gamma^{\sigma}+v^{\sigma}\right)\gamma_{\rho}\right]\varepsilon^{*\rho}_{v}P_+ \ . \quad (8)$$

Since $\gamma_\sigma\overline{F}^{\sigma}_{v} = 0$, the Dirac structure of $M^{(c)}_{\sigma\alpha\beta}$ could contain terms of the form $v_\sigma\gamma_\alpha\gamma_\beta$, $v_\sigma v_\alpha\gamma_\beta$, $g_{\sigma\alpha}v_\beta$ and $g_{\sigma\alpha}\gamma_\beta$. However, since the matrix element (7) is at zero recoil, one has

$$v_\sigma\overline{F}^{\sigma}_{v} = v_\beta P_+ i\sigma^{\alpha\beta}P_+ = 0 \quad (9)$$

and the only surviving term has the form $g_{\sigma\alpha}\gamma_\beta$. Therefore

$$M^{(c)}_{\sigma\alpha\beta} = \mu\ g_{\sigma\alpha}\gamma_\beta \quad (10)$$

and the mixing matrix element (7) is proportional to the coupling μ,

$$\frac{< D\left(\frac{3}{2}^-,1^-\right)(v,\varepsilon)|\mathcal{L}^{(c)}_{mag,v}(0)|D\left(\frac{1}{2}^-,1^-\right)(v,\varepsilon)>}{\sqrt{m_{D_{3/2}}\ m_{D_{1/2}}}} = -4\sqrt{\frac{2}{3}}\ \frac{\mu}{2m_c} \quad (11)$$

To see how in HQET the transition $B \to D\left(\frac{3}{2}^-,1^-\right)$ at first order in $1/m_Q$ at zero recoil is related to this mixing, let us use the formalism of Leibovich, Ligeti, Stewart and Wise [2], that was applied to $\frac{1}{2}^- \to \frac{3}{2}^+$ transitions. Leibovich et al have considered already the $B \to D\left(\frac{3}{2}^-,1^-\right)$ transition at zero recoil (Section IV of [2]). However, for our purpose, it will be instructive to use the general formalism, and consider here the matrix elements *at non-zero recoil* to compare both cases $\frac{1}{2}^- \to \frac{3}{2}^+$, $\frac{1}{2}^- \to \frac{3}{2}^-$. At the end of the calculation we will take the zero recoil limit for the $B \to D\left(\frac{3}{2}^-,1^-\right)$ transition.

The study of the semileptonic decay $\overline{B} \to D\left(\frac{3}{2}^-,1^-\right)\ell\overline{\nu}_\ell$ is not only of academic interest, since such orbitally excited state is expected at a mass $\lesssim 2.8$ GeV. However, we expect this state to be wide, since it can decay, among other modes, by S-wave into $D\left(\frac{3}{2}^+,1^+\right)+\pi$.

First, we must notice that at non-zero recoil, the $1/m_Q$ perturbations to the matrix elements

$$< D\left(\frac{3}{2}^-,1^-\right)(v')|[\overline{h}^{(c)}_{v'}\Gamma h^{(b)}_{v}](0)|B(v)> \quad (12)$$

are of three types : current perturbations and perturbations of the lagrangian $\mathcal{L}_{kin}$ and $\mathcal{L}_{mag}$. The leading order matrix element (12) vanishes at zero recoil [2], and the same happens for the $\mathcal{L}_{kin}$ perturbation, that behaves in powers of $(w-1)$ as the leading term.

Concerning the current perturbation matrix element, it also vanishes at zero recoil, as pointed out in ref. [2] (Section IV). This follows from the relation $\not{v} F_v^\sigma = F_v^\sigma = F_v^\sigma \not{v}$, that can be read from (5). This is at odds with the current perturbation matrix element for $B(v) \to D\left(\frac{3}{2}^+, 1^+\right)(v')$, that, in general, does not vanish at zero recoil [2].

Therefore, we will only consider matrix elements of the $\mathcal{L}_{mag}$ perturbation, but study in parallel the $B(v) \to D\left(\frac{3}{2}^\pm, 1^\pm\right)(v')$ transitions, to grasp the difference between both cases. As we will see below, due $\mathcal{L}_{mag}$, the transition $B(v) \to D\left(\frac{3}{2}^-, 1^-\right)(v')$, unlike $B(v) \to D\left(\frac{3}{2}^+, 1^+\right)(v')$, does not vanish at zero recoil.

Considering an arbitrary current $\overline{c}\Gamma b$, the relevant matrix elements are

$$\frac{1}{\sqrt{m_{D_{3/2}}\ m_B}} < D\left(\tfrac{3}{2}^\pm, 1^\pm\right)(v')|i\int dxT\left\{\mathcal{L}^{(c)}_{mag,v'}(x)[\overline{h}^{(c)}_{v'}\Gamma h^{(b)}_v](0)\right\}$$
$$+ i\int dxT\left\{\mathcal{L}^{(b)}_{mag,v}(x)[\overline{h}^{(c)}_{v'}\Gamma h^{(b)}_v](0)\right\}|B(v)>$$
$$= \frac{1}{2m_c}Tr\left[R^{(\pm)(c)}_{\sigma\alpha\beta}\overline{F}^{(\pm)\sigma}_{v'} i\sigma^{\alpha\beta}P'_+\Gamma H_v\right] + \frac{1}{2m_b}Tr\left[R^{(\pm)(b)}_{\sigma\alpha\beta}\overline{F}^{(\pm)\sigma}_{v'}\Gamma P_+ i\sigma^{\alpha\beta}H_v\right] \quad (13)$$

where the superindex $\pm$ in $F^{(\pm)\sigma}_{v'}$ indicates the parity of the state $D\left(\frac{3}{2}^\pm, 1^\pm\right)$, H_v corresponds to the pseudoscalar state :

$$H_v\left(\tfrac{1}{2}^-, 0^-\right) = P_+(-\gamma_5) \quad (14)$$

and $m_{D_{3/2}}$ is the mass of either the $\frac{3}{2}^+$ or the $\frac{3}{2}^-$ meson.

Using the conditions

$$F_v^\sigma v_\sigma = F_v^\sigma \gamma_\sigma = 0 \quad (15)$$

and the antisymmetry of $i\sigma^{\alpha\beta}$, that implies $P_+ v_\alpha i\sigma^{\alpha\beta}P_+ = P'_+ v'_\alpha i\sigma^{\alpha\beta}P'_+ = 0$, the parametrizations for $R^{(\pm)(c)}_{\sigma\alpha\beta}$, $R^{(\pm)(b)}_{\sigma\alpha\beta}$ follow :

$$R^{(\pm)(c)}_{\sigma\alpha\beta} = \eta_1^{(\pm)(c)} v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(\pm)(c)} v_\sigma v_\alpha\gamma_\beta + \eta_3^{(\pm)(c)} g_{\sigma\alpha}v_\beta + \eta_4^{(\pm)(c)} g_{\sigma\alpha}\gamma_\beta$$
$$R^{(\pm)(b)}_{\sigma\alpha\beta} = \eta_1^{(\pm)(b)} v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(\pm)(b)} v_\sigma v'_\alpha\gamma_\beta + \eta_3^{(\pm)(b)} g_{\sigma\alpha}v'_\beta + \eta_4^{(\pm)(b)} g_{\sigma\alpha}\gamma_\beta \quad (16)$$

where the η's depend on w. Other tensor structures give terms that, under the trace, are linearly dependent on these terms.

Owing to our remarks on the mixing (10) we have kept on purpose the term that has the tensor structure $g_{\sigma\alpha}\gamma_\beta$. As pointed out by Leibovich et al., this term is not independent from the others for $\frac{1}{2}^- \to \frac{3}{2}^+$ transitions. They correctly choose, for $\frac{1}{2}^- \to \frac{3}{2}^+$, the basis (omitting the (+) superindex)

$$
\begin{aligned}
R^{(c)}_{\sigma\alpha\beta} &= \eta_1^{(c)}\ v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(c)}\ v_\sigma v_\alpha\gamma_\beta + \eta_3^{(c)}\ g_{\sigma\alpha}v_\beta \\
R^{(b)}_{\sigma\alpha\beta} &= \eta_1^{(b)}\ v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(b)}\ v_\sigma v'_\alpha\gamma_\beta + \eta_3^{(b)}\ g_{\sigma\alpha}v'_\beta\ .
\end{aligned}
\tag{17}
$$

However, this is not the natural basis for $\frac{1}{2}^- \to \frac{3}{2}^-$ transitions. Indeed, from (13)-(16) one gets, for three terms of $R^{(\pm)(c)}_{\sigma\alpha\beta}$ in (16) the following trace identity, respectively for the transitions $\frac{1}{2}^- \to \frac{3}{2}^+$, $\frac{1}{2}^- \to \frac{3}{2}^-$,

$$
Tr\left\{\left[v_\sigma\gamma_\alpha\gamma_\beta + 2g_{\sigma\alpha}v_\beta + 2(1\pm w)g_{\sigma\alpha}\gamma_\beta\right]\overline{F}^{(\pm)\sigma}_{v'} i\sigma^{\alpha\beta}P'_+\Gamma H_v\right\} = 0\ . \tag{18}
$$

While the basis (17) is suitable for $\frac{1}{2}^- \to \frac{3}{2}^+$ transitions, this is not the case for $\frac{1}{2}^- \to \frac{3}{2}^-$, because $(1-w)g_{\sigma\alpha}\gamma_\beta$ *vanishes at zero recoil.*

The decay matrix elements of B mesons that are related to the mixing between $D\left(\frac{3}{2}^-, 1^-\right)$ and $D\left(\frac{1}{2}^-, 1^-\right)$, are the matrix element at zero recoil through the axial current :

$$
\frac{< D\left(\frac{3}{2}^-,1^-\right)(v,\varepsilon)|i\int dx T\left\{\mathcal{L}^{(c)}_{mag,v}(x)[\overline{h}^{(c)}_v\gamma_\mu\gamma_5 h^{(b)}_v](0)\right\}|B(v)) >}{\sqrt{m_{D_{3/2}}\ m_B}}
$$

$$
= \frac{1}{2m_c}Tr\left[R^{(-)(c)}_{\sigma\alpha\beta}\overline{F}^{(-)\sigma}_v i\sigma^{\alpha\beta}P_+\gamma_\mu\gamma_5 H_v\right] \tag{19}
$$

where we have used (13) and (16) and the contribution of $R^{(-)(b)}_{\sigma\alpha\beta}$ vanishes at zero recoil.

Using the decomposition (16) at zero recoil, we see that the terms $v_\sigma\gamma_\alpha\gamma_\beta$, $v_\sigma v_\alpha\gamma_\beta$, $g_{\sigma\alpha}v_\beta$ do not contribute because of the relations (9), and we are only left with the term $\eta_4^{(-)(c)}(1)g_{\sigma\alpha}\gamma_\beta$. One finds, after some Dirac algebra,

$$
\frac{< D\left(\frac{3}{2}^-,1^-\right)(v,\varepsilon)|i\int dx T\left\{\mathcal{L}^{(c)}_{mag,v}(x)[\overline{h}^{(c)}_v\gamma_\mu\gamma_5 h^{(b)}_v](0)\right\}|B(v) >}{\sqrt{m_{D_{3/2}}\ m_B}} = 2\sqrt{\frac{2}{3}}\frac{\eta_4^{(-)(c)}(1)}{2m_c}\varepsilon^*_\mu\ . \tag{20}
$$

On the other hand, one can insert intermediate states in the T-product and obtain

$$
\frac{< D\left(\frac{3}{2}^-,1^-\right)(v,\varepsilon)|i\int dx T\left\{\mathcal{L}^{(c)}_{mag,v}(x)[\overline{h}^{(c)}_v\gamma_\mu\gamma_5 h^{(b)}_v](0)\right\}|B(v) >}{\sqrt{m_{D_{3/2}}\ m_B}}
$$

$$= \varepsilon_\mu^* \frac{1}{\Delta E} \frac{< D\left(\frac{3}{2}^-, 1^-\right)(v,\varepsilon)|\mathcal{L}_{mag,v}^{(c)}(0)|D\left(\frac{1}{2}^-, 1^-\right)(v,\varepsilon) >}{\sqrt{m_{D_{3/2}} m_{D_{1/2}}}} \tag{21}$$

where ΔE is the level spacing $\Delta E = m_{D_{3/2}} - m_{D_{1/2}}$. Only the $(n = 0)$ ground state $D\left(\frac{1}{2}^-, 1^-\right)$ contributes to the sum because the matrix element is at zero recoil and one has $\xi(1) = 1$, $\xi^{(n)}(1) = 0$ $(n \neq 0)$. The factor in front of the r.h.s. of (21) comes from the calculation of the trace

$$Tr\left[\not{\varepsilon}^* P_+ \gamma_\mu \gamma_5 P_+ (-\gamma_5)\right] = -2\varepsilon_\mu^* \tag{22}$$

Therefore, comparing (20) and (21) one finds

$$2\sqrt{\frac{2}{3}} \frac{\eta_4^{(-)}}{2m_c} = \frac{1}{\Delta E} \frac{< D\left(\frac{3}{2}^-, 1^-\right)(v,\varepsilon)|\mathcal{L}_{mag,v}^{(c)}(0)|D\left(\frac{1}{2}^-, 1^-\right)(v,\varepsilon) >}{\sqrt{m_{D_{3/2}} m_{D_{1/2}}}} \tag{23}$$

In conclusion, we have shown that in HQET the transition between B and $D\left(\frac{3}{2}^-, 1^-\right)$ mesons through the axial current at zero recoil is proportional to the mixing between the states $D\left(\frac{3}{2}^-, 1^-\right)$ and $D\left(\frac{1}{2}^-, 1^-\right)$ due to the tensor force induced by $\mathcal{L}_{mag}$.

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