# Pentaquark Decay in QCD Sum Rules

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## Abstract

In a diquark-diquark-antiquark picture of the pentaquark we study the decay  $\Theta \to K^+ n$  within the framework of QCD sum rules. After evaluation of the relevant three-point function, we extract the coupling  $g_{\Theta nK}$  which is directly related to the pentaquark width. Restricting the decay diagrams to those with color exchange between the meson-like and baryon-like clusters reduces the coupling constant by a factor of four. Whereas a small decay width might be possible for a positive parity pentaquark, it seems difficult to explain the measured width for a pentaquark with negative parity.

#### I. INTRODUCTION

The experimental evidence for pentaquarks is now considered to be weak. However the strong statement that they do not exist can only be made after tests, which are presently being carried on [\[1\]](#page-9-0). Even if this state turns out to be unreal, its study generated many new theoretical ideas which can be valuable in the future [\[2\]](#page-9-1).

One of the most puzzling characteristics of the pentaquark is its extremely small width (much) below 10 MeV which poses a serious challenge to all theoretical models. Many explanations for this narrow width have been advanced [\[2](#page-9-1)]. In this work we calculate the pentaquark decay width within the framework of QCD sum rules (QCDSR) [\[3\]](#page-9-2). Several sum rule calculations have been performed for the mass of the pentaquark containing a strange quark [\[4,](#page-9-3) [5,](#page-9-4) [6,](#page-10-0) [7,](#page-10-1) [8\]](#page-10-2). These calculations are based on two-point functions with different interpolating currents. Surprisingly, all these determinations give similar masses with reasonable values. A common problem of all determinations is the large continuum contribution which has its origin in the high dimension of the interpolating currents and results in a large dependence on the continuum threshold. Another problem is the irregular behavior of the operator product expansion (OPE), which is dominated by higher dimension operators and not by the perturbative term as it should be.

Here we present a sum rule determination for the decay width based on a three-point function for the decay  $\Theta \to nK^+$ . In this way we can extract the coupling  $g_{\Theta nK}$  which is directly related to the pentaquark width. To describe the pentaquark we use the diquarkdiquark-antiquark model with one scalar and one pseudoscalar diquark in a relative S-wave.

In [\[9,](#page-10-3) [10\]](#page-10-4) it has been argued that such a small decay width can only be explained if the pentaquark is a genuine 5-quark state, i.e., it contains no color singlet meson-baryon contributions and thus color exchange is necessary for the decay. The analysis presented both in [\[9\]](#page-10-3) and in [\[10\]](#page-10-4) is only qualitative. The narrowness of the pentaquark width can then be attributed to the non-trivial color structure of the pentaquark which requires the exchange of, at least, one gluon. In this work we will also test quantitatively the hypothesis put forward in [\[9,](#page-10-3) [10](#page-10-4)] to see whether this mechanism is sufficient to explain the small width.

## II. CORRELATION FUNCTION

The investigation of the pentaquark decay width requires a three-point function which we define as

<span id="page-2-0"></span>
$$
\Gamma(p, p') = \int d^4x \, d^4y \, e^{-iqy} \, e^{ip \cdot x} \, \Gamma(x, y) \,,
$$
  

$$
\Gamma(x, y) = \langle 0|T\{\eta_N(x)j_K(y)\bar{\eta}_{\Theta}(0)\}|0\rangle \,,
$$
 (1)

where  $\eta_N$ ,  $j_K$  and  $\eta_\Theta$  are the interpolating fields associated with neutron, kaon and  $\Theta$ , respectively [\[11\]](#page-10-5).

We next consider the expression [\(1\)](#page-2-0) in terms of hadronic degrees of freedom and write the phenomenological side of the sum rule. Treating the kaon as a pseudoscalar particle, the interaction between the three hadrons is described by the following Lagrangian density:

<span id="page-2-1"></span>
$$
\mathcal{L} = ig_{\Theta nK} \bar{\Theta} \gamma_5 Kn \quad \text{for } P = +
$$
  

$$
\mathcal{L} = ig_{\Theta nK} \bar{\Theta} Kn \quad \text{for } P = -
$$
 (2)

Writing the correlation function [\(1\)](#page-2-0) in momentum space and inserting complete sets of hadronic states we obtain an expression which depends on the following matrix elements:

$$
-i V(p, p') = \langle n(p', s') | \Theta(p, s) K(q) \rangle,
$$
  
\n
$$
\langle 0 | \eta_N | n(p', s') \rangle = \lambda_N u^{s'}(p'),
$$
  
\n
$$
\langle K(q) | j_K | 0 \rangle = \lambda_K,
$$
  
\n
$$
\langle \Theta(p, s) | \bar{\eta}_{\Theta} | 0 \rangle = \lambda_{\Theta} \bar{u}^s(p) \qquad \text{for } P = +
$$
  
\n
$$
\langle \Theta(p, s) | \bar{\eta}_{\Theta} | 0 \rangle = -\lambda_{\Theta} \bar{u}^s(p) \gamma_5 \quad \text{for } P = -
$$
\n(3)

Using the simple Feynman rules derived from  $(2)$  we can rewrite  $V(p, p')$  as

$$
V(p, p') = -g_{\Theta nK} \bar{u}^{s'}(p') \gamma_5 u^s(p) \qquad P = +
$$
  

$$
V(p, p') = -g_{\Theta nK} \bar{u}^{s'}(p') u^s(p) \qquad P = -
$$
 (4)

The coupling constants  $\lambda_N$  and  $\lambda_\Theta$  can be determined from the QCD sum rules of the corresponding two-point functions.  $\lambda_K$  is related to the kaon decay constant through

<span id="page-2-2"></span>
$$
\lambda_K = \frac{f_K m_K^2}{m_u + m_s} \,. \tag{5}
$$

<span id="page-3-0"></span>Combining the expressions above we arrive at

$$
\Gamma_{phen} = \frac{-g_{\Theta nK} \lambda_{\Theta} \lambda_N \lambda_K}{(p^2 - m_N^2)(q^2 - m_{K^+}^2)(p^2 - m_{\Theta}^2)}
$$
  
  $\times \Gamma_E + \text{continuum}$  (6)

with

$$
\Gamma_E = \sigma^{\mu\nu}\gamma_5 q_\mu p'_\nu - im_N \not q \gamma_5 \n+ i(m_N \mp m_\Theta) \not p' \gamma_5 \n+ i\gamma_5(p'^2 \mp m_\Theta m_N - qp') \tag{7}
$$

We shall work with the  $\sigma^{\mu\nu}\gamma_5 q_\mu p'_\nu$  $\mathcal{L}_{\nu}$  structure because, as it was shown in [\[12\]](#page-10-6), this structure gives results which are less sensitive to the coupling scheme on the phenomenological side, i.e., to the choice of a pseudoscalar or pseudovector coupling between the kaon and the baryons.

We now come back to [\(1\)](#page-2-0) and write the interpolating fields in terms of quark degrees of freedom as

$$
j_K(y) = \bar{s}(y)i\gamma_5 u(y),
$$
  
\n
$$
\eta_N(x) = \epsilon^{abc} (d_a^T(x) C \gamma_\mu d_b(x)) \gamma_5 \gamma^\mu u_c(x),
$$
  
\n
$$
\bar{\eta}_{\Theta}(0) = -\epsilon^{abc} \epsilon^{def} \epsilon^{cfg} s_g^T(0) C
$$
  
\n
$$
\times [\bar{d}_{\epsilon}(0) \gamma_5 C \bar{u}_d^T(0)][\bar{d}_b(0) C \bar{u}_a^T(0)].
$$
\n(8)

The pentaquark current above (proposed in [\[6](#page-10-0)]) contains a pseudoscalar and a scalar diquark. With these diquarks the two point function might receive a significant contribution from instantons. In [\[13](#page-10-7)] we have studied a situation in which these instanton contributions affected the two-point function but gave a negligible contribution to the three-point function. Moreover, in [\[14](#page-10-8)] we have observed that instantons give a negligible contribution to heavy baryon weak decays. Motivated by these results, in this first calculation we shall neglect instantons.

Inserting the currents into [\(1\)](#page-2-0), the resulting expression involves the quark propagator in the presence of quark and gluon condensates, which is known from previous studies. Using it we arrive at a final complicated expression for the correlator, which is represented schematically by the sum of the diagrams of Fig. 1.

Let us consider the phenomenological side [\(6\)](#page-3-0) and, following [\[15](#page-10-9)], rewrite it generically as:

$$
\Gamma(q^2, p^2, p'^2) = \Gamma_{pp} + \Gamma_{pc1} + \Gamma_{pc2} + \Gamma_{cc}
$$
\n(9)

where  $\Gamma_{pp}(q^2, p^2, p'^2)$  stands for the pole-pole part and reads

$$
\Gamma_{pp} = \frac{-g_{\Theta nK} \lambda_{\Theta} \lambda_N \lambda_K}{(p^2 - m_{\Theta}^2)(p^2 - m_N^2)(q^2 - m_K^2)}
$$
(10)

The continuum-continuum term  $\Gamma_{cc}$  can be obtained as usual, with the assumption of quark-hadron duality [\[11](#page-10-5)].

The pole-continuum transition terms are contained in  $\Gamma_{pc1}$  and  $\Gamma_{pc2}$ . They can be explicitly written as a double dispersion integral:

$$
\Gamma_{pc1} = \int_{m_{K^*}^2}^{\infty} \frac{b_1(u, p^2) du}{(m_N^2 - p'^2)(u - q^2)},
$$
\n
$$
\Gamma_{pc2} = \int_{m_{N^*}^2}^{\infty} \frac{b_2(s, p^2) ds}{(m_K^2 - q^2)(s - p'^2)}.
$$
\n(11)

Since there is no theoretical tool to calculate the unknown functions  $b_1(u, p^2)$  and  $b_2(s, p^2)$ explicitly, one has to employ a parametrization for these terms. We will use two different parametrizations: one with a continuous function for the Θ and one where the pole term is singled out.

<span id="page-4-0"></span>We shall assume that the functions  $b_1$  and  $b_2$  have the following form:

$$
b_1(u, p^2) = \tilde{b_1}(u) \int_{m_{\Theta}^2}^{\infty} d\omega \frac{b_1(\omega)}{\omega - p^2}
$$
  

$$
b_2(s, p^2) = \tilde{b_2}(s) \int_{m_{\Theta}^2}^{\infty} d\omega \frac{b_2(\omega)}{\omega - p^2}
$$
 (12)

with continuous functions  $b_{1,2}(w)$ , starting from  $m_{\Theta}^2$ . This is our **parametrization A**. The functions  $\tilde{b}_1(u)$  and  $\tilde{b}_2(s)$  describe the excitation spectra of the kaon and the nucleon, respectively. After Borel transform, the pole-continuum term contains one unknown constant factor which can be determined from the sum rules.

In order to investigate the role played by the  $\Theta$  continuum, we shall now explicitly force the phenomenological side to contain only the pole part of the Θ, both in the pole-pole term and in the pole-continuum terms. This can formally be done by choosing  $b_1(\omega) = b_2(\omega) =$  $\delta(\omega - m_\Theta^2)$  in [\(12\)](#page-4-0) and the functions then read:

$$
b_1(u, p^2) = \frac{\tilde{b}_1(u)}{m_{\Theta}^2 - p^2},
$$
  
\n
$$
b_2(s, p^2) = \frac{\tilde{b}_2(s)}{m_{\Theta}^2 - p^2}.
$$
\n(13)

This is our **parametrization B**. In this case we have the  $\Theta$  in the ground state. Again, in the final expressions this gives additional constants which can be calculated.

#### III. SUM RULES

The sum rule may be written identifying the phenomenological and theoretical descriptions of the correlation function. As mentioned above, we shall work with the  $\sigma^{\mu\nu}\gamma_5 q_\mu p'_\nu$ ν structure. In the case of the three-point function considered here, there are two independent momenta and we may perform either a single or a double Borel transform. We first consider the choice:

$$
(I) \tq^2 = 0 \t p^2 = p'^2 \t\t(14)
$$

and perform a single Borel transform:  $p^2 = -P^2$  and  $P^2 \to M^2$ . In this case we take  $m_K^2 \simeq 0$  and single out the 1/q<sup>2</sup>-terms. The second choice is:

(II) 
$$
q^2 \neq 0
$$
  $p^2 = p'^2$ . (15)

Here we perform two Borel transforms:  $p^2 = -P^2$  and  $P^2 \to M^2$  and also  $q^2 = -Q^2$  and  $Q^2 \rightarrow M'^2$ . We have also considered the choice  $q^2 = p^2 = p'^2 = -P^2$ , performing one single Borel transform  $(P^2 \to M^2)$ . However, in the present calculation we were not able to find a stable sum rule. Introducing the notation  $G = -g_{\Theta nK} \lambda_{\Theta} \lambda_N \lambda_K$  and using (I) and (II) we obtain the following sum rules:

<span id="page-5-0"></span>Method I:

$$
\Gamma_{pp}(M^2) + \Gamma_{pc2}(M^2) = \int_0^{s_0} ds \, \rho_{th}(s) \, e^{-s/M^2} \tag{16}
$$

with

$$
\Gamma_{pp}(M^2) = G \frac{e^{-m_{\Theta}^2/M^2} - e^{-m_N^2/M^2}}{m_{\Theta}^2 - m_N^2}
$$
\n(17)

and for the pole-continuum part we obtain

$$
\Gamma_{pc2}(M^2) = A e^{-m_{N^*}^2/M^2}
$$
param. A  

$$
\Gamma_{pc2}(M^2) = A e^{-m_{\Theta}^2/M^2}
$$
param. B (18)

In both parametrizations the term  $\Gamma_{pc1}$  is exponentially suppressed and, as discussed in [\[15\]](#page-10-9), has been neglected. A is an unknown constant and can be determined from the sum rules.

## <span id="page-5-1"></span>Method II

$$
\Gamma_{pp}(M^2, M^{'2}) + \Gamma_{pc2}(M^2, M^{'2}) = \tag{19}
$$

$$
\int_0^{u_0} du \int_0^{s_0} ds \, \rho_{th}(s, u) \, e^{-s/M^2} \, e^{-u/M'^2} \tag{20}
$$

with

$$
\Gamma_{pp} = G e^{-m_K^2/M'^2} \frac{e^{-m_\Theta^2/M^2} - e^{-m_N^2/M^2}}{m_\Theta^2 - m_N^2}
$$
\n(21)

and with

$$
\Gamma_{pc2} = A \, e^{-m_K^2 / M'^2} \, e^{-m_{N^*}^2 / M^2} \tag{22}
$$

for parametrization A and

$$
\Gamma_{pc2} = A \, e^{-m_K^2 / M'^2} \, e^{-m_\Theta^2 / M^2} \tag{23}
$$

for parametrization B. Also in this case  $\Gamma_{pc1}$  is exponentially suppressed. In the above expressions  $\rho_{th}$  is the double discontinuity computed directly from the theoretical (OPE) description of the correlation function (see [\[11\]](#page-10-5) for details and also [\[16\]](#page-10-10)) and  $s_0$  is the continuum threshold of the nucleon defined as  $s_0 = (m_N + \Delta_N)^2$ .

## IV. RESULTS

The hadronic masses are  $m_N = 938$  MeV,  $m_{N^*} = 1440$  MeV,  $m_K = 493$  MeV and  $m_{\Theta} = 1540$  MeV. For each of the sum rules above (Eqs. [\(16\)](#page-5-0) and [\(19\)](#page-5-1)) we can take the derivative with respect to  $1/M^2$  and in this way obtain a second sum rule. In each case we have thus a system of two equations and two unknowns  $(G \text{ and } A)$  which can then be easily solved.

In the numerical analysis of the sum rules we use the following values for the condensates:  $\langle \overline{q}q \rangle = - (0.23 \pm 0.02)^3 \text{ GeV}^3, \langle \overline{s}s \rangle = 0.8 \langle \overline{q}q \rangle, \langle \overline{s}g_s \sigma. \text{G} s \rangle = m_0^2 \langle \overline{s}s \rangle$  with  $m_0^2 = 0.8 \text{ GeV}^2$ and  $\langle g_s^2 G^2 \rangle = 0.5 \text{ GeV}^4$ . The gluon condensate has a large error of about a factor 2, but its influence on the analysis is relatively small. The couplings constants  $\lambda_N$  and  $\lambda_{\Theta}$  are taken from the corresponding two-point functions:

$$
\lambda_N = (2.4 \pm 0.2) \times 10^{-2} \,\text{GeV}^3 \tag{24}
$$

$$
\lambda_{\Theta} = (2.4 \pm 0.3) \times 10^{-5} \,\text{GeV}^6 \tag{25}
$$

The coupling  $\lambda_K$  is obtained from [\(5\)](#page-2-2) with  $f_K = 160$  MeV,  $m_s = 100$  MeV and  $m_u = 5$ MeV:

$$
\lambda_K = 0.37 \,\text{GeV}^2 \,. \tag{26}
$$

In Fig. 1, among these OPE diagrams there are two distinct subsets. In the first (from 1a to 1g) there is no gluon line connecting the petals and therefore no color exchange. A diagram of this type we call color-disconnected. In the second subset of diagrams (1h, 1i and 1j) we have color exchange. If there is no color exchange, the final state containing two color singlets was already present in the initial state, before the decay, as noticed in [\[8\]](#page-10-2). In this case the pentaquark had a component similar to a  $K - n$  molecule. In the second case the pentaquark was a genuine 5-quark state with a non-trivial color structure. We may call this type of diagram a color-connected (CC) one. In our analysis we write sum rules for both cases: all diagrams and only color-connected. The former case is standard in QCDSR calculations and therefore we shall omit details and present only the results. The latter case implies that the pentaquark is a genuine 5-quark state and the evaluation of  $g_{\Theta nK}$  will thus be based only on the CC diagrams. We shall work in the Borel window given by  $1 \text{ GeV}^2 < M^2$ ,  $M'^2 < 1.5 \text{ GeV}^2$ . Since the strange mass is small, the dominating diagram is Fig. 1b of dimension three with one quark condensate. In the range considered, the dimension 5 condensates are substantially suppressed compared to this term.

We have found out that the contribution from the pole-continuum part is of a similar size as the pole part. For lower values of  $M^2$  around 1 GeV<sup>2</sup>, the pole contribution dominates, however, for larger values of  $M^2$  the importance of the pole-continuum contribution grows and eventually becomes larger than the pole part. This is an additional reason to restrict the analysis to small values for the Borel parameters.

We have evaluated the sum rules for the coupling constant computed with all diagrams of Fig. 1 and we have found that they are very stable. We give the values of the coupling extracted at  $M^2 = 1.5 \text{ GeV}^2$  and  $M'^2 = 1 \text{ GeV}^2$  in Table I. In what follows we shall present our results for the coupling constant  $g_{\Theta nK}$  obtained with the color connected diagrams only. In Fig. 2 we show the coupling, given by the solution of the sum rule I A [\(16\)](#page-5-0), as a function of the Borel mass squared  $M^2$ . Different lines show different values of the continuum threshold  $\Delta_N$ . As it can be seen,  $g_{\Theta nK}$  is remarkably stable with respect to variations both in  $M^2$  and in  $\Delta_N$ . In Fig. 3 we show the coupling obtained with the sum rule II A [\(19\)](#page-5-1). We find again fairly stable results which are very weakly dependent on the continuum threshold. In Fig. 4 we show the results of the sum rule I B. In Fig. 5 we present the result of the sum rule II B. The meaning of the different lines is the same as in the previous figures. The results are similar to the cases before.

In Table I we present a summary of our results for  $g_{\Theta nK}$  giving emphasis to the difference between the results obtained with all diagrams and with only the color-connected ones. For the continuum thresholds we have employed  $\Delta_N = \Delta_K = 0.5$  GeV.

|      |      | case $ g_{\Theta nK} $ (CC) $ g_{\Theta nK} $ (all diagrams) |
|------|------|--|
| I A  | 0.71 | 2.59   |
| II A | 0.82 | 3.59   |
| I B  | 0.84 | 3.24   |
| ПB   | 0.96 | 4.48   |

Table I: $g_{\Theta nK}$  for various cases

For our final value of  $g_{\Theta nK}$  we take an average of the sum rules I A - II B. It is interesting to observe that the influence of the continuum threshold is relatively small, especially when compared to the corresponding two-point functions.

Considering the uncertainties in the continuum thresholds, in the coupling constants  $\lambda_{K,N,\Theta}$  and in the quark condensate we get an uncertainty of about 50%. Our final result then reads:

<span id="page-8-0"></span>
$$
|g_{\Theta nK}|(\text{all diagrams}) = 3.48 \pm 1.8,
$$
  

$$
|g_{\Theta nK}|(\text{CC}) = 0.83 \pm 0.42.
$$
 (27)

Including all diagrams, the prediction for  $\Gamma_{\Theta}$  is then 13 MeV (652 MeV) for a positive (negative) parity pentaquark. In the CC case we get a width of 0.75 MeV (37 MeV) for a positive (negative) parity pentaquark. The measured upper limit of the width is around 5-10 MeV both in the Kn channel (considered here) and in the Kp channel.

We see that it is very difficult to obtain the measured decay width for a negative parity pentaquark.

#### V. SUMMARY AND CONCLUSIONS

We have presented a QCD sum rule study of the decay of the  $\Theta^+$  pentaquark using a diquark-diquark-antiquark scheme with one scalar and one pseudoscalar diquark. Based on the evaluation of the relevant three-point function, we have computed the coupling constant  $g_{\Theta nK}$ . In the operator product expansion we have included all diagrams up to dimension 5. In this particular type of sum rule a complication arises from the pole-continuum transitions which are not exponentially suppressed after Borel transformation and must be explicitly included. The analysis was made for two different pole-continuum parametrizations and in two different evaluation schemes. The results are consistent with each other. In addition, we have tested the ideas presented in [\[9,](#page-10-3) [10](#page-10-4)] by including only diagrams with color exchange. Our final results are given in eq. [\(27\)](#page-8-0).

We find that for a positive parity pentaquark a width much smaller than 10 MeV would indicate a pentaquark which contains no color-singlet meson-baryon contribution. For a negative parity pentaquark, even under the assumption that it is a genuine 5-quark state, we can not explain the observed narrow width of the Θ.

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FIG. 1: The main diagrams which contribute to the theoretical side of the sum rule in the relevant structure. a) - g) are the color-disconnected diagrams, whereas h) - j) are the color-connected diagrams. The cross indicates the insertion of the strange mass.



FIG. 2:  $|g_{\Theta nK}|$  in case I A with three different continuum threshold parameters. Solid line:  $\Delta_N=0.5$  GeV, dotted line:  $\Delta_N=0.4$  GeV, dash-dotted line:  $\Delta_N=0.6$  GeV.



FIG. 3:  $|g_{\Theta nK}|$  in case II A. Solid line:  $\Delta_N = 0.5$  GeV. Dotted line:  $\Delta_N = 0.4$  GeV. Dashed line:  $\Delta_N = 0.6 \text{ GeV}. M'^2 = 1 \text{ GeV}^2.$ 



FIG. 4:  $|g_{\Theta nK}|$  in case I B with three different continuum threshold parameters. Solid line:  $\Delta_N$  = 0.5 GeV, dotted line: $\Delta_N=$  0.4 GeV, dash-dotted line:  $\Delta_N=$  0.6 GeV.



FIG. 5:  $|g_{\Theta nK}|$  in case II B. Solid line:  $\Delta_N = 0.5$  GeV. Dotted line:  $\Delta_N = 0.4$  GeV. Dashed line:  $\Delta_N = 0.6 \text{ GeV}. M'^2 = 1 \text{ GeV}^2.$