

Enhancement of the hadronic b quark decays*

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A class of previously unknown strong-interaction corrections is found to enhance the rate of non-leptonic decays of the b quark by 5-8 percent. This effect decreases the predicted fraction of semi-leptonic decays and brings it into fair agreement with experimental results. As well as solving a long-standing puzzle of measurements disagreeing with the Standard Model prediction, our work suggests a way for future precise studies of non-leptonic b quark decays and their application to searching for “new physics.”

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The b quark is a relatively long-lived particle. As the lighter of the third-generation quarks, it can decay only into quarks from other generations, to which its couplings are suppressed in the Standard Model. For more than a decade, theoretical predictions disagreed with measurements of the relative probabilities of semi-leptonic and non-leptonic b quark decays. It has been speculated that this discrepancy may be due to some exotic “new physics” mechanism, not affected by the Standard Model suppression [1].

The main decays of the b quark are the semi-leptonic channels $b \rightarrow c l \bar{\nu}_l$ (with l being an electron, a muon, or a τ lepton), and the hadronic channels $b \rightarrow c \bar{u} d'$ and $b \rightarrow c \bar{c} s'$, as shown in Fig. 1 (d' and s' denote approximate flavor eigenstates coupling to u and c , respectively). In addition, b can also decay into an u quark, $b \rightarrow u X$, or through one of the radiative channels such as $b \rightarrow s \gamma$, which arise due to quantum loop effects. These decays are rare and will be ignored in this paper. The probability of a semi-leptonic decay such as $b \rightarrow c e \bar{\nu}_e$ is known as the semi-leptonic

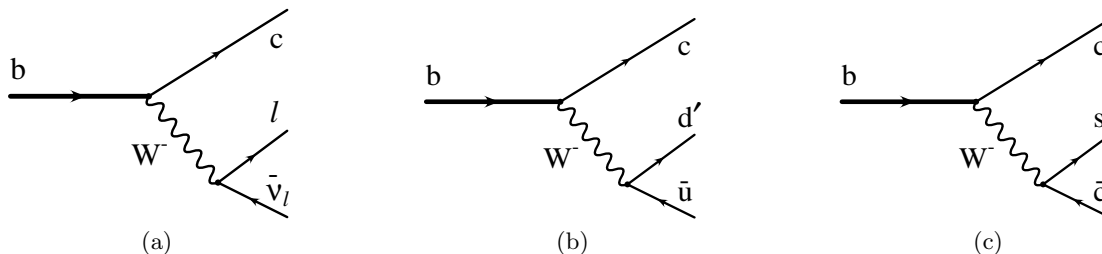


FIG. 1: Main decay channels of the quark b : (a) semi-leptonic, (b) non-leptonic with light quarks, (c) non-leptonic with an extra charm quark. d' and s' denote the approximate flavor eigenstates: combinations of d and s quarks which couple to u and c in weak decays.

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branching ratio B_{SL} . It can be expressed in terms of the partial decay rates Γ ,

$$B_{\text{SL}}(b \rightarrow ce\bar{\nu}_e) = \frac{\Gamma(b \rightarrow ce\bar{\nu}_e)}{\sum_{l=e,\mu,\tau} \Gamma(b \rightarrow cl\bar{\nu}_l) + \Gamma(b \rightarrow c\bar{u}d') + \Gamma(b \rightarrow c\bar{c}s') + \Gamma_{\text{rare}}} \quad (1)$$

The latest published measurement [2] found

$$B_{\text{SL}}^{\text{exp}}(B \rightarrow Xe^+\nu_e) = (10.91 \pm 0.26)\%. \quad (2)$$

Note that this experimental result refers to a B meson rather than to the b quark; the difference between their branching ratios is expected to be small, as is discussed below.

Theoretical predictions of B_{SL} have been significantly higher than the measured values. Ref. [1] thoroughly analysed possible non-perturbative effects and all perturbative ones known at that time [3], and concluded that B_{SL} should be not less than 12.5 percent. An important point made in that study was that the non-perturbative effects are small, in particular those that differentiate between the free quark and the meson decay rate.

Subsequently it was found [4, 5] that perturbative corrections enhance the decay channel $b \rightarrow c\bar{c}s'$ by about 30 percent, due to the slowness of the massive charm quarks in the final state. Such enhancement of the hadronic rate decreases the theoretical lower limit on B_{SL} by about one percentage point. At the time of its publication, such explanation was controversial [6], since it would increase the average number n_c of charm quarks produced in b decays, which seemed to contradict the data (see also a discussion in [7] which in addition studied spectator effects). Very recently a new measurement [8] found a larger value of n_c , so the enhancement of $b \rightarrow c\bar{c}s'$ is no longer out of question.

In this paper the value taken from a recent review [9] is adopted as a reference point for the theoretical prediction,

$$B_{\text{SL}}^{\text{theory}}(B) > 11.5\%. \quad (3)$$

This number is based on the analysis of perturbative and non-perturbative effects of [1] and an enhancement of the $b \rightarrow c\bar{c}s'$ channel. Despite the enhancement of $\Gamma(b \rightarrow c\bar{c}s')$, it still exceeds the experimental value, Eq. (2), by about 2.3 standard deviations (2.3σ). Assuming that the experimental number remains constant, what effects can change the theoretical limit and bring it into agreement with observations? First of all, whatever affects the semi-leptonic rate in Eq. (1), has a very similar impact on the non-leptonic rate and thus cancels in the ratio. Thus, the most important effects controlling B_{SL} are the corrections to the non-leptonic decays.

In order to bring theory and experiment into agreement within one sigma, the theoretical value of B_{SL} should decrease by a third of a percentage point. To get a rough estimate of the required change of the non-leptonic rate, consider a limit in which all leptons and light quarks are massless, and neglect all interactions among decay products. Then Eq. (1) gives $B_{\text{SL}} \simeq \frac{1}{3+6} = 11.1\%$. The numbers in the denominator account for the three semi-leptonic channels and the two non-leptonic channels; the widths of the latter are enhanced by a color factor of three. In order to lower this branching ratio by one-third of a percentage point, the non-leptonic width should increase by 4.6 percent. In this paper, a class of strong-interaction effects — that previously could not be evaluated — are found to provide just such an enhancement.

Since it has been established that the size of B_{SL} is controlled by the perturbative QCD corrections [1], we briefly review what diagrams describe those effects. To this end it is convenient to consider the imaginary parts of the forward scattering amplitudes, such as those in Fig. 2. There are two separate quark lines: that continuing from the incoming b quark, and a closed loop containing only lighter quarks. An analysis of B_{SL} is simplified by the almost exact cancellation of corrections due to gluon exchanges on the bc line, like the diagram in Fig. 2(a). These corrections are common to semi- and non-leptonic corrections and cancel in the ratio (1) (up to residual effects due to small phase space differences). Exchanges of gluons between the light quarks, such as Fig. 2(b), have already been studied in great detail in the context of τ lepton decays. Thus, the only class specific to our problem is the interaction between both quark lines, Fig. 2(c).

Since the light quark pair is produced as a color singlet, at least two gluons have to be exchanged in an interaction between the two quark lines. Such corrections arise only in the second order in the strong coupling constant α_s , and they are similar to electrodynamic (QED) interactions (no diagrams with non-abelian three-gluon vertices contribute at this order). Fig. 2(c) shows one of the twelve types of diagrams, which must be calculated.

The evaluation of diagrams of type (c) has been considered a daunting challenge. Indeed, they are four-loop diagrams that depend on two masses, m_W and m_b . We first explain how the hierarchy of mass scales, $m_b \ll m_W$, somewhat simplifies the task and then describe how the four-loop diagrams are computed.

Since the b -quark is much lighter than the W boson, only the leading term in the expansion in m_b/m_W is needed. In the language inspired by the asymptotic operation [10], two characteristic virtualities q^2 for each of the two gluons should be considered: their scales are m_W^2 and m_b^2 . The hard gluons ($q^2 \sim m_W^2$) modify the Wilson coefficients of the four-fermion effective operators. The soft ones ($q^2 \sim m_b^2$) correct their matrix elements.

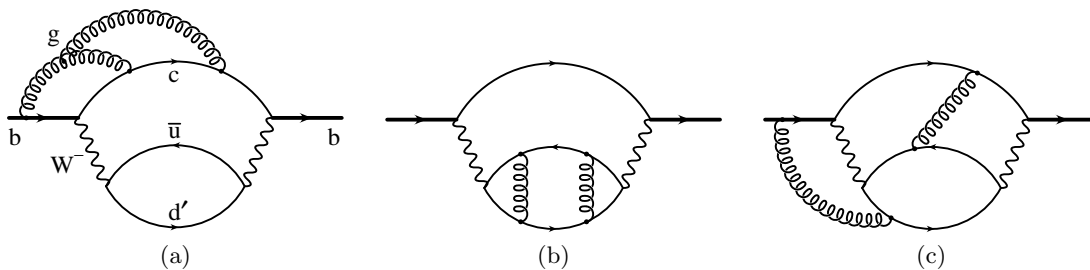


FIG. 2: Examples of the three types of QCD corrections to the *squares* of the non-leptonic decay amplitudes: (a) corrections on the heavy quark line, (b) on the light quark line, (c) between the lines. The solid and wavy lines corresponds to the same fields as in Fig. 1(b), and the springs denote gluons.

The soft and hard effects are not separately finite, hence the final result contains logarithms of the ratio of the two scales, $\ln^2 \frac{m_W}{m_b}$ and $\ln \frac{m_W}{m_b}$. In the past, these logarithmic terms were evaluated and even summed to all orders using the renormalization group equation [11]. However, the numerical value of the logarithm is not very large, $\ln \frac{m_W}{m_b} \simeq 2.8$, so the L^2 and L terms may not be sufficient for a reliable prediction of the non-leptonic decay rate. The importance of finding the non-logarithmic part, provided in this paper, has been stressed repeatedly.

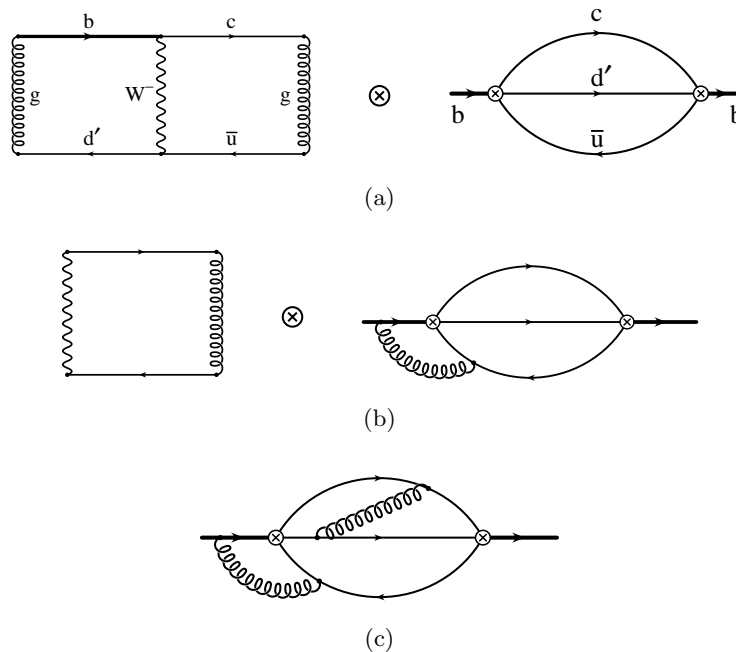


FIG. 3: Examples of factorized contributions to the four-loop diagram of Fig. 2(c) from various regions of virtuality of the two gluons: (a) hard-hard, (b) hard-soft, (c) soft-soft. In (a,b) there is also, not shown, a second W boson propagator, which reduces to an overall factor $1/m_W^2$. In (c) there are two such factors. The circles and crosses in the soft subgraphs indicate places from which hard subgraphs have been taken.

Contributions in which at least one gluon is hard are relatively easy to evaluate. The hard momentum flows in a closed loop (since all external momenta are soft), and this closed subgraph can be shrunk to a point in the remaining soft part of the diagram. Examples of how this is done in practice are shown in Fig. 3(a,b). The hard subgraphs have no external legs and are easy to compute. In hard-soft diagrams three-loop soft subdiagrams appear, for which a general solution has been found recently during a study of semi-leptonic decays [12].

The biggest challenge is posed by the soft-soft diagrams such as Fig. 3(c). Here the imaginary part of a genuine four-loop diagram is needed, albeit now containing only a single mass scale. A related task was solved in a series of papers on the muon and semi-leptonic $b \rightarrow u$ decays [13, 14, 15]. Those processes involve two non-interacting particles (neutrinos in the case of the muon decay and leptons in the case of QCD corrections to the $b \rightarrow u$ decay). Our present problem is more difficult because all particles can interact. For example, all fermion lines in Fig. 3(c) interact with gluons.

For the purpose of this calculation the algorithm proposed in [16] is adopted. Integration by parts [17] generates identities through which all needed integrals can be expressed in terms of a few so-called master integrals. Some of them are the same as in Ref. [14] and some new ones had to be determined, as will be described elsewhere. The large systems of linear equations needed in this reduction procedure are solved using symbolic manipulation software based on the BEAR package [18]. Parts of the calculations were performed using the package MINCER [19] and programs for solving three-loop diagrams developed in [20], using the computer algebra program FORM [21].

As a result, the two-gluon corrections resulting from interactions between the quark lines are obtained in a fully analytical form. The result can be presented as an enhancement of the hadronic decay width,

$$\frac{\Gamma(b \rightarrow c\bar{u}d')}{3\Gamma(b \rightarrow ce\bar{\nu})} = 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \cdot 4 \left(L^2 + \frac{15}{8}L + \delta_1 + \delta_2 \right) + \mathcal{O}(\alpha_s^3). \quad (4)$$

Here $L \equiv \ln \frac{M_W}{m_b}$ and δ_1 describes the two-loop corrections arising from interactions between the quarks \bar{u} and d' . For the purpose of this analysis, the value computed for the hadronic decays of the lepton τ [22] is adopted (for a recent review of higher order calculations for τ decays see [23]),

$$\delta_1 \simeq 1.3. \quad (5)$$

The last correction, δ_2 , is the main result of the present research. It arises from exchanges of two gluons between the bc and ud' lines and reads

$$\delta_2 = -\frac{9259}{5832} + \frac{17}{18}\pi^2 \ln 2 - \frac{5785}{11664}\pi^2 + \frac{13}{1440}\pi^4 + \frac{503}{648}\zeta(3) \simeq 1.8. \quad (6)$$

The two non-logarithmic corrections, $\delta_{1,2}$, increase the non-leptonic decay rate by $1.3\alpha_s^2$, or about 5 to 8 percent, if α_s is varied between 0.2 and 0.25. They increase the denominator of Eq. (1) and therefore lower the predicted semi-leptonic branching ratio, by 0.35 to 0.6 percentage points. Thus the theoretical prediction for B_{SL} goes down from the previous ‘‘reference point’’ of 11.5 percent in Eq. (3) to about 11 percent, certainly within one standard deviation from the experimental determination, Eq. (2).

In this study all masses of the quarks in the final state have been neglected. This is likely a very good approximation for the decay $b \rightarrow c\bar{u}d'$. For example, it is known how the coefficient of the logarithm L in Eq. (4) depends on m_c : it changes from the massless limit value of $15/8 = 1.875$ to 1.79 for the actual charm mass [11]. It is less clear what the impact of a non-zero m_c is in the decay $b \rightarrow c\bar{c}s'$. In this channel there are two massive charm quarks and they are moving slowly in much of the available phase space. This may greatly enhance Coulomb-like strong interactions in the final state. In the case of a single gluon exchange between \bar{c} and s' , the effect of m_c was found to increase the correction by more than a factor of four [4]. Thus, it would be very valuable to determine the impact of the charm mass on our correction δ_2 . Such a study can be carried out using the technique developed in this paper, but is technically even more challenging because of the need to compute four-loop diagrams like Fig. 3(c) with higher powers of propagators.

The emerging agreement between the theoretical value and the measurement of the semi-leptonic branching ratio confirms the Standard Model description of the heavy quark dynamics. Neither large non-perturbative effects that could potentially arise from QCD [1] nor exotic contributions to the electroweak decay mechanism are needed to bring theory and experiment into agreement. Further improvements of perturbative calculations as well as experimental studies of B_{SL} are very warranted. One may hope to bring the comparison of theory and experiment to about 1 percent level, and thus restrict or uncover ‘‘new physics’’ contributions to the heavy quark decays.

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