

# Supersymmetric Grand Unification: the quest for the theory \*

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With the advent of neutrino masses, it has become more and more acknowledged that SO(10) is a more suitable theory than SU(5): it leads naturally to small neutrino masses via the see-saw mechanism, it has a simpler and more predictive Yukawa sector. There is however a rather strong disagreement on what the minimal consistent SO(10) theory is, i.e. what the Higgs sector is. The issue is particularly sensitive in the context of low-energy supersymmetry.

## I. INTRODUCTION

Supersymmetric Grand Unification has been one of the main extensions of the Standard Model (SM) for now more than two decades. Today, however, it is in search of a universally accepted minimal, consistent model. With the growing evidence for neutrino masses [1], it is becoming more and more clear that the SU(5) theory is not good enough: it contains too many parameters in the Yukawa sector. The situation is much more appealing in the SO(10) scenario, which is custom fit to explain small neutrino masses in a simple and fairly predictive manner. The main dispute lies in the breaking of SO(10) down to the Minimal Supersymmetric Standard Model (MSSM), in the delicate question of the choice of the Higgs superfields.

Roughly speaking, there are two schools of thought: one that sticks to the small representations, which guarantees asymptotic freedom above  $M_{GUT}$ , but must make use of higher dimensional operators, suppressed by  $M_{Pl}$ ; one that argues in favor of the renormalizable theory only, even at the price of becoming strong between  $M_{GUT}$  and the Planck scale. Each program has its pros and cons. The first one in a sense goes beyond grand unification by appealing to the string picture in order to provide additional horizontal symmetries needed to simplify the theory plagued by many couplings. The second one is based on pure grand unification, with the hope that the Planck scale physics plays a negligible role. It is the second one that we discuss at length in this talk.

## II. WHY GRAND UNIFICATION AND WHY SUPERSYMMETRY ?

No excuse needs to be offered for the natural wish to unify the strong and electro-weak interactions. This appealing idea has two important generic features: proton decay and the existence of magnetic monopoles. They are

by themselves sufficient reason to pursue the unification scenario.

There are three important reasons to incorporate low-energy supersymmetry in this program: *i)* the hierarchy problem of the Higgs mass, *ii)* the gauge coupling unification, and *iii)* the Higgs mechanism in the form of radiative symmetry breaking. Let us briefly discuss them.

- Supersymmetry per se says nothing about the smallness of the Higgs mass (the hierarchy problem), it just keeps the perturbative effect small, the way that chiral symmetries protect small Yukawa couplings. The old feelings that this might not be such a big deal, since the cosmological constant does not get protected in a similar way, are becoming more widespread today.

- Gauge coupling unification of the MSSM [2] is a rather remarkable phenomenon, but its meaning is not completely clear. Namely, if one believes in a desert between  $M_W$  and  $M_{GUT}$ , then this becomes a crucial ingredient. The desert is a property of the minimal gauge group, SU(5), which is not a good theory of neutrino masses. In SO(10), on the other hand, supersymmetry is not essential at all; the theory works even better without supersymmetry since then it predicts intermediate scales in the range  $10^{10} - 10^{14} GeV$ , ideal for neutrino masses via the see-saw mechanism [3], and for leptogenesis [4].

It is worth stressing though that supersymmetric grand unification was anticipated already in 1981, and it gave a rationale for a heavy top quark with a mass around  $200 GeV$  (needed to increase the  $\rho$  parameter and help change  $\sin^2 \theta_W$  from its then accepted value of 0.20 to the current 0.23, see e.g. last paper in Ref.[2]).

- Radiative symmetry breaking and the Higgs mechanism [5]. The tachyonic property of the Higgs mass term has bothered people for a long time. It is of course purely a question of taste, for either sign of  $M_H^2$  is equally probable. Since the charged sfermion mass terms are definitely not tachyonic, in supersymmetry one could ask why is the Higgs scalar so special. The answer is rather simple: if the nature of the scalar mass terms is determined by some large scale, and if all  $m^2 > 0$ , it turns out that

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the Higgs doublet coupled to the top quark rather naturally becomes tachyonic at low scales due to the large top Yukawa coupling. This was kind of prophetic more than twenty years ago, and it could be a rationale for such a heavy top quark. Admittedly a little fine-tuning is still needed between the so-called  $\mu$  term and the stop mass, but whether this is a small or a large problem is still disputable.

Suppose we accept low energy supersymmetry as natural in grand unification. We then face the task of identifying the minimal consistent supersymmetric grand unified theory and then hope that it will be confirmed by experiment. Due to the miraculous gauge coupling unification in the MSSM, we are then tempted to accept the idea of the desert. Since the desert is a natural property of SU(5), it is not surprising that SU(5) was considered for a long time the main candidate for a supersymmetric GUT. Why not stick to this idea?

The point is that SU(5), at least in its simplest form, favors massless neutrinos. Higher dimensional operators can only provide small neutrino masses, not large enough to explain  $\Delta m_\odot^2 \simeq 10^{-5} eV^2$  and especially  $\Delta m_A^2 \simeq 10^{-3} eV^2$ . While we can try to remedy this in one way or another, the fact is that the realistic theory needs too many parameters to be predictive. Let us try to justify this claim by analyzing SU(5) in some detail.

### III. SUPERSYMMETRIC SU(5)

The minimal Higgs sector needed to break the symmetry completely down to  $U(1)_{em} \times SU(3)_c$  consists of the adjoint  $24_H$  and two fundamentals  $5_H$  and  $\bar{5}_H$ . The Higgs superpotential is quite simple

$$W_H = m(24_H^2) + \lambda(24_H)^3 + \mu \bar{5}_H 5_H + \alpha 5_H 24_H \bar{5}_H \quad (1)$$

and so is the Yukawa one

$$W_Y = y_d 10_F \bar{5}_F \bar{5}_H + y_u 10_F 10_F 5_H \quad (2)$$

since the charged fermions belong to  $\bar{5}_F$  and  $10_F$ .

The above theory is usually called the minimal supersymmetric SU(5) theory. It apparently has a small number of parameters:

- 3 real  $y_d$  (after diagonalization)
- $2 \times 6 = 12$  real  $y_u$  ( $y_u$  is a symmetric matrix in generation space)
- 2 real  $\mu, m$  (after rotations)
- $2 \times 2 = 4$  real  $\lambda, \alpha$

In total, 21 real parameters. The trouble is that the theory fails badly [6]. Neutrinos are massless, and thus  $V_\ell = 1$ ; furthermore, the relations  $m_e = m_d$  at  $M_{GUT}$

fail, except for the third generation. The most conservative approach would be to blame the failure in the absence of higher dimensional operators, this way no change in the structure of the theory is needed.

Neutrino masses are then given by the Weinberg-type operator

$$W_{y_\nu} = y_\nu \frac{\bar{5}_F \bar{5}_F \bar{5}_H \bar{5}_H}{M_{Pl}} \quad (3)$$

giving

- $6 \times 2 = 12$  new real parameters in  $y_\nu$ .

Similarly, we must add higher dimensional operators to (2),

$$\begin{aligned} \Delta W_y = & \frac{(24_H \bar{5}_H)}{M_{Pl}} 10_F 10_F + \frac{(10_F 10_F 24_H)}{M_{Pl}} \bar{5}_H \\ & + \frac{(24_H \bar{5}_H)}{M_{Pl}} 10_F \bar{5}_F + \frac{(24_H \bar{5}_H)}{M_{Pl}} 10_F 5_H \end{aligned} \quad (4)$$

(we have omitted SU(5) indices, and represented contraction with indices in  $24_H$  with a bracket). This means

- $9 \times 2 \times 2 + 6 \times 2 \times 2 = 60$  real parameters

and then most predictability is gone.

What remains? First,  $m_b = m_\tau$  is still there, still a success. Gauge couplings unify as we know, but the GUT scale is not predicted precisely as often claimed in the literature. The point is that, for the sake of consistency, one should add higher dimensional operators to (1), so that one expects

$$\Delta W_H = c_1 \frac{Tr 24_H^4}{M_{Pl}} + c_2 \frac{(Tr 24_H^2)^2}{M_{Pl}} \quad (5)$$

and if the coupling  $\lambda$  in (1) were to be small, these terms would become important. But  $\lambda$  is a Yukawa type coupling, i.e. it is self-renormalizable, so it can be naturally small. This point is worth discussing further.

At the renormalizable tree level, one gets the same masses for the color octet and the  $SU(2)_L$  triplet in  $24_H$ :  $m_8 = m_3$ . This is almost always assumed when the running from  $M_{GUT}$  to  $M_W$  is studied. Now, if  $\lambda$  is small, the  $c_i$  terms in (5) can dominate; if so, one gets  $m_3 = 4m_8$ . This fact alone suffices to increase  $M_{GUT}$  by an order of magnitude above the usually quoted value  $M_{GUT}^0 \simeq 10^{16} GeV$  (calculated with  $c_i = 0$ ). Similarly, the masses of the colored triplets  $T$  and  $\bar{T}$  in  $5_H$  and  $\bar{5}_H$  would get increased by a factor of about 30, and the  $d = 5$ -induced proton life-time by about  $10^3$ . More precisely, one obtains [7]

$$m_T = m_T^0 \left( \frac{m_3}{m_8} \right)^{5/2}, \quad (6)$$

$$M_{GUT} = M_{GUT}^0 \left( \frac{M_{GUT}^0}{2m_8} \right)^{1/2}, \quad (7)$$

where the superscript <sup>0</sup> denotes the tree-level value  $m_3 = m_8$ . In this case

$$m_8 \simeq \frac{M_{GUT}^2}{M_{Pl}} \quad (8)$$

so that

$$M_{GUT}^4 \simeq (M_{GUT}^0)^3 M_{Pl} \quad (9)$$

With  $M_{GUT}^0 \simeq 10^{16} GeV$ , this means

$$M_{GUT} \simeq 10 M_{GUT}^0, \quad m_T \simeq 32 m_T^0 \quad (10)$$

It should be stressed that  $\lambda$  small is natural technically, as much as a small electron Yukawa coupling. Taking  $\lambda \sim O(1)$  and ruling out the theory would be equivalent to finding that the SM does not work with all the Yukawa couplings being of order one.

Taking into account non-renormalizable interactions can thus save the theory. It is important to recall that without them, the minimal SU(5) does not make sense anyway, predicting as it does  $m_\nu = 0$  and  $m_d = m_e$ ; once this is corrected the theory is still valid.

Of course, if one prefers the renormalizable theory, one needs new states such as  $45_H$  (in order to correct  $m_d = m_e$ ), or  $15_H$  to give neutrino masses, or three (at least two) singlet right-handed neutrinos. This introduces even more uncertainties in the computations of  $M_{GUT}$  and  $\tau_P$ . In short, the minimal realistic supersymmetric SU(5) theory is not yet ruled out. It is indispensable to improve the experimental limit on  $\tau_P$  by two-three orders of magnitude. Grand unification needs desperately a new generation of proton decay experiments.

In the supersymmetric version of SU(5), there is yet another drawback. As much as the MSSM, it allows for the  $d = 4$  proton decay through terms like

$$\Delta W = m \lambda' 10_F \bar{5}_F \bar{5}_F \quad (11)$$

which contains both

$$\lambda' (\nu^c D^c D^c + Q L D^c) \quad (12)$$

This is a disaster (unless  $\lambda' \lesssim 10^{-12}$ ). A way out is assumed through the imposition of R-parity, or equivalently matter parity [8]  $M : F \rightarrow -F, H \rightarrow H$ , where  $F$  stands for the fermionic (matter) superfields and  $H$  for the Higgs ones. Grand unification ought to do better than this, and SO(10) does it as we shall see.

In any case, SU(5) does a poor job in the neutrino sector and in the charged fermion sector it is either incomplete [9] or it has too many parameters. One would have to include extra horizontal symmetries, and this route is in some sense beyond grand unification and often needs strings attached. If we stick to the pure grand unification, we better move on to SO(10).

#### IV. TOWARDS UNIFICATION: PATI-SALAM SYMMETRY

Quark-Lepton unification can be considered a first step towards the complete SO(10) unification of a family of fermions in a single representation. Many interesting features of SO(10) GUTs, such as a renormalizable see-saw and R-parity conservation, are already present in partial unification based on the Pati-Salam [10] group  $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$ , so it is instructive to review the situation there. Later, when we turn to SO(10), decomposition of representation under the Pati-Salam subgroup will prove to be the most useful.

To simplify the discussion, imagine a two-step breaking of the PS symmetry down to the MSSM

$$\begin{aligned} & SU(2)_L \times SU(2)_R \times SU(4)_c \\ \longrightarrow & SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c \\ \longrightarrow & SU(2)_L \times U(1)_Y \times SU(3)_c \end{aligned} \quad (13)$$

The first step breaks  $G_{PS}$  down to its maximal subgroup, the LR (Left-Right) group [11], and it is simply achieved through the vev of an adjoint representation (the numbers in parenthesis indicate the  $G_{PS}$  representations)

$$A = (1, 1, 15) \quad (14)$$

In turn, the breaking of the LR group can be achieved by having  $SU(2)_R$  triplets fields, with  $B - L = 2$ , acquiring a vev. Triplets will couple to fermions and give a mass to right-handed neutrino, providing the see-saw mechanism at the renormalizable level. Right-handed doublets could also do the job, but then non-renormalizable operators have to be invoked, which means effective operators resulting from a new theory at a higher scale, but this theory we will discuss explicitly in the next section.

There is a more profound reason for preferring the triplets. They have an even  $B - L$  number, and thus preserve matter parity as we defined above. This in turn means R-parity is not broken at a high scale. But then it can be easily shown that it cannot be broken afterwards, at the low energy supersymmetry breaking or electroweak scale. More precisely, a spontaneous breakdown of R-parity through the sneutrino VEV (the only candidate) would result in the existence of a pseudo-Majoron with its mass inversely proportional to the right-handed neutrino mass. This is ruled out by the Z decay width [12, 13]. This fact is completely analogous to the impossibility of breaking R-parity spontaneously in the MSSM, where the Majoron is strictly massless.

In terms of PS representations, the LR triplets are contained in the fields

$$\Sigma(3, 1, 10), \bar{\Sigma}(3, 1, \bar{10}), \Sigma_c(1, 3, \bar{10}), \bar{\Sigma}_c(1, 3, 10) \quad (15)$$

The matter supermultiplets are

$$\psi(2, 1, 4), \psi_c(1, 2, \bar{4}) \quad (16)$$

and the minimal light Higgs multiplet is

$$\phi(2, 2, 1) \quad (17)$$

The most general superpotential for the fields (15) is

$$W = mTrA^2 + MTr(\Sigma\bar{\Sigma} + \Sigma_c\bar{\Sigma}_c) + Tr(\Sigma A\bar{\Sigma} - \Sigma_c A\bar{\Sigma}_c) \quad (18)$$

where we assume the following transformation properties under Parity

$$\Sigma \rightarrow \Sigma_c, \quad \bar{\Sigma} \rightarrow \bar{\Sigma}_c, \quad A \rightarrow -A \quad (19)$$

We choose  $A$  to be a parity-odd field in order to avoid flat directions connecting left- and right-breaking minima.

It is straightforward to show that the SM singlets in  $A$ ,  $\Sigma_c$  and  $\bar{\Sigma}_c$  take vevs in the required directions to achieve the (in principle two-step) symmetry breaking

$$\langle A \rangle = M_c \quad \langle \Sigma_c \rangle = M_R \quad \langle \bar{\Sigma}_c \rangle = M_R \quad (20)$$

with

$$M_c \simeq M, \quad M_R \simeq \sqrt{Mm} \quad (21)$$

As discussed in detail in [14, 15], the  $SU(2)_R$ -breaking vev lies in a flat direction that connect them with charge-breaking vacua. It can be eliminated if the soft breaking terms break also  $SU(2)_R$ . If not, one would have to appeal to operators coming from a more complete theory as studied in the next section. The interesting point here is that the breaking in the minimal model leaves a number of fields potentially light [16]. There is a larger, accidental  $SU(3)$  symmetry broken down to  $SU(2)$  by the right-handed triplet fields, hence five Nambu-Goldstone bosons. But the gauge symmetry  $SU(2)_R \times U(1)_{B-L}$  is broken down to  $U(1)_Y$ , so that three of them are eaten, leaving us with states  $\delta_c^{++}, \bar{\delta}_c^{++}$  that acquire a mass only at the scale of supersymmetry breaking. These states are common in supersymmetric theories that include the Left-Right group, and have been subject of experimental search [17]. In a similar way, a color octet in  $A$  has a mass of order  $M_R^2/M_c$ , and could in principle be light.

The unification constraints give the interesting possibility

$$10^3 GeV \leq M_R \leq 10^7 GeV \quad 10^{12} GeV \leq M_c \leq 10^{14} GeV$$

opening up the possibility of the LHC discovering them at the  $TeV$  scale. For larger  $M_R$ , which would be necessary if one wants to fit neutrino masses without additional fine-tuning, these particles become less accessible to experiment. However, the large number of fields in this theory implies the loss of perturbativity at a scale around  $10M_c$ , and non-renormalizable effects suppressed by this new fundamental scale can be shown to guarantee that they have comparable masses [15]. Namely, if these effects are included, the only consistent possibility is the single-step breaking

$$M_R \simeq M_c \simeq 10^{10} GeV \quad (22)$$

Surely the most interesting feature of a low scale of PS symmetry breaking is the possibility of having  $U(1)_{B-L}$  monopoles, with mass  $m_M \simeq 10M_c$ . If produced in a phase transition via the Kibble mechanism, the requirement that their density be less than the critical density then implies  $M_c \leq 10^{12} GeV$ . We see that the single-step breaking at  $M_c \sim M_R \sim 10^{10} GeV$  (in a theory including non-renormalizable terms) offers the interesting possibility of potentially detectable intermediate mass monopoles, as long as one manages to get rid of the false vacuum problem of supersymmetric theories.

One final note about PS symmetry and neutrino masses. In LR theories the see-saw mechanism is in general non-canonical, or type II. That is, there is a direct left-handed neutrino mass from the induced vev of the left-handed triplet fields in  $\Sigma$  (which we shall call  $\Delta$ ) [18, 19, 20]

$$\propto \langle \Delta \rangle \simeq M_W^2/M_R$$

Namely, in non-supersymmetric theories the symmetry allows for a coupling in the potential

$$\Delta V = \lambda \Delta \phi^2 \Delta^c + M^2 \Delta^2 \quad (23)$$

resulting in

$$\langle \Delta \rangle = \lambda \frac{\langle \phi \rangle^2 M_R}{M^2} \simeq \lambda \frac{M_W^2}{M_R} \quad (24)$$

In supersymmetry such terms are of course not present, but one could have interactions with, for example, a heavy field  $S$  transforming as  $(1, 1, 3)$  under  $G_{PS}$

$$W = \phi^2 S + \Delta \Delta^c S + M S^2 \quad (25)$$

Integrating out  $S$  would then give a contribution

$$\Delta W = \frac{1}{M} \Delta \phi^2 \Delta^c \quad (26)$$

producing the required small vev. Or one could have a couple of heavy fields  $X = (2, 2, \bar{10})$  and  $\bar{X} = (2, 2, 10)$ , which through terms like

$$W = \phi \Delta X + \phi \Delta^c \bar{X} + M X \bar{X} \quad (27)$$

would give the same effect. These representations in fact are the ones appearing in the minimal  $SO(10)$  theory of the next section.

The absence of the  $S, X, \bar{X}$  fields in the minimal PS theory guarantees a type I see-saw at the supersymmetric level. Breaking of supersymmetry can generate a nonvanishing but negligible vev for  $\Delta$  [13]:

$$\langle \Delta \rangle \simeq \left( \frac{m_{3/2}}{M_c} \right)^2 \frac{m_D^2}{M_R} \quad (28)$$

which contributes by a tiny factor  $(m_{3/2}/M_c)^2 \leq 10^{-14}$  to the usual see-saw mass term  $m_\nu \simeq m_D^2/m_{\nu R}$ . In short, the minimal PS model has a clean, type I see-saw.

In spite of providing only a partial unification, PS theory has interesting features, namely potentially light states and the possibility of intermediate monopoles, that could be a way of differentiating it from other theories at a high scale. We are however interested in grand unification, so let us move on.

## V. SO(10) GRAND UNIFIED THEORY

If not for anything else, but for the fact that matter parity is a finite gauge rotation, SO(10) would be a better candidate for a supersymmetric GUT. But, as is well known, it also unifies a family of fermions, has charge conjugation as a gauge transformation, has right-handed neutrinos and through the see-saw mechanism leads naturally to small neutrino masses. And, most important, at the renormalizable level, if one is willing to accept large representations, it has fewer parameters than SU(5). We will elaborate on this point as we go along.

The issue here, and the main source of dispute among the experts in the field, is the choice of the Higgs sector. Before deciding on this, a comment on  $d = 4$  proton decay in the MSSM is in order. The basic problem is the impossibility of distinguishing the leptonic and the Higgs doublets, both being superfields. This persists in SU(5) where you have both  $5_F$  and  $5_H$  superfields. In SO(10) fermions belong to  $16_F$  and the "light" Higgs to  $10_H$ . This difference should be taken seriously, and all efforts should be made to maintain it.

Not all researchers agree on this. Certainly not the people who pursue minimality by choosing small representations, like the set  $(45_H, 16_H, \overline{16}_H)$ , in order to break SO(10) down to  $SU(3)_c \times SU(2)_L \times U(1)_Y$ . This way, through  $\langle 16_H \rangle = \langle 16_F \rangle \neq 0$ , matter parity will be broken at  $M_{GUT}$ ; hence the catastrophic  $d = 4$  proton decay. One is then forced to postulate extra discrete symmetries in order to save the theory. In any case, more flavor symmetries are needed, since both the symmetry breaking and the fermion masses need higher-dimensional, Planck suppressed operators whose number is rather large (at least thirteen complex couplings in  $W_H$ , the Higgs superpotential). The Yukawa superpotential takes the form

$$\begin{aligned} W_Y = & y_{10} 16_F \Gamma 16_F 10_H \\ & + \frac{1}{M_{Pl}} [c_1 (16_F \Gamma 16_F) (16_H \Gamma 16_H) \\ & + c_2 (16_F \Gamma 16_F) (\overline{16}_H \Gamma \overline{16}_H) \\ & + c_3 16_F \Gamma^3 16_F 45_H 10_H \\ & + c_4 16_F \Gamma^5 16_F \overline{16}_H \Gamma^3 \overline{16}_H] \end{aligned} \quad (29)$$

At  $M_{GUT}$ , one arrives at the prediction

$$m_b = m_\tau \left( 1 + c \frac{M_{GUT}}{M_{Pl}} \right) \quad (30)$$

which works very well as we know.

Also, the see-saw takes the so-called type I form. From (29),

$$m_{\nu_R} \simeq c_4 \frac{M_{GUT}^2}{M_{Pl}} \simeq 10^{12} - 10^{14} \text{ GeV} \quad (31)$$

which fits nicely the light neutrino masses. The type II contribution, obtained when  $16_H$  gets a small vev  $\sim M_W$ ,

$$M_\nu^{II} \simeq \frac{M_W^2}{M_{Pl}} \simeq 10^{-5} - 10^{-6} \text{ eV} \quad (32)$$

is too small to explain either atmospheric or solar  $\nu$  data ( maybe relevant for small mass splits in the case of degenerate neutrinos ).

Once flavor symmetries are added, one can do the texture exercise and look for the most appealing model. But this program goes beyond the scope of grand unification. We ought to try to construct the minimal realistic supersymmetric GUT without invoking any new physics.

### A. The pure renormalizable supersymmetric SO(10)

Such a theory is easily built [21, 22, 23, 24] with large representations in the Higgs sector

$$210_H, 126_H + \overline{126}_H, \overline{10}_H$$

with this content the theory is not asymptotically free any more above  $M_{GUT}$  [25], and the SO(10) gauge couplings becomes strong at the scale  $\Lambda_F \lesssim 10 M_{GUT}$ . The Higgs superpotential is surprisingly simple

$$\begin{aligned} W_H = & m_{210} (210_H)^2 + m_{126} \overline{126}_H 126_H + m_{10} (10_H)^2 \\ & + \lambda (210_H)^3 + \eta 126_H \overline{126}_H 210_H \\ & + \alpha 10_H 126_H 210_H + \overline{\alpha} 10_H \overline{126}_H 210_H \end{aligned} \quad (33)$$

With  $\langle 210_H \rangle \neq 0$  and  $\langle 126_H \rangle = \langle \overline{126}_H \rangle \neq 0$ , SO(10) gets broken down to the MSSM, and then  $\langle 10_H \rangle$  completes the job in the usual manner.

The Yukawa sector is even more simple

$$W_Y = 16_F (y_{10} 10_H + y_{126} \overline{126}_H) 16_F \quad (34)$$

with only 3 real (say)  $y_{10}$  couplings after diagonalization, and  $6 \times 2 = 12$  symmetric  $y_{126}$  couplings, 15 in total.

From the  $\alpha$  and  $\overline{\alpha}$  terms one gets

$$\begin{aligned} W_H = & \dots + \alpha (2, 2, 1)_{10} (2, 2, 15)_{126} (1, 1, 15)_{210} + \\ & \overline{\alpha} (2, 2, 1)_{10} (2, 2, 15)_{\overline{126}} (1, 1, 15)_{210} + \dots \end{aligned} \quad (35)$$

Now, the success of gauge coupling unification in the MSSM favors a single step breaking of SO(10), so that  $\langle (1, 1, 15)_{210} \rangle \simeq M_{GUT}$ . In other words, the light Higgs is a mixture [26] of (at least)  $(2, 2, 1)_{10}$  and  $(2, 2, 15)_{\overline{126}}$ ; equivalently

$$\langle (2, 2, 15)_{\overline{126}} \rangle \simeq \langle (2, 2, 1)_{10} \rangle \quad (36)$$

Since  $(2, 2, 15)_{\overline{126}}$  is an adjoint of  $SU(4)_c$ , being traceless it give  $m_\ell = -3m_d$ , unlike  $\langle(2, 2, 1)_{10}\rangle$ , which implies  $m_\ell = m_d$ . In other words, the  $\langle 10_H \rangle$  must be responsible for the  $m_b \simeq m_\tau$  relation at  $M_{GUT}$ , and the  $\langle \overline{126}_H \rangle$  for the  $m_\mu \simeq 3m_s$  relation at  $M_{GUT}$ . In this theory, the Georgi-Jarlskog program becomes automatic.

Of course, we don't know anymore why  $m_b \simeq m_\tau$ , or why  $10_H$  dominates; admittedly a loss. But not all is lost. Since  $\langle 10_H \rangle = \langle(2, 2, 1)\rangle$  is a Pati-Salam singlet, the difference between down quark and charged lepton mass matrices must come purely from  $\langle \overline{126}_H \rangle$

$$M_d - M_e \propto y_{126} \quad (37)$$

Suppose the see-saw mechanism is dominated by the so-called type II: this is equivalent to neutrino masses being due to the triplet  $(3, 1, \overline{10})_{126}$ , with

$$\langle(3, 1, \overline{10})_{126}\rangle_{\overline{126}} \simeq \frac{M_W^2}{M_{GUT}} \quad (38)$$

In other words

$$M_\nu \equiv M_\nu^{II} \simeq y_{126} \langle(3, 1, \overline{10})_{126}\rangle \quad (39)$$

or

$$M_\nu \propto M_d - M_e \quad (40)$$

In order to illustrate the point, consider only the 2nd and 3rd generations. In the basis of diagonal  $M_e$  we have

$$M_\nu \propto \begin{pmatrix} m_\mu - m_s & \epsilon_{de} \\ \epsilon_{de} & m_\tau - m_b \end{pmatrix} \quad (41)$$

With a small mixing  $\epsilon_{de}$ , we see that a large atmospheric mixing is only possible  $m_b \simeq m_\tau$ [27]. In other words, the experimental fact of  $m_b \simeq m_\tau$  at  $M_{GUT}$ , and large  $\theta_{atm}$  seem to favor the type type II see-saw.

On the other hand, it can be shown, in the same approximation of 2nd and 3rd generations only, that if type I dominates it gives a small  $\theta_{atm}$  [28]. This is a very interesting point, for it may make it possible to determine the nature of the see-saw in this theory, in fact, it can be shown [28] that the two types are really inequivalent.

The complete picture requires detailed studies of the three-generation case, and numerical studies have been performed. A type II see-saw is still supported [29], with the interesting prediction of a large  $\theta_{13}$  and a hierarchical neutrino mass spectrum. A small contribution of  $120_H$  [30] or higher dimensional operators[31] allow for better fits. Adding CP phases still may give room for type I [32] (for earlier work on type I see[33]).

## B. Unification constraints

It is certainly appealing to have an intermediate see-saw mass scale  $M_R$ , between  $10^{12} - 10^{15}$  GeV or so. In the non-renormalizable case, with  $16_H$  and  $\overline{16}_H$ , this is

precisely what happens:  $M_R \simeq cM_{GUT}^2/M_{Pl} \simeq c(10^{13} - 10^{14})$  GeV. In the renormalizable case, with  $126_H$  and  $\overline{126}_H$ , one needs to perform a renormalization group study using unification constraints. While this is in principle possible, in practice it is hard due to the large number of fields. The stage has recently been set, for all the particle masses were computed[34, 35], and the preliminary studies show that the situation may be under control[36]. It is interesting that the existence of intermediate mass scales lowers the GUT scale[34, 37] (as was found before in models with  $54_H$  and  $45_H$ [38]), allowing for a possibly observable  $d = 6$  proton decay.

Notice that a complete study is basically impossible. In order to perform the running, you need to know particle masses precisely. Now, suppose you stick to the principle of minimal fine-tuning. As an example, you fine-tune the mass of the  $W$  and  $Z$  in the SM, then you know that the Higgs mass and the fermion masses are at the same scale

$$m_H = \frac{\sqrt{\lambda}}{g} m_W, \quad m_f = \frac{y_f}{g} m_W \quad (42)$$

where  $\lambda$  is a  $\phi^4$  coupling, and  $y_f$  an appropriate fermionic Yukawa coupling. Of course, you know the fermion masses in the SM model, and you know  $m_H \simeq m_W$ .

In an analogous manner, at some large scale  $m_G$  a group  $G$  is broken and there are usually a number of states that lie at  $m_G$ , with masses

$$m_i = \alpha_i m_G \quad (43)$$

where  $\alpha_i$  is an approximate dimensionless coupling. Most renormalization group studies typically argue that  $\alpha_i \simeq O(1)$  is natural, and rely on that heavily. In the SM, you could then take  $m_H \simeq m_W$ ,  $m_f \simeq m_W$ ; while reasonable for the Higgs, it is nonsense for the fermions (except for the top quark). In supersymmetry *all* the couplings are of Yukawa type, i.e. self-renormalizable, and thus taking  $\alpha_i \simeq O(1)$  may be as wrong as taking all  $y_f \simeq O(1)$ . While a possibly reasonable approach when trying to get a qualitative idea of a theory, it is clearly unacceptable when a high-precision study of  $M_{GUT}$  is called for.

## C. Proton decay

As you know,  $d = 6$  proton decay gives  $\tau_p(d = 6) \propto M_{GUT}^4$ , while ( $d = 5$ ) gives  $\tau_p(d = 5) \propto M_{GUT}^2$ . In view of the discussion above, the high-precision determination of  $\tau_p$  appears almost impossible in  $SO(10)$  (and even in  $SU(5)$ ). Preliminary studies [39] indicate fast  $d = 5$  decay as expected.

We are ignoring the higher dimensional operators of order  $M_{GUT}/M_{Pl} \simeq 10^{-2} - 10^{-3}$ . If they are present with the coefficients of order one, we can forget almost everything we said about the predictions, especially in the Yukawa sector. However, we actually know that the presence of  $1/M_{Pl}$  operators is not automatic (at least

not with the coefficients of order 1). Operators of the type (in symbolic notation)

$$O_5^p = \frac{c}{M_{Pl}} 16_F^4 \quad (44)$$

are allowed by SO(10) and they give

$$O_5^p = \frac{c}{M_{Pl}} [(QQQL) + (Q^c Q^c Q^c L^c)] \quad (45)$$

These are the well-known  $d = 5$  proton decay operators, and for  $c \simeq O(1)$  they give  $\tau_p \simeq 10^{23} yr$ . Agreement with experiment requires

$$c \leq 10^{-6} \quad (46)$$

Could this be a signal that  $1/M_{Pl}$  operators are small in general? Alternatively, you need to understand why just this one is to be so small. It is appealing to assume that this may be generic; if so, neglecting  $1/M_{Pl}$  contributions in the study of fermion masses and mixings is fully justified.

#### D. Leptogenesis

The see-saw mechanism provides a natural framework for baryogenesis through leptogenesis, obtained by the out-of-equilibrium decay of heavy right-handed neutrinos[4]. This works nicely for large  $M_R$ , in a sense too nicely. Already type I see-saw works by itself, but the presence of the type II term makes things more complicated. One cannot be a priori sure whether the decay of right-handed neutrinos or the heavy Higgs triplets is responsible for the asymmetry, although the hierarchy of Yukawa couplings points towards  $\nu_R$  decay. In the type II see-saw, the most natural scenario is the  $\nu_R$  decay, but with the triplets running in the loops [40]. It appears that the sign of the asymmetry gets correctly predicted for the type 2 seesaw, an impressive result [41].

#### VI. SUMMARY AND OUTLOOK

We have argued in favor of SO(10) as the minimal consistent supersymmetric grand unified theory. It includes all the interesting features of Left-Right and Pati-Salam symmetries, it is the ideal setting for a see-saw mechanism, and has the MSSM with automatic R-parity as the low energy limit. It can give connections with low energy phenomenology, such as the one relating  $b - \tau$  unification with neutrino mixings, besides being able to provide realistic charged fermion spectrum.

As a gauge symmetry accommodating all fermions of one generation in a single representation, there is little doubt on the convenience of SO(10). The question of the Higgs sector is the one unsolved: one can choose between two different approaches. One can insist on perturbativity all the way to the Planck scale and choose small representations, using then  $1/M_{Pl}$  operators to generate the

physically acceptable superpotential; it is then necessary to use textures to simplify the theory. In this sense, this programme appeals to physics beyond grand unification. The other approach is to stick to the pure SO(10) theory, at the expense of using very large representations. The couplings then become strong at  $\lambda_F = 10M_{GUT}$ , but the theory has the advantage of requiring only a small number of couplings, and is a complete theory of matter and non-gravitational interactions.

The important question is rather if these versions of the theory can be tested in the near future. Work is in progress by several groups on the possibility of establishing testable constraints on neutrino masses and mixings, proton decay, and the implementation of the leptogenesis scenario. In the pure SO(10) approach, with less parameters, proving the theory wrong might be just a question of time.

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#### Added note:

There has been a lot of activity in the subject of supersymmetric SO(10) since these talks were presented. In particular, two groups made numerical studies in favor of the minimal theory discussed here ([42, 43]). The complete study of proton decay and leptogenesis remain an important challenge. Meanwhile, the search for the minimal predictive SO(10) theories carried on to the split-supersymmetric and non-supersymmetric (or even supersplit-supersymmetric[44]) versions, since the gauge coupling unification allows for both. For example, one could get rid of  $\mathbf{126}_H$  in favor of  $\mathbf{16}_H$  and the two-loop radiative mechanism for the right-handed neutrino mass. There are no loops for the superpotential in supersymmetry, and in this case the ideal scenario becomes split supersymmetry, with light gauginos and higgsinos and heavy fermions as close to  $M_{GUT}$  as allowed by phenomenological constraints [45, 46]. This implies giving up completely on naturalness, and then one can as well resort to the ordinary, non-supersymmetric SO(10). For the construction of minimal predictive models in this scenario, see [47].

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