The $\tilde{U}(12)$ -Classification Scheme, Static *U*(4)**-Spin Symmetry for Light-Quarks and "Exotic" Hadrons**

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Abstract. Several years ago we have proposed a manifestly covariant $\tilde{U}(12)_{SF}$ -classification scheme of hadrons, which maintains the successful $SU(6)_{SF} \otimes O(3)_L$ framework in Non-Relativistic Quark Model and is also reconcilable with the mechanism of Spontaneously-Broken Chiral Symmetry in Relativistic Field Theoretical Models. The essential point here is to notice the overlooked freedom of the $SU(2)_\rho$ -spin for (confined) light quarks, which leads to existence of new type of "exotic" hadrons, called "chiralons" and to a selection rule, ρ_3 -line rule, on the spectator quark line.

A series of puzzling new hadrons recently observed such as *X*(3872)-meson family and Θ(1540) and so on are possibly classified mostly as chiralons.

PRESENT STATUS ON HADRON SPECTROSCOPY

(Conventional Two View-Points on Hadron Classification) The non-relativistic view is based upon NRQM and has the successful $SU(6)_{SF} \otimes O(3)_L$ framework; while the relativistic view resorts to RFTM and realizes the mechanism of SBCS, an indispensable notion in the low-energy hadron physics. On the other hand, both have their own serious difficulties; the former is lacking Lorentz covariance and out of notion chiral symmetry; while the latter is unable to treat internal excitation of hadrons.

(Proposal of $\tilde{U}(12)_{SF}$ *-Classification Scheme)* This[1] is a manifestly-covariant extension of NRQM in conformity with the chirality γ -transformation on the light quarks, and accordingly is reconciled with the mechanism of SBCS. The scheme has a unitary symmetry, $U(12)_{SF} \otimes O(3)_L$, in the rest-frame of hadrons, represented in the covariant $U(12)_{SF} \otimes O(3,1)_L$ space; that is, the hadron Wave Function, which is tensors in the above space, has the static symmetry $U(12)_{SF} \supset SU(3)_F \otimes U(4)_S$, embedded in $\tilde{U}(12)_{SF} \supset SU(3)_F \otimes \tilde{U}(4)_{DS}$, where $U(4)_S \supset SU(2)_\rho \otimes SU(2)_\sigma$ and the $U(4)_{S}(\tilde{U}(4)_{DS})$ being a unitary group(pseudo-unitary Lorentz group) on the 4 components of Dirac spinor(the γ -matrices≡ $\sigma \otimes \rho$). The basic vectors of the "new" freedom of $SU(2)_\rho$ are given by two eigenstates of ρ_3 with respective eigen value $r = \pm$; "Paulispinor" $\Phi_+(X)$ with $j^P = \frac{1}{2}$ 2 ⁺, and "Chiral-spinor" $\Phi_-(X)$ with $j^P = \frac{1}{2}$ 2 − . The chiral spinor $\Phi_-(X)$ with "exotic" quantum numbers leads to existence of "exotic-hadrons"².

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² We had defined the two kinds of hadron states: The chiral states are described by tensors with at least one chiral spinor-index, while the Pauli states are by those with only Pauli spinor-index. We call especially

FIGURE 1. Genaral hadrons in $\tilde{U}(12)$ -classification scheme

(Observation of New Hadrons), seemingly to be out of the conventional classification scheme, has recently been reported successively. They have features : (F1) Their mass is mostly close to the threshold of their observed or intermediate decay channels, and (F2) their decay width is unexpectedly narrow. The purpose of this work is to point out that these "new hadrons" be promising candidates of chiralons.

OVERVIEW OF HADRONS IN $\tilde{U}(12)$ **-SCHEME**

(Schematic Structures of General Hadrons) in $\tilde{U}(12)$ -classification scheme are shown in Fig[.1](#page-1-0) systematically for conventional $(q\bar{q})$ -mesons and (qqq) -baryons and also for multi-quark hadrons resorting on Joined-Spring Quark Model. It had been proposed so as to give color-singlet and triality-zero hadrons a long time ago. In this Fig. we have, in addition to the internal space-time structure, given the structure on flavor, color and the new $SU(2)_\rho$ -spin freedom of constituent quarks(, where necessary), and also made some tentative assignment of relevant new hadrons, taking into account theoretical expectations from the static $U(4)$ -spin symmetry.

(Existence of Chiral States and Tentative Assignments) The $U(4)$ _S-spin WF of hadrons contains a relevant tensor product of basic vectors in $SU(2)_\rho$ -space. The basic-vectors are two kinds of Dirac spinors, which are the Fourier conjugates of $\Phi_{\pm}(X)$, with definite four-volocity $v_{\mu} = P_{\mu}/M$ of relevant hadrons(, see later sections). In JSQM(where the symmetry due to quark statistics is somewhat restricted by connected springs) the relevant hadron WF is represented by those of quarks and of di-quarks with the quantum numbers j^P as

$$
q_{\pm} : 1/2^{\pm}; \, d(q_{+}q_{+}); 0^{+}, 1^{+}; \, d^{\chi}(q_{+}q_{-}); 0^{-}, 1^{-}; \text{ and } d^{\chi\chi}(q_{-}q_{-}); 0^{+}, 1^{+}.\qquad(1)
$$

The existence of chiral spinor q _− with $j^P = 1/2^-$ is the origin of exotics and leads to chiral states which show anyhow exotic properties.

WF of $(q\bar{q})$ -mesons and (qqq) -baryons The $SU(2)_0$ -spin WF of $[\pi(140)]/[\sigma(600)]$ nonets comes from their being linear representation of S.B. chiral symmetry. Similarly WF of $D_{sJ}^*(0^+;2317)/D_{sJ}(1^+;2460)$ reflects that they are chiral partners of $D_s(0^-;1968)/D_s^*$ (1⁻;2112). The WF of [ρ]/[ω] being Paulons is due to their electromagnetic properties. The WF for Roper resonance *N*(1440) and *SU*(3)-singlet Λ(1405)

the hadrons represented purely by the Pauli/Chiral states as Paulons/Chiralons.

is supposed because of their comparatively lighter mass.

Multi-quark "Exotic hadrons" The $X(3872)$ family($X(3872)$, $Y(3940)$, \cdots) are considered to be good candidate of chiralons in the system of $[(cn_+) \cdot (\overline{cn_-})]$ tetra-quarks. The *X*(3872) had the mass close-to-threshhold of its decay chanel, $J/\psi + \rho$, ω , and unexpectedly small width Γ < 2.3MeV. In Fig. we have picked up also the other observed new hadrons with properties of close-to-threshold mass and of the smaller-than-expected width.

OVERLOOKED FREEDOM OF *SU*(2)ρ**-SPIN**

(Summary of Covariant Description of Composite Hadrons) It should be first noted that we are not treating a dynamical bound-state problem but presenting a kinematical framework, where the c-number Dirac-spinor as a mathematical tool, called urciton, is simulating the role of physical quarks.

We set up the hadron WF which transforms as a relevant tensor product of the c-number quark field and its Pauli conjugate, $\psi_A(x)$ ans $\bar{\psi}^B(x)$ $(A = (\alpha, a)$ etc. $\alpha(a)$ denotes Dirac spinor(flavor) index), and start from the Yukawa-type Klein-Gordon equation to be satisfied by WF

$$
[(\frac{\partial}{\partial X_{\mu}})^2 - \mathcal{M}^2(r_{\mu}, \partial/\partial r_{\mu})] \Phi_{A_1 \dots}^{B_1 \dots}(X, r, \dots) = 0,
$$
 (2)

where the \mathcal{M}^2 is depending on relative coordinates of constituents (X_μ/r_μ) being C.M./relative coordinates), and assumed to be diagonal on A_1 and B_1 etc. In this talk we concern only ground states neglecting the dependence on $r_µ$ of WF. Then WF of hadrons $\Phi(X)$ is given as solutions of the "local" K.G.Eq.[\(2\)](#page-2-0), and the positive(negative)frequency Fourier conjugates of WF, become, through the second-quantization, to represent systematically the annihilation(creation) operator of all hadrons(anti-hadrons) composed in the relevant multi-quark system. The Fourier conjugates of Pauli-conjugate WF become similarly the creation(annihilation) operators. This leads to the crossing relation or substitution law of hadrons, an important attribute of hadrons.

The $\tilde{U}(4)_{DS}$ WF of hadrons with $v_{\mu} = P_{\mu}/M$ ($P_{\mu}(M)$); hadron momentum(mass)) is a tensor product of basic vectors, urciton-spinors, in $SU(2)_\rho$ -space. They are defined through the two (Pauli and chiral) types of Dirac spinor $\Phi_{\alpha}(X) = \{\Phi_{+\alpha}, \Phi_{-\alpha}(X)\}^3$, defined by the solutions of K.G. equation as

$$
\begin{aligned}\n(\frac{\partial^2}{\partial X_\mu^2} - M^2) \Phi_\alpha(X) &= \left[(\gamma_\mu \partial_\mu + M)(\gamma_\mu \partial_\mu - M) \Phi(X) \right]_\alpha = 0, \\
(\gamma_\mu \partial_\mu + |M|) \Phi_+(X) &= 0; \quad (\gamma_\mu \partial_\mu - |M|) \Phi_-(X) = 0\n\end{aligned} \tag{3}
$$

³ Whether the chiral spinor is required in the actual spectrum is judged only through comparison with experiments. However, for the confined light quarks inside of the lower mass hadrons it is indispensable to describe the relevant chiral symmetry. In describing hadrons with them both "mass conjugate" members of $W(P)$ and of $\bar{W}(P)$ are required, since the hole theory, where $U_-(P) = V_+(P)$ and $V_-(P) = U_+(P)$, is not applicable to the confined quarks independently of the remainders(, Note that particle and anti-particle have mutually conjugate-quantum numbers, for example, $q(\bar{q})$ belongs to $\frac{3}{2c}(3_c^*)$ on the color). On the other hand it is not the case for heavy-quarks. It may be evident from Eq.(4) that the (\pm) sign of Φ_{\pm} becomes equal to that of ρ_3 -spin in hadron frame.

They are related mutually as $\Phi_{\pm,\alpha}(X) = -\gamma_5 \Phi_{\mp,\alpha}(X)$, and the complete set of the basic vectors $W_\alpha(P) = \{U_{\pm\alpha}(P)\}\$ and $\bar{W}^\beta(P) = \{\bar{V}^\beta_\pm(P)\}\$ are given by Fourier conjugates of $\Phi(X) = {\Phi_+(X)}$ and, as those of their Pauli conjugate $\bar{\Phi}(X)$.

$$
\Phi_{\pm,\alpha}(X) \equiv \sum_{P_{\mu}(P_0>0)} U_{\pm\alpha}(P_{\mu}) e^{iPX}, \quad \bar{\Phi}^{\beta}_{\pm}(X) \equiv \sum_{P_{\mu}(P_0>0)} \bar{V}^{\beta}_{\pm}(P_{\mu}) e^{iPX}, \tag{4}
$$

where $(\bar{V}_{\pm}{}^{t})^{\beta}(P_{\mu}) = \mp C^{\beta\alpha}U_{\pm\alpha}(P_{\mu})$ ($C = \gamma_4\gamma_2 = -i\rho_1\sigma_2$ is charge- conjugation matrix).

MULTI-QUARK EXOTIC HADRONS AND *X*(3872) **FAMILY**

(Properties of Multi-Quark Hadrons) The mass of ground states is generally given as: $M = M^{(0)} + \delta M$; $M^{(0)} = \sum_i m_i$ (sum of constituent masses), where $M^{(0)}$ is mass in the ideal limit (being the $U(12)_{SF}$ -symmetric on light-quarks), while δM represents the effects of broken chiral-symmetry($\delta^{\chi}M$) and of perturbative QCD($\delta^{OGE}M$). Accordingly the total mass $M^{(0)}$ is degenerate through all types of hadron and/or hadron systems at their thresholds if they as a whole have the same quark-flavor configurations. This leads to F1 one feature of puzzling new hadrons. For an example of *X*(3872),

$$
M_T^{(0)}((cq)(\overline{cq})) = M_{\psi}^{(0)}(c\overline{c}) + M_M^{(0)}(q\overline{q}) = M_D^{(0)}(c\overline{c}) + M_{\overline{D}}^{(0)}(\overline{c}q)
$$
(5)

Now we focus our attention on the fission process with small phase volume, as in the above example, among ground state hadrons. In this process the transition matrix element M is considered to come purely from the overlapping $I_{O.L.}$ of WF as $M\!=\!g^{(0)}\!\times\!I_{O.L.}$ $(g^{(0)};$ a dimensional parameter), and $I_{O.L.}$ consists of a product of those on each quark line as $I_{O.L.} = \prod_i I_{O.L.}[q_i]$. The $I_{O.L.}[q]$ is just the inner product between WF of initial and final quarks. Then, because of the static $U(4)$ -spin symmetry for light-quarks, the transiton between urcitons with different r-value is forbidden at threshold, while suppressed strongly in the region close to threhold. This, a kind of selection rule to be called ρ_3 -line rule, leads to F2, another feature of new hadrons. The suppression factor is, concerning the $SU(2)_\rho$ -space, $\varepsilon(P) = |P|/(M+E(|P|)) < 1$ (P, E, and *M* being momentum, energy, and mass of the relevant final hadrons). In this connection the example of $X(3872)$ family are instructive: Firstly the mass of *X*(3872) is almost equal to the threshold of $(D(1870), \bar{D}^*(2010))$ and is within the threshold region of $(\rho(\omega), J/\psi)$. Both the decay channels are doubly forbidden by the ρ_3 -line rule. Experimentally the decay into the latter channel has been obseved with a small width $\Gamma < 2.3$ MeV, but not into the former. This fact is understood by the ρ_3 -line rule, considering the relation between respective suppression factors, $\epsilon^{\hat{A}}(\mathbf{P})_{D\bar{D}^*}<\epsilon^A(\mathbf{P})_{J/\psi\cdot\rho(\omega)}\ll 1$. Secondly the mass of *Y*(3.943) is well above the open-charm thresholds (3.872MeV) of $D\bar{D}^*$ and near to the threshold(3.880) of (ω , J/ψ), of which both decay-channels are similarly double-forbidden by ρ_3 -line rule. Experimentally its decay into the $\omega J/\psi$ has been observed with a rather narrow width $\Gamma \approx 90$ MeV, while not into the channel $D\bar{D}^*$. This situation may be also understandable from the difference of the suppression factors, $\varepsilon_{D\bar{D}^*} < \varepsilon_{J/\psi \cdot \omega}$. *(Level Structures of Tetra-quark* $[(cq)(\overline{cq})]$ -*System)* The quantum numbers of groundstate multiplets are obtained from those of di-quarks and anti-diquarks, $d(cq)$ and $d(\overline{cq})$ (see, Eq.(1)). The relevant $SU(2)_\rho$ -space WF are classified into three-groups; $T_{\chi\chi}(d^{\chi}d^{\chi})$, $T_{P\chi}(d^{\chi}d^P/d^Pd^{\chi})$ and $T_{PP}(d^{\chi}d^Pd^P)$ (where $d^{\chi} \equiv (cq_{-})$ and $d^P \equiv (cq_{+})$) with

 $j^p(T) = j^p(d) \otimes j^p(\bar{d})$). In order to obtain the more realistic mass-spectra is necessary

to estimate the symmetry breaking effects $\delta M = \delta^{\chi} M + \delta^{OGE} M$, which are approximated by the respective sum of those of constituents, $\delta M_T = \delta M_d + \delta M_{\bar{d}}$.

Concerning $\delta^{\chi}M \equiv M^{\chi} - M^P$, applying the L σ M for the Yukawa interaction of light quarks inside of $d(cq)$ -diquarks and of $D(c\bar{q})$ mesons, we get the relation,

$$
\delta^{\chi} M_d(cq) = -\delta^{\chi} M_D(c\bar{q}) \quad (\delta^{\chi} M_D(c\bar{n}) \propto a, \ \delta^{\chi} M_D(c\bar{s}) \propto b) \tag{6}
$$

where $a \equiv \langle n\bar{n}\rangle_0$ and $b \equiv \langle s\bar{s}\rangle_0$ are vacuum eqpectation values of respective scalar densities. Using the experimental values of $\delta^{\chi} M_D(c\bar{s})$ and of the ratio a/b we obtain[2] the values; $\delta^{\chi}M_d(cn) = -242\text{MeV}$, $\delta^{\chi}M_d(cs) = -348\text{MeV}$. This implies that the masses of $T[(cq)(\overline{cq})]$ -system are in order of $M_T(\chi\chi) < M_T(P\chi) < M_T(PP)$ with the difference $\delta^{\chi}M_d(cq)$, Eq.(6). It is notable that the negative relative-sign between both sides of Eq.(6) reflects the Charge-Conjugation property of thr c-number scalar-density.

Concerning $\delta^{OGE}M$, we set up the effective spin-current interaction between c-quarks and light-quarks as

$$
\langle H^{J} \rangle \approx \langle \bar{c} \sigma_{\mu\nu} c \rangle \langle \bar{q}_{\pm} F_{U} \sigma_{\mu\nu} q_{\pm} \rangle \sim \langle \sigma^{(c)} \rangle \langle \sigma^{(q)} \rangle; \tag{7}
$$

where the middle term is color-gauge invariant and becomes static $U(4)_S$ -symmetric due to the role of unitarizer F_U . This leads to the relation of $\delta^J M \equiv M(J=1) - M(J=0)$,

$$
\delta^{J} M_d(cq) = (1/2) \delta^{J} M_D(c\bar{q}) = 71 \text{MeV} \quad \text{for } q = (n, s)
$$
 (8)

where the positive relative-sign of the first equality in Eq. (7) reflects the C.C. property of the c-number tensor-density and the factor 1/2 comes from the color freedom. The value of Eq.(8) is obtained from the experimental values in the $D(c\bar{q})$ meson system. Here it may be notable that the mass defference between $X(3872)$ and $Y(3940)$ is close to the estimated one Eq.(8). The more detailed investigations on $X(3872)$ family are reported in this conference[3]. Finally, we give brief comments on the related works: The diquark and anti-diquark picture of *T*-quark system was first discussed in Ref.[4], but without considerations on chiral symmetry. Actually the relevant level structure of [4] is equivalent to that of our T_{PP} system. The $D^0\bar{D}^{*0}$ molecular picture[5] also explains(or starts from) the feature of new hadrons(F1). However, its WF has very loose structure whereas in our case it is, supposed to be, tight due to color confining force.

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