# Energy loss and dynamical evolution of quark $p_T$ spectra

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Average energy loss of light quarks has been calculated in a two stage equilibrium scenario where the quarks are executing Brownian motion in a gluonic heat bath. The evolution of the quark $p_T$ spectra is studied by solving Fokker-Planck equation in an expanding plasma. Results are finally compared with experimentally measured pion $p_T$ spectrum at RHIC.

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Production of high transverse momentum ($p_T$) particles in heavy ion collision, in recent years, has assumed special interest. This is related with the phenomenon of high $p_T$ particle suppression – dubbed as jet quenching. Actually energetic partons while passing through plasma lose energy which degrades the population of high $p_T$ hadrons. Experimentally such suppression has been observed at Relativistic Heavy Ion Collider (RHIC) at high $p_T$ domain [1]. This phenomenon of 'jet quenching' can be used to extract the properties of early stage of the plasma, temporarily produced in the high energy heavy ion collision [2].

To study the modified $p_T$ spectrum of hadrons it is essential to estimate parton energy loss in the thermal bath of quarks and gluons. Partons in a plasma can dissipate energy in two ways, either by two body scattering (collisional loss) or via the emission of gluons (radiative loss). Significant progress has been made in recent years to calculate partonic energy loss [2–9]. In many of these calculations, first, path length dependent energy loss is estimated, which, consequently is used to modify the fragmentation functions that depopulate high $p_T$ hadrons [11]. Present approach is different in the sense that here we dynamically evolve the quark $p_T$ spectra for a given initial distribution. Importance of this has already been discussed in refs. [3,12] where the focus has been on the radiative loss. Even though this is the main mechanism of energy loss, under certain kinematic conditions, collisional loss could be comparable, or even be more, to its radiative counterpart [9]. This is particularly so in case of heavy quark because of the dead cone effect [10].

To estimate energy loss, we inject quarks with a given energy distribution and study the broadening of the same as the system expands and cools. In the present model gluons are thermalized at a much smaller time scale than quarks. Such two stage equilibration scenario, albeit in a different context, was considered sometime ago [13,14]. Possibility of earlier thermalization (even faster) of (soft) gluons is further accentuated, if the gluons are assumed to be in color glass condensate (CGC) state initially. In

tion time scale for RHIC and LHC energies are estimated to be $t_i = 1.40$ GeV$^{-1}$, $t_i = 0.62$ GeV$^{-1}$ respectively [15].

Under such simplifying scenario, it thus reduces to the problem where quarks are executing Brownian motion in the gluonic heat bath. The evolution of the quarks are, therefore, governed by the Fokker-Planck equation (FPE). So we avoid solving full Boltzmann kinetic equation (BKE) and approximate relevant collision integral in terms of appropriately defined drag and diffusion coefficients [16–18].

The drag and diffusion co-efficients are calculated using techniques of finite temperature perturbative quantum chromodynamics. The former is related to the quark energy loss, while the latter is related to the square of the momentum transfer [16,17]. Both of these quantities show infrared divergences as the collisions are dominated by soft ($t$-channel) gluons. To cure this problem, we screen the interaction via hard thermal loop (HTL) corrected propagator which makes long range Coulomb interaction finite.

To arrive at the relevant FP equation from BKE we assume that there is no external force and therefore,

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p}\cdot\nabla_{\mathbf{r}}\right) f(\mathbf{p},\mathbf{x},t) = C[f(\mathbf{p},\mathbf{x},t)] \qquad (1)$$

Here, quarks have a phase space distribution which evolves in time and the collision term is evaluated by considering ultra-relativistic scattering of the quarks and gluons which eventually are expressed in terms of transport coefficients. Considering that the system expands in the longitudinal direction Eq.(1) takes the following form [19]:

$$\frac{\partial f(\mathbf{p},z,t)}{\partial t} + v_{pz}\frac{\partial f(\mathbf{p},z,t)}{\partial z} = C[f(\mathbf{p},z,t)]. \qquad (2)$$

Here, $v_{pz} = p_z/E_p$ (for light partons $E_p = |p|$). This equation can be simplified further for the central rapidity region which is boost invariant in rapidity, which implies

$$f(\mathbf{p_T},p_z,z,t) = f(\mathbf{p_T},p'_z,\tau). \qquad (3)$$

the proper time. Using the Lorentz transformation relation $\partial\tau/\partial z|_{z=0} = 0$, $\gamma_{z=0} = 1$ and $\partial p'_z/\partial z|_{z=0} = -p/t$, one finds

$$v_{pz}\frac{\partial f}{\partial z} = -\frac{p_z}{t}\frac{\partial f}{\partial p_z} \quad (4)$$

Therefore the Boltzmann equation takes the following form

$$\frac{\partial f(\mathbf{p_T}, p_z, t)}{\partial t}|_{p_z t} = \left(\frac{\partial}{\partial t} - \frac{p_z}{t}\frac{\partial}{\partial p_z}\right) f(\mathbf{p_T}, p_z, t) \quad (5)$$

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{t}\frac{\partial}{\partial p_z}\right) f(\mathbf{p_T}, p_z, t) = C[f(\mathbf{p_T}, p_z, t)]. \quad (6)$$

Evidently in Eq. 6, the second term on the left hand side represents the expansion while the right hand side characterizes the collisions. The latter can be written in terms of the differential collision rate $W_{\mathbf{p},\mathbf{q}}$

$$C[f(\mathbf{p_T}, p_z, t)] = \int d^3q[W_{p+q;q}f(\mathbf{p}+\mathbf{q}) - W_{p;q}f(\mathbf{p})] \quad (7)$$

which quantifies the rate of change of the quark momentum from $\mathbf{p}$ to $\mathbf{p}-\mathbf{q}$, $W_{\mathbf{p},\mathbf{q}} = d\Gamma(\mathbf{p},\mathbf{q})/d^3q$, where $\Gamma$ represent scattering rates.

In a partonic plasma, small angle collisions, with parametric dependence of $O(g^2T)$, are more frequent than the large angle scattering rate. The latter goes as $\sim O(g^4T)$. Therefore the distribution function does not change much over the mean time between two soft scatterings. This allows us to approximate $f(\mathbf{p}+\mathbf{q}) \simeq f(\mathbf{p})$. In contrast, $W_{\mathbf{p},\mathbf{q}}$, being sensitive to small momentum transfer, falls off very fast with increasing $q$. Therefore, we write

$$W_{\mathbf{p+q},\mathbf{q}}f(\mathbf{p}+\mathbf{q}) \simeq W_{\mathbf{p},\mathbf{q}}f(\mathbf{p}) + q_i\frac{\partial}{\partial p_i}(W_{\mathbf{p},\mathbf{q}}f) + \frac{1}{2}q_iq_j\frac{\partial^2}{\partial p_i\partial p_j}(W_{\mathbf{p},\mathbf{q}}f) \quad (8)$$

With these approximation, Eq. 6 can be written as

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{t}\frac{\partial}{\partial p_z}\right) f(\mathbf{p_T}, p_z, t) = \frac{\partial}{\partial p_i}A_i(\mathbf{p})f(\mathbf{p}) + \frac{1}{2}\frac{\partial}{\partial p_i\partial p_j}[B_{ij}(\mathbf{p})f(\mathbf{p})], \quad (9)$$

where we have defined the following kernels,

$$A_i = \int d^3qW_{\mathbf{p},\mathbf{q}}q_i \quad (10)$$

$$B_{ij} = \int d^3qW_{\mathbf{p},\mathbf{q}}q_iq_j \quad (11)$$

$$A_i = \frac{\nu}{16p(2\pi)^5}\int\frac{d^3k'}{k'}\frac{d^3k}{k}\frac{d^3q}{p'}d\omega q_i|\mathcal{M}|^2_{t\to 0}f(k)(1+f(k'))$$

$$B_{ij} = \frac{\nu}{16p(2\pi)^5}\int\frac{d^3k'}{k'}\frac{d^3k}{k}\frac{d^3q}{p'}d\omega q_iq_j|\mathcal{M}|^2_{t\to 0}f(k) (1+f(k'))\delta^3(\mathbf{q}-\mathbf{k'}+\mathbf{k})\delta(\omega-\mathbf{v_{k'}}\cdot\mathbf{q})\delta(\omega-\mathbf{v_k}\cdot\mathbf{q}), \quad (13)$$

First we consider Compton scattering $(gq(\bar{q}) \to gq(\bar{q}))$ for which

$$|\mathcal{M}|^2 = g^4\left[\frac{s^2+u^2}{t^2} - \frac{4}{9}\frac{s^2+u^2}{us}\right] \quad (14)$$

In terms of explicit momentum variables it reduces to,

$$|\mathcal{M}|^2_{t\to 0} = 8g^4\frac{p'k'pk}{(q^2-\omega^2)^2}(1-cos\theta_{pk})(1-cos\theta_{p'k'}). \quad (15)$$

We label the incoming four momenta of the test and bath particles as $P$ and $K$ respectively and the corresponding final momenta are $P'$ and $K'$. In Eq.(14) $s, u$ and $t$ are Mandelstam variables with $t = \omega^2 - q^2$ and $s = -(u+t)$. The scattering angles, following ref. [20], can be written as

$$cos\theta_{pq} = \frac{\omega}{q} + \frac{t}{2pq}$$
$$cos\theta_{kq} = \frac{\omega}{q} - \frac{t}{2pq} \quad (16)$$

The parton parton collision is dominated by the soft momentum transfer. Therefore, we consider small angle scatterings to estimate the leading contribution, In this limit $t \to 0$ and $s = -u$. Hence the collision is dominated by the $t$ channel, $|\mathcal{M}|^2 \to g^4 2s^2/t^2$. We also have $cos\theta_{kq} \sim cos\theta_{k'q} \sim \omega/q \sim cos\theta_{pq} \sim cos\theta_{p'q}$ and $cos\theta_{pk} = \omega^2/q^2 + (1-\omega^2/q^2)cos\phi_{pk}$. With these azimuthal angle averaged matrix element becomes,

$$\langle|\mathcal{M}|\rangle^2_{t\to 0} \simeq 12g^4p'k'pk(1-\omega^2/q^2)^2/(q^2-\omega^2)^2 \simeq 12g^4p^2k^2/q^4 \quad (17)$$

It might be noted that Eq. 12 is symmetric under $k \leftrightarrow k'$, which allows one to write,

$$A(p^2) = \frac{\nu}{16p^2(2\pi)^5}\int\frac{d^3k'}{k'}\frac{d^3k}{k}\frac{d^3q}{p'}d\omega p\cdot q\langle\mathcal{M}\rangle^2_{t\to 0} \frac{1}{2}[f(k)(1+f(k')) - f(k')(1+f(k))] \delta^3(\mathbf{q}-\mathbf{k'}+\mathbf{k})\delta(\omega-\mathbf{v_{k'}}\cdot\mathbf{q})\delta(\omega-\mathbf{v_k}\cdot\mathbf{q}). \quad (18)$$

recognizing the fact that $\omega$ is small compared to the momentum, we have

Furthermore,

$$
\begin{aligned}
&-\int k^2 \frac{\partial f(k)}{\partial k} dk \\
&= \int 2kf(k)dk = \frac{\pi^2 T^2}{3}. \qquad (20)
\end{aligned}
$$

With the help of these identities, the drag coefficient can easily be calculated at the leading log order:

$$A(p^2) = \frac{\nu \pi \alpha_s^2 T^2}{6p} \mathcal{L}, \qquad (21)$$

where $\mathcal{L} = \int \frac{dq}{q}$ [15]. Evidently, $\mathcal{L}$ is infrared singular. Such divergences do not arise if close and distant collisions are treated separately. For very low momentum transfer the concept of individual collision breaks down and one has to take collective excitations of the plasma into account. Hence there should be a lower momentum cut off above which bare interactions might be considered. While for soft collisions medium modified hard thermal loop corrected propagator should be used [8,9]. It is evident that Eq. 12 actually gives $dE/dt$ or the energy loss rate [9] that can be related to the drag coefficient.

$B_{ij}$ can be decomposed into longitudinal and transverse components:

$$B_{ij} = B_t(\delta_{ij} - \frac{p_i p_j}{p^2}) + B_l \frac{p_i p_j}{p^2} \qquad (22)$$

Explicit calculation shows that the off diagonal components of $B_{ij}$ vanish.

$$
\begin{aligned}
B_{t,l} = &\frac{\nu g^4}{(2\pi)^4} \int \frac{d^3k d^3q d\omega}{2k2k'2p2p'} \delta(\omega - \mathbf{v_p} \cdot \mathbf{q}) \delta(\omega - \mathbf{v_k} \cdot \mathbf{q}) \\
&\langle \mathcal{M} \rangle^2_{t \to 0} f(k)[1 + f(k) + \omega \frac{\partial f}{\partial k}] q^2_{t,l}. \qquad (23)
\end{aligned}
$$

Here, in the small angle limit $q_l \simeq \omega$ and $q_t \simeq \sqrt{q^2 - q_l^2}$. With all these, in the leading log approximation

$$B_t = \frac{2\nu \pi \alpha_s^2}{3} T^3 \mathcal{L} \qquad (24)$$

$$B_l = \frac{\nu \pi \alpha_s^2}{3} T^3 \mathcal{L} \qquad (25)$$

Here the fluctuation-dissipation theorem, is found to be satisfied automatically. This connects drag and momentum diffusion constants giving rise to Einstein's relation $B(p^2) = 2TEA(p^2)$, where $B_{ij}(p^2) = \delta_{ij} B(p^2)$. To remove arbitrariness related to the momentum cutoff scales hidden in $\mathcal{L}$, hard thermal loop corrected propagator is used [9].

In the coulomb gauge, we can define $D_{00} = \Delta_l$ and

$$\Delta_l(q_0, q)^{-1} = q^2 - \frac{3}{2}\omega_p^2 \left[ \frac{q_0}{q} ln \frac{q_0 + q}{q_0 - q} - 2 \right] \qquad (26)$$

$$\Delta_t(q_0, q)^{-1} = q_0^2 - q^2 + \frac{3}{2}\omega_p^2 \left[ \frac{q_0(q_0^2 - q^2)}{2q^3} ln \frac{q_0 + q}{q_0 - q} - \frac{q_0^2}{q^2} \right] \qquad (27)$$

With this, matrix element in the limit of small angle scattering, we get the following expression for the squared matrix element,

$$
\begin{aligned}
|\mathcal{M}|^2 = &g^4 C_{qq} 16 (EE_1)^2 |\Delta_l(q_0, q) \\
&+ (v \times \hat{q}).(v_1 \times \hat{q}) \Delta_t(q_0, q)|^2 \qquad (28)
\end{aligned}
$$

with $v = \hat{p}$, $v_1 = \hat{p_1}$ and $C_{qq}$ is the color factor. With the screened interaction, the drag and diffusion constants can be calculated along the line of ref. [9].

Now to calculate average energy loss of the light quarks in an expanding partonic plasma we inject test quarks having following distribution at time $t_i$,

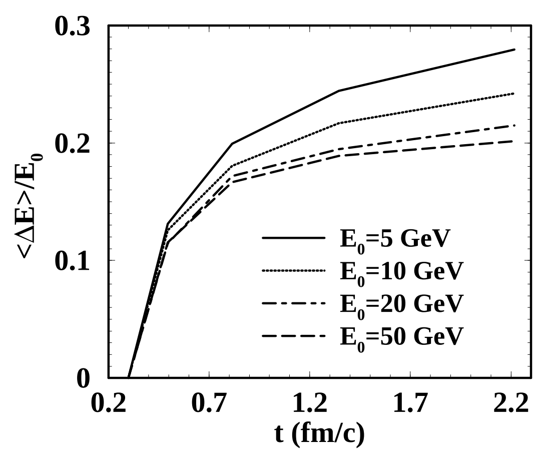
$$f(p_T, p_z, t = t_i) = N \delta^2(p_T - p_{T0}) \delta(p_z - p_{z0}) \qquad (29)$$

Bjorken cooling law [21], $\tau_i T_i^{1/c_s^2} = \tau T^{1/c_s^2}$ is used to describe the space-time evolution of the plasma. Here $c_s$ is the velocity of sound.

As the time progresses the system expands and $f(p_T, p_z, t)$ evolves according to Eq. 9, which is solved numerically. The average energy loss is given by

$$\langle \Delta E \rangle = \langle E \rangle - E_0 \qquad (30)$$

where $\langle E(t) \rangle = \int d^3p E f(p_T, p_z, t) / \int d^3p f(p_T, p_z, t)$. $E_0$ corresponds to the peak of the initial spectrum, *i.e.* $E_0 = < E(t_i) >$.



In Fig. 1, we present results for the energy loss as a function of time for various initial energies with means indicated in the legend. It might be noted that at early times fractional energy loss is independent of the average incoming energy.

Next we investigate time evolution of quark $p_T$ spectra. Assuming that the high $p_T$ partons materialize into hadrons outside the system and the hadronization process does not affect the shape of the $p_T$ distribution drastically. Under these circumstances the pion $p_T$ spectra may be taken to be proportional to quark $p_T$ spectra for a given time. Therefore, we take the initial $p_T$ distribution of quark to be proportional to the pion $p_T$ spectrum as measured in $p$-$p$ collision [23]:

$$
\begin{aligned}
f(p_T, p_z, t = t_i) &= \frac{N_0}{p_T}\frac{dN(y=0)}{d^2p_T dy} \\
&= \frac{\bar{N}_0}{p_T}\frac{1}{(1+\frac{p_T}{p_0})^\nu}, \qquad (31)
\end{aligned}
$$

where, $\nu = 9.97$ and $p_0 = 1.212$ and $N_0(\bar{N}_0)$ is the normalization constant. The final spectrum is obtained by assuming the transition temperature, $T_c \sim 190$ MeV at a time $t \sim 2$ fm/c. The results are compared with the measured $p_T$ spectrum of pions (for $p_T \gtrsim 2$ GeV) at RHIC [22] in Fig.2. The data is well described for $p_T = 3 - 8$ GeV.

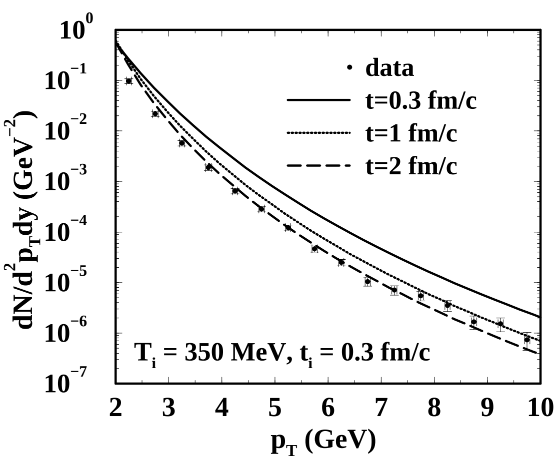



FIG. 2. Time evolution of the transverse momentum distributions.

In conclusion, in the present work, we estimate quark energy loss considering stochastic nature of the interaction by solving FP equation. Relevant drag and diffusion coefficients have been calculated in the soft collision limit. To highlight the importance of collisional energy loss radiative processes is excluded. We have analysed the pion $p_T$ spectrum measured at RHIC by the PHENIX collaboration [22]. The two body scattering is found to give reasonable amount of quenching required to explain the data. Recently transverse momentum spectrum for the $D$ meson has also been measured via single electron $p_T$ distribution [10,24]. The present formalism can be applied to analyse the data reported in ref. [24]. Such investigations are currently under progress [25].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