

The Effect of Kaon Condensation on Quark-Antiquark Condensate in Dense Matter

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Abstract

Assuming that at sufficiently high densities the constituent quarks become relevant degrees of freedom, we study within the framework of a chiral quark model the influence of s-wave K^- condensation on the quark-antiquark condensates. We find that, in linear density approximation, the presence of a K^- condensate quenches the $\bar{u}u$ condensate, but that the $\bar{d}d$ condensate remains unaffected up to the chiral order under consideration. We discuss the implication of the suppressed $\bar{u}u$ condensate for flavor-dependent chiral symmetry restoration in dense matter

1 Introduction

One of the challenging current problems in nuclear physics is to elucidate the behavior of nuclear matter under extremely high-density and/or high-temperature environments. It is theoretically expected that, at very high baryon densities (even at low temperatures), chiral symmetry is likely to be restored, and that baryon matter can be converted into quark matter; for a review, see e.g. Ref.[1]. Reasonable estimates also suggest the possible formation of a kaon condensate at high densities [2, 3, 4, 5, 6, 7, 8]. The existence of quark matter and/or a kaon condensate can have important consequences for the structure of compact stars and for the cooling behavior of a remnant star after supernova explosion and the subsequent formation of a neutron star.

Kaon condensation has been studied extensively since Kaplan and Nelson’s seminal work [2] appeared in the mid 1980’s. In a tree-order calculation in chiral perturbation theory (ChPT) based on an $SU(3)_L \times SU(3)_R$ chiral Lagrangian, Kaplan and Nelson showed that s-wave kaon condensation could occur at a density around $3\rho_0$, where ρ_0 is the normal nuclear density. It was subsequently pointed out that electrons with high chemical potential would help speed the condensation process [3]. An improved ChPT treatment of kaon condensation that goes beyond tree-order calculations and that is consistent with the empirical kaon-nucleon interactions was subsequently proposed [3,4,5], and the results indicated that the critical density ρ_c^K could lie in the range $2\rho_0 < \rho_c^K < 4\rho_0$; for a review, see e.g. Ref. [6]. A two-loop calculation of the s-wave pion and kaon self-energies in nuclear matter was carried out in Refs. [9, 10, 11]. These investigations suggested that the effective pion mass in matter is likely to be relatively stable as a function of the density. The behavior of the in-medium K^- effective mass, m_K^* , just below the K^-p threshold is less clear since it is strongly affected by the non-perturbative in-medium dynamics of the sub-threshold $\Lambda(1405)$ -resonance [5, 12, 13] and by the $\bar{K}N$ coupling to the $\pi\Sigma$ channel, see e.g. Ref.[12, 13]. This topic was discussed in detail in Ref. [14], where Pauli blocking and nucleon-nucleon short-range correlations were also taken into account in estimating m_K^* . The effects of kaon-nucleon and nucleon-nucleon correlations on m_K^* and kaon condensation in nuclear medium were studied in Ref. [15], according to which these correlations move the critical nuclear density for kaon condensation above $6\rho_0$. The competition of pion and kaon condensation and the phase diagram of a three-flavor Nambu-Jona-Lasinio model at finite temperature T and finite quark chemical potentials were investigated in Ref. [16]. At sufficiently high densities, strange-quark degrees of freedom may become relevant [17], and the presence of strange matter can push the onset of kaon condensation to higher densities and, for some choices of the input parameters that are hard to pin down, even out of the physically relevant density regime [17].

Meanwhile there have recently been interesting developments concerning deeply bound kaonic nuclear states and kaonic atoms [18]. These probes however pertain to the (near-)zero-density environment, and one should keep in mind that mechanisms extrapolated or inferred from the zero-density regime may not be operative at high densities, in particular, in the proximity of chiral phase transition where kaon condensation supposedly takes place. For instance, according to a renormalization group flow formalism, which is an appropriate framework to use when phase transitions are involved, certain terms in the Lagrangian that describe multi-body correlations may become “irrelevant” operators, playing a negligible role near the transition region [19]. This can be the case with the four-baryon interaction terms that play a significant role in the energy-density region where the above-mentioned $\Lambda(1405)$ -resonance is prominent. In other words, it is possible that these four-baryon terms are only important at very low densities

far from the high densities required for a phase transition to occur (be that the critical density for kaon condensation or chiral restoration). We also note that lattice studies of matter in heat bath indicate that the relevant fermionic degrees of freedom near the chiral transition temperature are the constituent quarks rather than the baryons. Furthermore, recent developments in holographic dual QCD [20, 21] at finite temperature indicate that, whereas chiral symmetry is definitely broken when confinement exists, the converse is not necessarily true, and that the chiral symmetry restoration scale can be much higher than the deconfinement scale. It has been shown [21] that, in a regime where chiral symmetry is broken but confinement still persists, the relevant mass scale is the constituent quark mass, not the current quark mass. Although it is perhaps unsafe to naively apply the same reasoning to the case at hand (that is, to a high-density case), a study based on skyrmion matter [22] indicates that, at high densities, basic changes in the matter structure can lead to in-medium interactions that are significantly different from those in free space. An interesting possibility is to adopt the holographic QCD models with baryons [23] to delve into kaon condensate, which, however, will not be pursued in the present work.

It is to be noted that the chiral quark model (χ QM) enables us to take into account – at least partially – the features discussed above, and we consider it illuminating to study (possible) kaon condensation in dense matter in the framework of χ QM. The relevance of χ QM in the neighborhood of chiral phase transition was discussed in detail in Refs. [24, 25, 26, 27]. In analogy to the temperature-induced chiral restoration where χ QM is invoked [28], the relevant degrees of freedom above a certain value of density, say, $\bar{\rho} > \rho_0$ ¹ can be taken to be the constituent quarks whose masses are generated via the “dressing” with the “soft” component of the gluon field, with the “hard” component hardly participating in the process. Thus one can think of the constituent quarks as quasi-particles resulting from highly nonperturbative vacuum re-structuring caused by the medium (high temperature and/or density) and hence encapsulating certain aspects of many-body correlations. As a result of this “dressing”, the constituent quarks, we may call them quaquarks, can be expected to interact weakly among themselves and with the (hard component of the) gluon field. This means that χ QM has the advantage of providing a systematic chiral power counting for the interactions of the constituent quarks with the Goldstone bosons and, in addition, allowing a weak-coupling expansion for interactions with the gluons [29]. This aspect gives a justification for us to ignore gluonic contributions in calculating the effective potential (see below). This approach to kaon condensation anchored on a chiral quark Lagrangian valid above the density $\bar{\rho}$ is consistent with – and complements – the top-down approach [31] based on expansion around the vector manifestation fixed point (which coincides with the chiral restoration point) of the Harada-Yamawaki hidden local symmetry theory [32].

In the present work we study K^- condensation in the framework of χ QM [29], and undertake the first investigation of how and to what extent a possible kaon condensate distorts the Fermi seas of the quasi-quarks and what influences this distortion can have on chiral symmetry restoration in dense matter. The issues investigated here are important in connection with phase transitions leading to color superconductivity; the behavior of the transition in the chiral quark

¹Studies based on the effective field theory treatment of nuclear matter indicate that $\bar{\rho}$ can be somewhat greater than normal matter density ρ_0 [27, 30]. The precise value of $\bar{\rho}$ which cannot be pinned down at present is not likely to be important for our purposes.

picture could be quite different from that of the standard scenario, where the transition is presumed to occur from a Fermi liquid state. In the present exploratory study, we describe quark matter as a free Fermi gas of quasi-quarks. This treatment does not explicitly take into account the strong correlations believed to be present at ordinary (low) nuclear matter density, at which three quarks are clustered into color-singlet nucleons that are spatially separated by strong short-range (nucleon-nucleon) repulsion. However, there exists the expectation that, unlike the baryons, the constituent quarks are not susceptible to strong short-range correlations [28], and hence the neglect of correlation effects is likely to be a less serious problem in the χ QM approach than in the baryonic picture. Although this problem warrants further examinations, we limit ourselves here to the Fermi gas model and investigate (within the confine of this model) the consequences of the χ QM Lagrangian – assumed to be valid above a few times the nuclear matter density – on kaon condensation and also the effects of kaon condensation (if it occurs) for the quark-antiquark condensate.

The paper is organized as follows. In section 2 we first give a brief recapitulation of χ QM, and then we demonstrate that, within the framework of linear density approximation, the use of χ QM for describing K^- condensation essentially reproduces the results obtained both in heavy-baryon chiral perturbation theory (HBChPT) [6] and in a formalism that describes fluctuations around the vector manifestation fixed point of hidden local symmetry theory [31]. In section 3 we examine the effects of K^- condensation on the quark-antiquark condensate $\langle \bar{q}q \rangle$ in the framework of χ QM and in HBChPT. Section 4 is dedicated to summary. In Appendix A, we present a brief discussion on the power counting rules in χ QM.

2 Kaon condensation in the chiral quark model

The chiral quark model (χ QM) we employ here is defined by the Lagrangian [29]

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_M + \mathcal{L}_{m_\phi}, \quad (1)$$

where the chiral-symmetry invariant part \mathcal{L}_0 is given by

$$\begin{aligned} \mathcal{L}_0 = & \bar{\psi}(i\not{D} + \not{Y})\psi + g_A \bar{\psi} \not{A} \gamma_5 \psi - M_0 \bar{\psi} \psi \\ & + \frac{1}{4} f_\pi^2 \text{Tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) - \frac{1}{2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \dots \end{aligned} \quad (2)$$

where $G^{\mu\nu}$ is the QCD field tensor. The covariant derivative is defined by

$$D_\mu = \partial_\mu + igG_\mu; \quad G_\mu = G_\mu^a T^a; \quad (3)$$

where G_μ^a is the gluon field with $a = 1, \dots, 8$ and

$$\begin{aligned} V_\mu &= \frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger); \quad A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger); \\ \psi &= \begin{pmatrix} u \\ d \\ s \end{pmatrix}; \quad \xi = e^{(i\Pi/f_\pi)}; \quad \Sigma = \xi \xi \end{aligned}$$

with

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (4)$$

Here $f_\pi \simeq 93$ MeV, and $M_0 \approx 350$ MeV denotes the part of the constituent quark mass generated by spontaneous chiral symmetry breaking which constitutes the bulk of the effective quark mass. The chiral symmetry-breaking term \mathcal{L}_M is given by

$$\mathcal{L}_M = -\frac{1}{2}c_1\bar{\psi}(\xi^\dagger\mathcal{M}\xi^\dagger + \xi\mathcal{M}\xi)\psi \quad (5)$$

where

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix},$$

and $c_1 \approx 1$ [29], which is fixed from the mass difference (~ 150 MeV) between the s quark and the u (or d) quark. We will assume $c_1 = 1$ in this work. \mathcal{L}_{m_ϕ} in Eq. (1), which is responsible for the finite Goldstone boson masses, takes the form

$$\mathcal{L}_{m_\phi} = \frac{1}{2}f_\pi^2\text{Tr}(\mu\mathcal{M}\Sigma^\dagger) + h.c. , \quad (6)$$

where μ is a parameter with the dimension of mass.

χ QM offers a systematic chiral power counting expansion in describing the interactions of constituent quarks with Goldstone bosons (summarized in Appendix A). To estimate higher-loop corrections, we need to set up in-medium power counting rules in χ QM.² To introduce in-medium counting rules applicable to dense matter, we assume that $p \sim k_F$, where p is a typical momentum scale and k_F is the quark Fermi momentum. Since the u - and d quark Fermi momentum at $4\rho_0$ (symmetric matter) is about 430 MeV, the expansion parameter related to the Fermi momentum is $k_F/\Lambda \sim 0.43$ ($\Lambda = 1$ GeV). With the assumption $p \sim k_F$, it is easy to establish that the in-medium counting rules agree with the free-space counting rules; see Eq.(A.4) in Appendix A. It is to be noted that in ChPT involving the nucleons, in-medium chiral counting contains a subtle issue related to the ‘‘heavy’’ nucleon mass [33]. No such issue arises in χ QM, since the constituent quark masses are of the same order as the typical momentum scale p . In addition, χ QM allows for a perturbative expansion of the constituent-quark gluon interaction (with $\alpha_s \approx 0.28$) [29]. This scheme contains no free parameters to the chiral order we will consider in this work and to lowest order in α_s . Those terms in the Lagrangian Eq. (1) which are relevant for our discussion of kaon condensation in quark matter can be written as

$$\begin{aligned} \mathcal{L}_K &= \frac{i}{4f_\pi^2}[\bar{u}(K^+ \not{\partial}K^- - (\not{\partial}K^+)K^-)u + \bar{s}(K^- \not{\partial}K^+ - (\not{\partial}K^-)K^+)s] \\ &+ \frac{1}{2f_\pi^2}(m_u + m_s)[\bar{u}K^+K^-u + \bar{s}K^-K^+s]. \end{aligned} \quad (7)$$

² In-medium chiral counting rules in the hadronic picture have been discussed at length in Refs. [33].

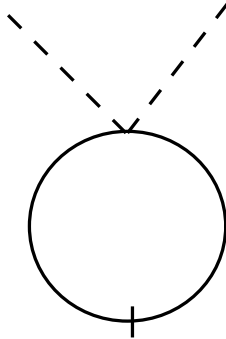


Figure 1: The kaon self-energy contribution in quark matter at lowest chiral order. The dashed line stands for the kaon, and the solid line for the quark. The quark propagator line marked with “|” denotes the density-dependent part of the quark propagator (at zero temperature), $-2\pi\delta(k^2 - M_f^2)\theta(k_0)N_F(k_0)|_{T\rightarrow 0}$, where $N_F(k_0)$ is the Fermi-Dirac distribution function and M_f is the mass of the constituent quark of flavor f .

In order to establish a basis for our work, we first describe kaon condensation in the framework of χ QM with the purpose of comparing our results with those obtained in HBChPT [6]. We calculate the K^- self-energy using the Lagrangian \mathcal{L}_K in Eq.(7). The lowest-order contributions come from the graphs shown in Fig. 1 and Fig. 2. As we will show below, however, the contribution from Fig. 2 is negligible compared to the one from Fig. 1. Higher chiral-order contributions will not be discussed in this work.

We consider kaons in medium that solely consists of up and down quarks with no strange-quarks in the Fermi sea. Our quark matter is assumed to be symmetric with respect to the u- and d-quarks, so the quark densities in the present case are characterized by $\rho_u = \rho_d \equiv \rho_q$ and $\rho_s = 0$, while the baryon density ρ_B is given by $\rho_B = \frac{2}{3}\rho_q$. To establish connection with the previous works, we will demonstrate that, within the framework of the linear density approximation, the χ QM approach to kaon condensation leads to results similar to those obtained in HBChPT[3, 4, 5, 6] and those obtained in an expansion around the vector manifestation fixed point [31]. To compare with the HBChPT calculations, we need to use non-relativistic approximation in evaluating the kaon self-energy in Fig. 1. Since the constituent quark mass, $M_0 \approx 350$ MeV, is smaller than the chiral scale, $\Lambda \simeq 1$ GeV, and is of the order of k_F , this non-relativistic approximation might not be very reliable. It is used here *only* for the sake of comparison with previous works. Using the lagrangian in Eq.(7), we find

$$-i\Sigma_K(q_0) = i \left[\frac{3}{4} \frac{(m_u + m_s)}{f_\pi^2} + \frac{3}{4} \frac{q_0}{f_\pi^2} \right] \rho_B \quad (8)$$

The in-medium kaon mass m_K^* is then obtained by solving the dispersion equation

$$m_K^{*2} = m_K^2 + \Sigma_K(q_0 = m_K^*), \quad (9)$$

where $m_K \approx 500$ MeV is the free kaon mass. We define $x = \frac{m_K^*}{m_K}$ and $c = \rho_B/\rho_0$, where $\rho_0 = 0.17$ fm $^{-3}$ is the normal nuclear matter density, to rewrite the dispersion equation as

$$x^2 + 0.24cx + 0.12c - 1 = 0, \quad (10)$$

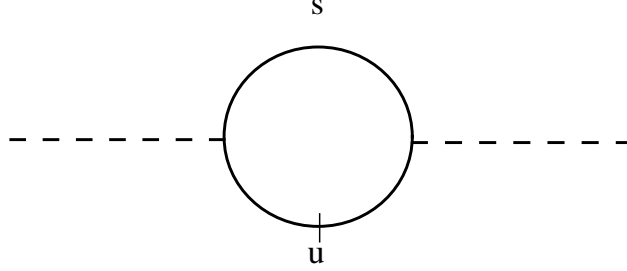


Figure 2: Kaon self-energy. The dashed lines stand for kaons and the solid lines for quarks. The symbol “|” denotes the density-dependent part of the quark propagator.

where we have used for the current quark masses $m_u \approx 6$ MeV, $m_s \approx 240$ MeV as in [6]. Solving this equation for typical values of c , we arrive at

$$\begin{aligned}
 c = 1 & : m_{K^-}^* \approx 410 \text{ MeV} \\
 c = 2 & : m_{K^-}^* \approx 330 \text{ MeV} \\
 c = 3 & : m_{K^-}^* \approx 260 \text{ MeV} \\
 c = 4 & : m_{K^-}^* \approx 193 \text{ MeV}.
 \end{aligned} \tag{11}$$

We note that the HBChPT calculation in Ref. [6] finds $m_{K^-}^* \approx 360$ MeV for $c = 1$. Since the electron chemical potential μ_e is known to have values $200 \text{ MeV} < \mu_e < 300 \text{ MeV}$ in the density range $\rho_B = (2-4)\rho_0$ [4], one may infer that kaon condensation takes place around $\rho_B \sim (3-4)\rho_0$. In the above we ignored the contribution from the diagram in Fig.2 and furthermore, to facilitate comparison with HBChPT, we used the heavy-fermion approximation for the quark propagator. We now examine whether the contribution of Fig. 2 to s-wave kaon condensation is indeed negligible. We also study the consequences of the relativistic treatment of the quark propagator. To this end, we calculate *relativistically* the contributions of the diagrams in Figs. 1 and 2. For SU(2) symmetric matter, the kaon self-energy, $-i\Sigma_K^{ex}(q_0)$, corresponding to Fig. 2 is given by

$$\Sigma_K^{ex}(q_0) = \frac{g_A^2}{2\pi^2 f_\pi^2} q_0^2 \int_0^{k_F} d\bar{k} \left(\frac{\bar{k}^2}{k_0} \right) \left(\frac{2k_0^2 - M_u^2 + q_0 k_0 - M_u M_s}{q_0^2 + 2q_0 k_0 + \Delta M^2} \right), \tag{12}$$

where $\bar{k} = |\vec{k}|$, $k_0 = \sqrt{\bar{k}^2 + M_u^2}$ and $\Delta M^2 = M_u^2 - M_s^2$, with M_u and M_s being the masses of the constituent u - and s -quarks, respectively. We again solve the dispersion equation, $m_{K^-}^{*2} = m_K^2 + \Sigma_K(q_0 = m_{K^-}^*)$, with the kaon self-energy Σ_K which includes the contributions of the diagrams in Fig. 1 and Fig. 2. The results are given in Eq.(13). For each row, the number that appears to the left (right) of the arrow is the result that excludes (includes) the contribution of Fig. 2.

$$\begin{aligned}
 c = 1 & : m_{K^-}^* \approx 415 \text{ MeV} \rightarrow 424 \text{ MeV} \\
 c = 2 & : m_{K^-}^* \approx 341 \text{ MeV} \rightarrow 361 \text{ MeV} \\
 c = 3 & : m_{K^-}^* \approx 278 \text{ MeV} \rightarrow 313 \text{ MeV} \\
 c = 4 & : m_{K^-}^* \approx 225 \text{ MeV} \rightarrow 283 \text{ MeV}.
 \end{aligned} \tag{13}$$

Comparison of the results in Eq.(11) with those in Eq.(13) reveals that the relativistic corrections are small, ranging from about 1% to $\sim 17\%$. Treating the diagram in Fig.1 relativistically

increases the value of $m_{K^-}^*$ compared to the value of the non-relativistic treatment, especially for the higher densities $\rho_B \sim (3-4)\rho_0$. As mentioned, the change in the value $m_{K^-}^*$ that occurs as we add the contribution of Fig. 2 to that of Fig. 1 is indicated by the arrow in Eq.(13). It is seen that the effect of the diagram in Fig.2 is a little bit bigger than the relativistic corrections, and it could push up the critical density for K^- condensation slightly.

3 Kaon Condensation and Quark-antiquark Condensate.

As discussed above, it is possible that a K^- condensate exists at high matter densities. To confirm the existence of K^- condensation in the framework of χ QM, we need to consider the higher order corrections in a systematic way. In this section, however, we assume that K^- condensation is realized in dense nuclear matter. Now, if a kaon condensate is formed, the values of various quantities that characterize the physical condition of dense matter can also change. In this subsection we study the influence of K^- condensation (if it occurs) on the quark-antiquark condensate $\langle \bar{q}q \rangle_{\rho_B}$ in dense matter. The purpose is to examine the effects of a postulated kaon condensate on the possible restoration of chiral symmetry in dense matter. We will continue to consider symmetric matter with no strangeness specified by $\rho_u = \rho_d \equiv \rho_Q = (3/2)\rho_B$ and $\rho_s = 0$.

3.1 Quark-antiquark condensate in the chiral quark model

We begin with a brief summary of the quark-antiquark condensate at low density. From the energy density of quark matter calculated in the standard manner, we can obtain the in-medium quark condensate with the help of the Hellmann-Feynman theorem and the Gell-Mann-Oakes-Renner (GMOR) relation, $2m_q \langle \bar{q}q \rangle_{vac} = -m_\pi^2 f_\pi^2$, where $\langle \mathcal{O} \rangle_{vac}$ represents the vacuum expectation value of the operator \mathcal{O} . The resulting expression for the quark condensate in medium is

$$\langle \bar{q}q \rangle_{\rho_B} = \langle \bar{q}q \rangle_{vac} + \frac{1}{2} \frac{d\tilde{\epsilon}}{dm_q} \quad (14)$$

where $\tilde{\epsilon}$ is the energy density, and m_q is the current quark mass, $m_q \equiv (m_u + m_d)/2$. We have assumed here that SU(2) isospin symmetry is conserved and defined $\bar{q}q \equiv (\bar{u}u + \bar{d}d)/2$. At low density, Eq. (14) leads to the model independent result

$$\frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_{vac}} \simeq 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2} \rho_B, \quad (15)$$

where σ_N is the nucleon sigma-term, see, *e.g.*, Ref.[34] for details. The evaluation of corrections to Eq.(15) requires model calculations.³

We now consider the effects a kaon condensate can have on the quark-antiquark condensate in matter. We will find that the existence of a K^- condensate leads to asymmetry

³ In one such model calculation [34] based on the hadronic picture with the parameters M_N , m_π and $g_{\pi NN}$ characterizing the Lagrangian, Eq. (14) is evaluated from

$$\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{q}q \rangle_{vac} + \frac{1}{2} \left[\frac{dM_N}{dm_q} \frac{\partial \tilde{\epsilon}}{\partial M_N} + \frac{dm_\pi}{dm_q} \frac{\partial \tilde{\epsilon}}{\partial m_\pi} + \frac{dg_{\pi NN}}{dm_q} \frac{\partial \tilde{\epsilon}}{\partial g_{\pi NN}} \right]. \quad (16)$$

between $\langle \bar{u}u \rangle_{\rho_B}$ and $\langle \bar{d}d \rangle_{\rho_B}$. In the following therefore we treat $\langle \bar{u}u \rangle_{\rho_B}$ and $\langle \bar{d}d \rangle_{\rho_B}$ as independent quantities. In χ QM characterized by the Lagrangian Eq.(2), we are inspired by [34] to write:

$$\langle \bar{q}_f q_f \rangle_{\rho_B} = \langle \bar{q}_f q_f \rangle_{vac} + \frac{1}{2} \left[\frac{\partial \tilde{\epsilon}}{\partial m_f} + \frac{dM_f}{dm_f} \frac{\partial \tilde{\epsilon}}{\partial M_f} + \frac{dm_\phi^2}{dm_f} \frac{\partial \tilde{\epsilon}}{\partial m_\phi^2} \right] \quad (17)$$

where $\phi = \pi, \eta, \text{ or } K$,⁴ while $f = u, d$, and $\langle \bar{q}_u q_u \rangle = \langle \bar{u}u \rangle$, $\langle \bar{q}_d q_d \rangle = \langle \bar{d}d \rangle$; $\tilde{\epsilon}$ is the energy density, and m_ϕ is the finite Goldstone boson mass generated by explicit chiral symmetry breaking. Here we assume that the coupling constants, g_A, g and f_π defined in Eqs.(2) and (3), do not depend on the current quark mass, e.g. $dg/dm_f = 0$. If we assume the presence of a K^- condensate in χ QM, a consideration similar to the one used in Ref. [3] leads to the following energy density:

$$\begin{aligned} \tilde{\epsilon} = & \frac{3}{4\pi^2} \sum_{f=u,d} [k_F^f (k_F^{f2} + M_f^2)^{3/2} - \frac{1}{2} M_f^2 k_F^f \sqrt{k_F^{f2} + M_f^2} - \frac{1}{2} M_f^4 \ln \frac{k_F^f + \sqrt{k_F^{f2} + M_f^2}}{M_f}] \\ & - \frac{1}{2} f_\pi^2 \mu_e^2 \sin^2 \theta + 2m_K^2 f_\pi^2 \sin^2 \frac{\theta}{2} - \mu_e \rho_u \sin^2 \frac{\theta}{2} - m' \rho_s^u \sin^2 \frac{\theta}{2}, \end{aligned} \quad (18)$$

where ρ_s^u is the scalar density of the u quark and $m' = m_u + m_s$. Furthermore, the ‘‘chiral angle’’ θ in V-spin space is given by $\theta \equiv \sqrt{2}v/f_\pi$, where v is the magnitude of the K^- condensate. Considering the energy density in χ QM given in Eq.(18), we write Eq. (17) in terms of $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$:

$$\langle \bar{u}u \rangle_{\rho_B} = \langle \bar{u}u \rangle_{vac} + \frac{1}{2} \left[\frac{\partial \tilde{\epsilon}}{\partial m_u} + \frac{dM_u}{dm_u} \frac{\partial \tilde{\epsilon}}{\partial M_u} + \frac{dm_K^2}{dm_u} \frac{\partial \tilde{\epsilon}}{\partial m_K^2} \right] \quad (19)$$

$$\langle \bar{d}d \rangle_{\rho_B} = \langle \bar{d}d \rangle_{vac} + \frac{1}{2} \left[\frac{dM_d}{dm_d} \frac{\partial \tilde{\epsilon}}{\partial M_d} + \frac{dm_K^2}{dm_d} \frac{\partial \tilde{\epsilon}}{\partial m_K^2} \right]. \quad (20)$$

In the expression for the energy density Eq.(18), $\langle \bar{d}d \rangle_{\rho_B}$ is independent of the K^- condensate amplitude θ , which will be shown explicitly below. We therefore focus on $\langle \bar{u}u \rangle_{\rho_B}$. In order to evaluate Eq.(19) and deduce the density dependence of the quark-antiquark condensate, we require specific information about $\frac{dM_f}{dm_f}$ and $\frac{dm_\phi^2}{dm_f}$.

To determine dM_f/dm_f , we compare Eq.(17) with the model-independent result in Eq.(15) at low density. Ignoring interactions among the constituent quarks and the contribution from constituent quark kinetic energy, which seems to be reasonable at low density [34], we obtain the following energy density in χ QM,

$$\tilde{\epsilon} = M_u \rho_u + M_d \rho_d, \quad (21)$$

where $\rho_f \equiv (k_F^f)^3/\pi^2$. For symmetric matter we obtain

$$\frac{\langle \bar{u}u \rangle_{\rho_B}}{\langle \bar{u}u \rangle_{vac}} = 1 - \frac{3\sigma_u}{2m_\pi^2 f_\pi^2} \rho_B, \quad (22)$$

⁴In the last term of Eq. (17), we need to sum over all Goldstone bosons such as contributions from pions [35]. In the present work, however, we only include the contribution from kaon condensate to highlight its specific effects on quark-antiquark condensate.

where $\sigma_u \equiv m_u dM_u/dm_u$, and we have used the GMOR relation. Comparing Eq. (22) with Eq. (15), we arrive at $\sigma_u = 2\sigma_N/3$. For the d quark we obtain $\sigma_d = 2\sigma_N/3$.⁵ To proceed we now assume that this relation is also valid up to a few times the normal nuclear matter density ρ_0 . For a numerical estimate we use the value $\sigma_N = 30$ MeV [36].

Next we evaluate dm_K^2/dm_f . The mass term in Eq. (6) leads to $m_K^2 = \mu(m_u + m_s)$, from which we obtain [34]⁶

$$\frac{dm_K^2}{dm_u} = \frac{m_K^2}{m_u + m_s} \quad (= \mu), \quad \frac{dm_K^2}{dm_d} = 0. \quad (23)$$

The amplitude of the kaon condensate θ can be determined by extremizing Eq. (18) with respect to θ . The result is

$$\cos \theta = \frac{1}{f_\pi^2 \mu_e^2} (m_K^2 f_\pi^2 - 0.5 \mu_e \rho_u - 0.5 m' \rho_s^u). \quad (24)$$

Now we consider in-medium $\langle \bar{d}d \rangle$ in a kaon-condensed phase. From Eq. (20) and Eq. (23), we obtain

$$\langle \bar{d}d \rangle_{\rho_B} = \langle \bar{d}d \rangle_{vac} + \frac{1}{2} \frac{dM_d}{dm_d} \frac{\partial \tilde{\epsilon}}{\partial M_d}. \quad (25)$$

This equation together with the energy density in Eq. (18) show that the K^- condensate will not affect the $\bar{d}d$ condensate. To see the effects of kaon condensation on the u quark condensate $\langle \bar{u}u \rangle$, we rewrite Eq. (19) as

$$R \equiv \frac{\langle \bar{u}u \rangle_{\rho_B}}{\langle \bar{u}u \rangle_{vac}} = 1 - \frac{m_u}{m_\pi^2 f_\pi^2} \left[\frac{\partial \tilde{\epsilon}}{\partial m_u} + \frac{\sigma_u}{m_u} \frac{\partial \tilde{\epsilon}}{\partial M_u} + \frac{m_K^2}{m'} \frac{\partial \tilde{\epsilon}}{\partial m_K^2} \right], \quad (26)$$

where we have used Eq.(23) and have taken $m_u = m_q$. The above expressions lead to

$$\begin{aligned} \frac{\partial \tilde{\epsilon}}{\partial m_u} &= f_\pi^2 \mu_e^2 \cos \theta \frac{\partial \cos \theta}{\partial m_u} - m_K^2 f_\pi^2 \frac{\partial \cos \theta}{\partial m_u} \\ &\quad + \frac{1}{2} \mu_e \rho_u \frac{\partial \cos \theta}{\partial m_u} - \rho_s^u \sin^2 \frac{\theta}{2} + \frac{1}{2} m' \rho_s^u \frac{\partial \cos \theta}{\partial m_u}, \\ \frac{\partial \tilde{\epsilon}}{\partial M_u} &= \frac{3}{2\pi^2} (M_u k_F^u \sqrt{k_F^u{}^2 + M_u^2} - M_u^3 \log \frac{k_F^u + \sqrt{k_F^u{}^2 + M_u^2}}{M_u}) \end{aligned}$$

⁵As mentioned in the introduction, the precise value of $\bar{\rho}$ is not well determined at present, but this is not so important in the present work. What matters here is the fact that the relevant degrees of freedom near chiral symmetry restoration are the quasiquarks rather than the baryons.

⁶A comment is in order here on how the nonlinear quark mass dependence of the Goldstone boson mass affects Eq. (23). The following terms give m_q^2 corrections to the Goldstone boson masses,

$$\frac{f_\pi^2}{4} c_1 \text{Tr}(\mathcal{M}^\dagger \Sigma) \text{Tr}(\mathcal{M}^\dagger \Sigma) + \frac{f_\pi^2}{4} c_2 \text{Tr}(\mathcal{M}^\dagger \Sigma \mathcal{M}^\dagger \Sigma) + \frac{f_\pi^2}{4} c_3 \text{Tr}(\mathcal{M}^\dagger \Sigma) \text{Tr}(\mathcal{M} \Sigma^\dagger) + h.c.,$$

where c_1 , c_2 and c_3 are dimensionless parameters assumed to be of the order of 1. These terms lead to

$$\frac{dm_K^2}{dm_u} = \frac{m_K^2}{m_u + m_s} \left\{ 1 + 2(c_1 + c_2 + c_3) \frac{(m_u + m_s)^2}{m_K^2} + (c_1 + c_3) \frac{m_d(m_u + m_s)}{m_K^2} \right\}.$$

These corrections are suppressed by a factor $(m_q/m_K)^2$ compared to the leading term in Eq. (23).

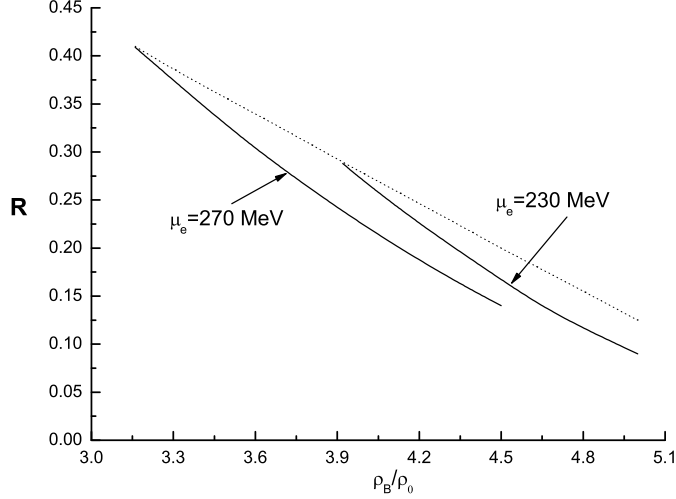


Figure 3: $R \equiv \langle \bar{u}u \rangle_{\rho_B} / \langle \bar{u}u \rangle_{vac}$ calculated with (solid line) and without (dotted line) kaon condensation taken into account. The results are given for two different values of the electron chemical potential, $\mu_e = 230$ and 270 MeV.

$$\begin{aligned}
& + f_\pi^2 \mu_e^2 \cos \theta \frac{\partial \cos \theta}{\partial M_u} - m_K^2 f_\pi^2 \frac{\partial \cos \theta}{\partial M_u} + \frac{1}{2} \mu_e \rho_u \frac{\partial \cos \theta}{\partial M_u} \\
& + \frac{1}{2} m' \rho_s^u \frac{\partial \cos \theta}{\partial M_u} - m' \sin^2 \frac{\theta}{2} \frac{\partial \rho_s^u}{\partial M_u} \\
\frac{\partial \tilde{\epsilon}}{\partial m_K^2} & = f_\pi^2 \left(1 - \frac{m_K^2}{\mu_e^2} \right) + \frac{1}{2 \mu_e} \rho_u + \frac{m'}{2 \mu_e^2} \rho_s^u, \tag{27}
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial \cos \theta}{\partial m_u} & = -\frac{\rho_s^u}{2 f_\pi^2 \mu_e^2}, \\
\frac{\partial \cos \theta}{\partial M_u} & = -\frac{m'}{2 f_\pi^2 \mu_e^2} \frac{\partial \rho_s^u}{\partial M_u} \\
\frac{\partial \rho_s^u}{\partial M_u} & = \frac{3}{2 \pi^2} \left[k_F^u \sqrt{k_F^{u2} + M_u^2} + \frac{2 M_u^2 k_F^u}{\sqrt{k_F^{u2} + M_u^2}} - 3 M_u^2 \ln \frac{k_F^u + \sqrt{k_F^{u2} + M_u^2}}{M_u} \right]. \tag{28}
\end{aligned}$$

If we use for illustrative purposes $\mu_e = 270$ MeV, $\rho_B = 3.3 \rho_0$, $\sigma_N = 30$ MeV and $\cos \theta = 0.89$ in Eq. (26), we arrive at $R \sim 0.35$. This should be compared with $R \sim 0.39$ that would result if there is no kaon condensation ($\theta = 0$). The results for R are also shown in Fig. 3 for typical cases of $\sigma_N = 30$ MeV and $\mu_e = 230$ and 270 MeV.⁷ Fig. 3 demonstrates that the presence of a kaon condensate leads to a faster decrease of $\langle \bar{u}u \rangle$ with increasing density. By contrast, the d quark condensate, $\langle \bar{d}d \rangle$, is not affected by the presence of K^- condensation to the lowest order in density under consideration. We therefore expect that in the kaon condensed

⁷For simplicity, the chemical potential μ_e is treated here as an external parameter, although in a full treatment it should be determined self-consistently, see, *e.g.*, Ref. [4, 37].

phase $\langle \bar{u}u \rangle / \langle \bar{d}d \rangle \rightarrow 0$ as ρ_B increases. It is generally expected that chiral symmetry restoration characterized by $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = 0$ occurs in dense matter.⁸ Our results indicate, however, that at densities between ρ_c^K and $\rho_c^{\chi SR}$ ($\rho_c^K < \rho_B < \rho_c^{\chi SR}$), kaon condensation may lead to a phase characterized by $\langle \bar{u}u \rangle \simeq 0$, $\langle \bar{d}d \rangle \neq 0$, which represents partial “flavor-dependent restoration” of chiral symmetry.

Finally, we observe that a K^- condensate gives rise to difference between the masses of the constituent u and d quarks. It is easy to show

$$\begin{aligned} M_u^* &= M_0 + m_u - \frac{m'}{2f_\pi^2} v^2 \\ M_d^* &= M_0 + m_d \quad (= M_d) . \end{aligned} \quad (29)$$

Thus the presence of a K^- condensate induces additional $SU(2)$ -isospin symmetry breaking on top of the small explicit isospin breaking due to $m_u - m_d \neq 0$.

3.2 Quark-antiquark condensate in HBChPT.

The results shown in previous subsection were obtained with the use of the energy density calculated in χ QM *without* taking into account the beta-equilibrium condition or the charge-neutrality condition. In order to check whether the imposition of these constraints affects our results, we evaluate in this section the quark-antiquark condensates in a kaon condensed phase using the energy density that has been calculated in HBChPT with the beta-equilibrium and charge-neutrality conditions taken into account [3, 4, 37]. The energy density ϵ_{HB} in the K^- condensed phase obtained from HBChPT is [3, 4, 37]:

$$\begin{aligned} \epsilon_{\text{HB}} &= \frac{3}{5} E_F^{(0)} u^{\frac{5}{3}} \rho_0 + V(u) \\ &+ u \rho_0 (1 - 2x)^2 S(u) - \frac{\mu^2}{2} f_\pi^2 \sin^2 \theta + 2m_K^2 f_\pi^2 \sin^2 \frac{\theta}{2} + \mu u \rho_0 x \\ &- \mu u \rho_0 (1 + x) \sin^2 \frac{\theta}{2} + (2a_1 x + 2a_2 + 4a_3) m_s u \rho_0 \sin^2 \frac{\theta}{2} - \frac{\mu^4}{12\pi^2} , \end{aligned} \quad (30)$$

where $E_F^{(0)} = (p_F^{(0)})^2 / (2m_N)$ and $p_F^{(0)} = (3\pi^2 \rho_0 / 2)^{1/3}$ are the Fermi energy and the Fermi momentum at normal nuclear matter density, x denotes the proton fraction $\rho_p = x \rho_B$, and u is defined by $\rho_B = u \rho_0$.⁹ $V(u)$ is the charge-symmetric contribution of the nuclear interactions, while $S(u)$ is the symmetry energy parameterized as

$$S(u) = (2^{2/3} - 1) \frac{3}{5} E_F^{(0)} (u^{2/3} - F(u)) + S_0 F(u) , \quad (31)$$

where $S_0 \simeq 30$ MeV and we choose $F(u) = u$. To concentrate on the effects of kaon condensation on the quark-antiquark condensate, we may ignore the first two terms in Eq. (30), which are independent of kaon condensation. Furthermore, for simplicity, we ignore the muon Fermi sea.

⁸Another order parameter for chiral symmetry breaking is the pion decay constant f_π in the chiral limit $m_q = 0$. Even if $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = 0$, chiral symmetry might be still broken, as long as $f_\pi \neq 0$ [33]. Such a pseudo-gap phenomenon is observed in dense skyrmion crystal matter [22].

⁹To conform with the expressions in the literature, we adopt the symbol u here instead of c used in section 2.

The energy of the charge-neutral ground state for a given baryon density is determined by extremizing ϵ_{HB} with respect to x , μ and θ

$$\frac{\partial \epsilon_{\text{HB}}}{\partial x} = 0, \quad \frac{\partial \epsilon_{\text{HB}}}{\partial \mu} = 0, \quad \frac{\partial \epsilon_{\text{HB}}}{\partial \theta} = 0. \quad (32)$$

We refer to Ref. [4] for the explicit expressions for these constraints. The numerical solutions for the above equations can be found in Tables 3, 4 and 5 of Ref. [4]. Following the same procedure as used in subsection 3.1, we obtain

$$\begin{aligned} \langle \bar{u}u \rangle &= 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2} \left[\frac{\partial \epsilon_{\text{HB}}}{\partial m_N} + \frac{m_u}{\sigma_N} \frac{m_K^2}{m'} \frac{\partial \epsilon_{\text{HB}}}{\partial m_K^2} \right] \\ \langle \bar{d}d \rangle &= 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2} \frac{\partial \epsilon_{\text{HB}}}{\partial m_N}, \end{aligned} \quad (33)$$

where

$$\frac{\partial \epsilon_{\text{HB}}}{\partial m_N} = -\frac{1}{m_N} u \rho_0 (1-2x)^2 (2^{2/3} - 1) \frac{3}{5} E_F^{(0)} (u^{2/3} - u) \quad (34)$$

$$\frac{\partial \epsilon_{\text{HB}}}{\partial m_K^2} = 2 f_\pi^2 \sin^2 \frac{\theta}{2}. \quad (35)$$

Here $\sigma_N = m_q dm_N / dm_q$ with $m_q = (m_u + m_d)/2$. We assume that $\sigma_N = \sigma_N^{(u)} (m_u dm_N / dm_u) = \sigma_N^{(d)} (m_d dm_N / dm_d)$. It is now straightforward to deduce the quark-antiquark condensate in a kaon-condensed phase with the use of ϵ_{HB} along with the numerical tables (Tables 3, 4 and 5) in Ref. [4]. The results are given in Table 1 for the representative values of input parameters, $\sigma_N = 30$ MeV, $a_1 m_s = -67$ MeV, $a_2 m_s = 134$ MeV and $a_3 m_s = (-134, -222, -310)$ MeV. Note that we have ignored the density dependence of the quark-antiquark condensate that is independent of kaon condensation. Table 1 indicates that the main conclusion of subsection 3.1 remains essentially unchanged by the imposition of the β -equilibrium and charge-neutrality conditions; namely, $\frac{\langle \bar{u}u \rangle_{\rho_B}}{\langle \bar{d}d \rangle_{\rho_B}} < 1$ in kaon condensed phase.

Table 1 : The ratios of quark-antiquark condensates in kaon condensed phase. Here $R_u = \frac{\langle \bar{u}u \rangle_{\rho_B}}{\langle \bar{u}u \rangle_{vac}}$ and $R_d = \frac{\langle \bar{d}d \rangle_{\rho_B}}{\langle \bar{d}d \rangle_{vac}}$. $a_3 m_s$ and μ are given in units of MeV.

$a_3 m_s$	u	μ	R_u	R_d
-134	7.18	93.7	0.834	0.9998
-134	7.68	73.5	0.822	0.9999
-222	4.08	98.7	0.865	0.9997
-222	4.58	38	0.812	0.9999
-310	2.92	86	0.875	0.9999

We notice that $\langle \bar{d}d \rangle_{\rho_B}$ in Table 1 exhibits slight dependence on the kaon condensate, whereas $\langle \bar{d}d \rangle_{\rho_B}$ in subsection 3.1 shows no such dependence. This difference can be easily explained by the fact that the symmetry energy $S(u)$, which is responsible for the θ -dependence of $\langle \bar{d}d \rangle_{\rho_B}$, cannot arise in tree-level or one-loop-order calculations in the chiral quark model, and

therefore the effects subsumed in $S(u)$ are missing in the energy density calculated in subsection 3.1.

Before closing this subsection, we discuss the effects of spontaneous isospin violation summarized in Eq. (29) on the nucleon mass. In the simplest valence quark picture in which the proton (neutron) contains two constituent u -quarks and one constituent d -quark (one d -quark and two u -quarks), we expect from Eq. (29) that the in-medium proton and neutron masses in a kaon condensed phase decreases with the density faster than in a normal phase (without kaon condensation).¹⁰ This feature is in qualitative agreement with the result in Ref. [38]. The nucleon effective masses in a kaon condensed phase is studied in the context of a relativistic mean-field model in Ref. [38], and it was found that the nucleons can have different effective masses in normal and kaon condensed phases.

4 Summary

We have discussed s -wave K^- condensation in the framework of the chiral quark model, assuming that, in the density regime close to the critical density, the relevant degrees of freedom are the constituent quark degrees of freedom. We have primarily investigated the effects of charged kaon condensation on the quark-antiquark condensate and we have found that a K^- condensate in quark matter suppresses the quark-antiquark condensate for the u quark, $\langle \bar{u}u \rangle$, but leaves $\langle \bar{d}d \rangle$ unaffected in the lowest order approximation adopted here. This suggests the possibility that a partial chiral symmetry restoration in the medium with a K^- condensate may be flavor dependent, *i.e.*, $\langle \bar{u}u \rangle / \langle \bar{d}d \rangle \ll 1$ for increasing density. This raises an interesting question as to whether or not the vector manifestation fixed point one finds in approaching a chiral restoration point from normal Fermi liquid remains intact if the chiral restoration point is approached from a kaon condensed state with its distorted Fermi seas of quarks.

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Appendix A

In this appendix, we state the power counting rules for χ QM. Since we can treat the gluons perturbatively with $\alpha_s \approx 0.28$ [29], it suffices to focus on the Goldstone bosons and quarks.

The most general vertex in χ QM in a cutoff regularization scheme takes the form [29],

$$(2\pi)^4 \delta^4(\sum p_i) \left(\frac{\pi}{f_\pi}\right)^A \left(\frac{\psi}{f_\pi \sqrt{\Lambda}}\right)^B \left(\frac{g G_\mu}{\Lambda}\right)^C \left(\frac{p}{\Lambda}\right)^D f_\pi^2 \Lambda^2, \quad (\text{A.1})$$

where for notational simplicity, we write $\Lambda = \Lambda_{\chi SB} = 4\pi f_\pi$.

¹⁰A more detailed study on the nucleon mass and a neutron-proton mass difference in kaon condensed matter will be reported in a future publication.

As far as the Goldstone boson sector is concerned, this counting rule is the same as the one used in standard chiral perturbation theory(ChPT). Including quarks is straightforward, since the constituent quark mass, M_f , can be considered small compared to $\Lambda_{\chi SB} \sim 1$ GeV, i.e. $M_f \sim p$ where p is a typical momentum scale. Each quark propagator contributes -1 power of p , each Goldstone boson propagator contributes -2 power of p , each derivative and quark mass in the interaction terms contribute $+1$ power of p , and each four-momentum integration contributes $+4$ powers of p .

All the factors put together, the chiral index D of a given amplitude with L loops, I_{GB} internal meson lines, I_Q quark lines, N^{GB} mesonic vertices and N^{GBQ} meson-quark vertices is given by

$$D = 4L - 2I_{GB} - I_Q + \sum_n n N_n^{GB} + \sum_d d N_d^{GBQ}. \quad (\text{A.2})$$

For connected diagrams, we can use the topological relation

$$L = I_{GB} + I_Q - \sum_n (N_n^{GB} + N_n^{GBQ}) + 1 \quad (\text{A.3})$$

to get

$$D = 2L + 2 + I_Q + \sum_n (n - 2) N_n^{GB} + \sum_d (d - 2) N_d^{GBQ}. \quad (\text{A.4})$$

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