

Dark Energy and Neutrino CPT Violation

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Abstract

In this paper we study the dynamical CPT violation in the neutrino sector induced by the dark energy of the Universe. Specifically we consider a dark energy model where the dark energy scalar derivatively interacts with the right-handed neutrinos. This type of derivative coupling leads to a cosmological CPT violation during the evolution of the background field of the dark energy. We calculate the induced CPT violation of left-handed neutrinos and find the CPT violation produced in this way is consistent with the present experimental limit and sensitive to the future neutrino oscillation experiments, such as the neutrino factory.

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I. INTRODUCTION

The recent observational data from type Ia supernovae[1], cosmic microwave background (CMB) radiation[2] and large scale structure (LSS)[3] have provided strong evidences for a spatially flat and accelerated expanding universe at the present time. In the context of Friedmann-Robertson-Walker cosmology, this acceleration is attributed to the domination of a component, dubbed dark energy[4]. The simplest candidate for dark energy seems to be a remnant small cosmological constant. However, many physicists are attracted by the idea that dark energy is due to a dynamical component, such as the quintessence [5, 6, 7, 8], the K-essence[9, 10], the phantom[11] and the quintom[12, 13, 14, 15, 16].

Being a dynamical component, the dark energy is expected to interact with matters. Recently there have been a lot of interests in the literature[17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] in studying the possible interactions between the neutrinos and the dark energy. An interesting prediction of these models is that the neutrino masses are not constant, but vary as a function of time and space[18, 19]. This prediction on the variation of the neutrino masses can be verified in the present and future experiments. For example, the neutrino mass evolution with time can be tested in the measurement of the time delay of the neutrinos emitted from the short gamma ray bursts[33]. Another interesting possibility is to detect the neutrino mass variation in space via the neutrino oscillations[19, 20, 34, 35, 36]. In this paper we study the possibility of CPT violation in the neutrino sector induced by the evolution of the dark energy scalar field.

In Ref.[17] the authors have made a proposal for the dynamical CPT violation by introducing a derivative coupling of the dark energy scalar to the left-handed fermions of the standard model¹:

$$\mathcal{L}_{eff} \sim \partial_\mu \phi \bar{l}_L \gamma^\mu l_L, \quad (1)$$

where ϕ is the dark energy scalar, for instance the quintessence, l_L by gauge invariance of the $SU(2)_L \times U(1)_Y$ is the doublet of the left-handed lepton $l_L = (\nu_L, e_L)^T$. During the evolution of the universe the time derivative of the scalar field does not vanish $\dot{\phi} \neq 0$ which gives rise to a CPT violation. This type of CPT violation as shown in [17] helps understand the

¹ There are several other papers that deal with CPT violation originating from spacetime-varying scalars[37, 38, 39]

matter anti-matter asymmetry of the universe; however, since the laboratory experimental limit on the CPT violation in electrons is so stringent that the induced CPT violation in the neutrino sector will be much below the sensitivity for the current and future experiments. Phenomenologically, a comparably large neutrino CPT violation is helpful to explain the possible CPT-violating neutrino oscillations. For example, the neutrino CPT violation has been proposed [40, 41, 42, 43] to account for the LSND anomaly[44], which can be tested at the upcoming MiniBooNE experiment[45]. It is noteworthy that whether the LSND result is confirmed or not by the future experiments, the possibility of CPT violation consistent with the other neutrino oscillation experiments[46, 47, 48] is intriguing and can be tested in the future neutrino factory experiment[48, 49].

In this paper we consider specifically a model where the dark energy scalar couples derivatively to the right-handed neutrino and calculate the induced CPT violation in left-handed neutrinos due to the mixing between the right-handed neutrinos and the left-handed neutrinos. Our results show that with an appropriate choice of the model parameter the CPT violation in the neutrino sector is consistent with the present experimental data and can be tested in the future neutrino factory experiment without conflicting to the experimental limit on the electron CPT violation.

II. MODEL OF NEUTRINO CPT VIOLATION

The model under investigation in this paper includes a scalar field ϕ which drives the universe acceleration and derivatively couples to the right-handed neutrinos $N_i = \nu_{Ri} + \nu_{Ri}^C (i = 1, \dots), \sim \frac{f_{ii}}{\Lambda} \bar{\nu}_{Ri} \partial_\mu \phi \gamma^\mu \nu_{Rj}$ with f the parameter which characterizes the strength of this type of interaction and is expected to be of order $\mathcal{O}(1)$, and Λ the energy scale characterizing the dynamics which generates this effective interaction. In our model, the right-handed neutrinos, being a part of the dark energy sector, are taken at the $\mathcal{O}(\text{eV})$ scale and $\Lambda \ll M_W$ to give observable CPT violating effects. Similar $\mathcal{O}(\text{eV})$ scale right-handed neutrino models are studied in [19, 20]. The derivative coupling will lead to a cosmological CPT violation in the right-handed neutrino sector during the evolution of the background dark energy field. The mixing between the left-handed neutrino and the right-handed neutrino is induced through a gauge invariant and renormalizable Yukawa interaction, $y_{\alpha j} \bar{l}_{L\alpha} \tilde{H} \nu_{Rj} (\alpha = e, \mu, \tau$ denotes the flavor indices), where y are the Yukawa couplings, l and \tilde{H} are the SM

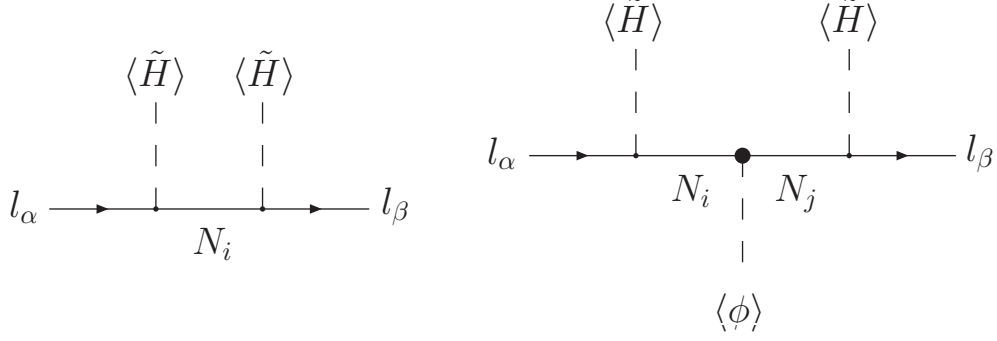


FIG. 1: Seesaw mechanism (the left diagram) and CPT violation in neutrinos (the right diagram) at tree level.

lepton doublet and Higgs doublet, respectively.

The relevant Lagrangian can be written as

$$\begin{aligned}
-\mathcal{L} &= y_{\alpha j} \bar{l}_{L\alpha} \tilde{H} \nu_{Rj} + \frac{1}{2} M_i \bar{\nu}_{Ri}^C \nu_{Ri} + h.c. + \frac{f_{ij}}{\Lambda} \bar{\nu}_{Ri} \partial_\mu \phi \gamma^\mu \nu_{Rj} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \\
&= y_{\alpha j} \bar{l}_{L\alpha} \tilde{H} N_j + h.c. + \frac{1}{2} M_i \bar{N}_i N_i + \frac{1}{2} \bar{N}_i A_{\mu ij} \gamma^\mu N_j + \frac{1}{2} \bar{N}_i S_{\mu ij} \gamma^\mu \gamma_5 N_j - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \quad (2)
\end{aligned}$$

with $V(\phi)$ the potential for ϕ . We have adopted the following definition in Eq.(2),

$$A_\mu = \frac{1}{2}(f - f^T) \frac{1}{\Lambda} \partial_\mu \phi, \quad (3)$$

$$S_\mu = \frac{1}{2}(f + f^T) \frac{1}{\Lambda} \partial_\mu \phi, \quad (4)$$

so the matrix A_μ is antisymmetric while S_μ is symmetric.

Integrating out the right-handed Majorana neutrinos, we obtain

$$-\mathcal{L}_\nu = \frac{1}{2} \bar{\nu}_\alpha m_\nu^{\alpha\beta} \nu_\beta + \frac{1}{2} \bar{\nu}_\alpha a_\mu^{\alpha\beta} \gamma^\mu \nu_\beta + \frac{1}{2} \bar{\nu}_\alpha s_\mu^{\alpha\beta} \gamma^\mu \gamma_5 \nu_\beta. \quad (5)$$

with

$$m_\nu = -m_D \frac{1}{M} m_D^T, \quad (6)$$

$$a_\mu = \frac{1}{2}(b_\mu - b_\mu^T) \quad (7)$$

and

$$s_\mu = -\frac{1}{2}(b_\mu + b_\mu^T), \quad (8)$$

where $\nu = \nu_L + \nu_L^C$ is the left-handed Majorana neutrino. Here $M = \text{diag}(M_1, M_2, \dots)$, $m_D = yv$ with $v \simeq 174\text{GeV}$ and b_μ is defined as

$$b_\mu = m_D \frac{1}{M} (A_\mu - S_\mu) \frac{1}{M} m_D^\dagger. \quad (9)$$

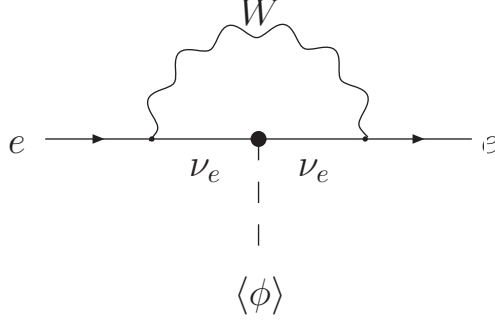


FIG. 2: CPT violation in electrons at one loop level.

We can see that the last two terms in Eq.(5) will produce the CPT violation in neutrinos for a non-vanishing $\dot{\phi}$. It is easy to show that the CPT violating term above can be given in a simple form[49, 50, 51, 52]

$$\mathcal{L}_{CPTV} = -\bar{\nu}_{L\alpha} b_{\mu}^{\alpha\beta} \gamma^{\mu} \nu_{L\beta} . \quad (10)$$

For the electron the CPT violation is induced by the W- and neutrino-loop². The coupling between the axial vector background and the electron current, $c_{\mu} \bar{e} \gamma^{\mu} \gamma_5 e$, is strongly constrained by the present experiments[54]. For $\Lambda \ll M_W$, we obtain

$$c_{\mu} \simeq \frac{1}{8\pi} \frac{\alpha}{\sin^2 \theta_W} \frac{\Lambda^2}{M_W^2} s_{\mu} , \quad (11)$$

where the loop integral was cutoff at Λ as the effective theory is only valid below Λ [53], $\alpha \simeq \frac{1}{137}$ is the fine structure constant, $\sin^2 \theta_W \simeq 0.23$ is the Weinberg angle, and $M_W \simeq 80\text{GeV}$ is the mass of W gauge boson.

Taking the CPT violating parameters a_{μ} and s_{μ} to be much smaller than the masses of the left-handed Majorana neutrinos, which will be shown latter to be the actual case, we can simplify Eq.(9) as

$$b_{\mu} \sim \tilde{b} \frac{m}{M} \frac{1}{\Lambda} \partial_{\mu} \phi \quad (12)$$

and similarly for Eq.(11) we have

$$c_{\mu} \sim \tilde{c} \frac{1}{8\pi} \frac{\alpha}{\sin^2 \theta_W} \frac{\Lambda^2}{M_W^2} \frac{1}{\Lambda} \partial_{\mu} \phi$$

² The $\tilde{H} - N$ exchange also contributes to the CPT violation in electrons and neutrinos[53], which is, however, strongly suppressed in our model since the masses of the right-handed Majorana neutrinos and also the Yukawa couplings are very small.

$$\sim 1.3 \times 10^{-3} \tilde{c} \frac{\Lambda}{M_W^2} \partial_\mu \phi, \quad (13)$$

where m , M denote the mass scale of the left- and right-handed Majorana neutrinos, \tilde{b} and \tilde{c} are two dimensionless parameters. It should be noted that the magnitude of \tilde{b} and \tilde{c} are at the same order of $f \sim \mathcal{O}(1)$ in Eq.(2).

For the homogeneous scalar field, we can express the time component of $\partial_\mu \phi$ in the following way,

$$\dot{\phi} = [(1 + w_\phi)\rho_\phi]^{1/2} \quad (14)$$

with the equation of state defined by

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \quad (15)$$

and the energy density

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi). \quad (16)$$

The values of w_ϕ and ρ_ϕ are constrained by the cosmological observations[2], $w_\phi < -0.78$ and $\rho_\phi \simeq 73\% \rho_c$ with $\rho_c \simeq 4.2 \times 10^{-47} \text{GeV}^4$ the critical energy density of the Universe at present. Accordingly we have for $\dot{\phi}$

$$\dot{\phi} \lesssim 2.6 \times 10^{-24} \text{GeV}^2. \quad (17)$$

The CPT violating parameters of the neutrinos and the electrons are now given by

$$b_0 \sim \tilde{b} \frac{m}{M} \frac{1}{\Lambda} \dot{\phi} \quad (18)$$

and

$$c_0 \sim 1.3 \times 10^{-3} \tilde{c} \frac{\Lambda}{M_W^2} \dot{\phi}. \quad (19)$$

III. EXPERIMENTAL TEST FOR THE CPT VIOLATION

The neutrino CPT violation can be tested in the neutrino oscillation experiments. So we begin our discussions with the effective Hamiltonian that governs the propagation of neutrinos. The evolution of neutrinos is determined by the Schrödinger equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \simeq H_{eff}(x) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad (20)$$

where the Hamiltonian can be written as[50]

$$\begin{aligned}
H_{eff}(x) &= \frac{1}{2|\vec{p}|}(m_\nu m_\nu^\dagger) + \frac{1}{|\vec{p}|}(b_\mu p^\mu) + \sqrt{2}G_F \text{diag}(N_e(x), 0, 0) \\
&\simeq \frac{1}{2p^0}(m_\nu m_\nu^\dagger) + b_0 - |\vec{b}| \cos \theta + \sqrt{2}G_F \text{diag}(N_e(x), 0, 0)
\end{aligned} \tag{21}$$

with m_ν and $b_\mu = (b_0, -\vec{b})$ defined in Eq.(6) and Eq.(9), respectively. $p^\mu = (p^0, \vec{p})$ is the four momentum of the neutrinos, θ is the angle between \vec{p} and \vec{b} , and $\sqrt{2}G_F N_e(x)$ is the usual MSW term[55].

We define b_i to be the eigenvalues of the matrix b_0 and $\delta b_{ij} = b_i - b_j$. Then δb_{21} is constrained by fitting the data from the solar neutrino experiments and the KamLAND reactor neutrino experiment[46], which gives

$$\delta b_{21} < 3.1 \times 10^{-20} \text{GeV} . \tag{22}$$

In addition, the bound of δb_{32} is also obtained from the Super-K and K2K data[47, 48]

$$\delta b_{32} < 5 \times 10^{-23} \text{GeV} . \tag{23}$$

And it has been shown the future neutrino factory experiment[48, 49] will be sensitive to δb_{32} as small as

$$\delta b_{32} \sim 3 \times 10^{-23} \text{GeV} . \tag{24}$$

From the cosmological observations[2] and the neutrino oscillation experimental results[56, 57, 58, 59], we take the parameters of our model as $\rho_\phi \sim 3 \times 10^{-47} \text{GeV}^4$ and $w_\phi \sim -0.9$ and the mass scale of the neutrinos as $m \sim 10^{-2} \text{eV}$. For the mass of right-handed neutrino (dark fermion) we take for example $M = 1 \text{eV}$ [19]. One can see the values of the CPT violation parameters δb_{21} and δb_{32} are predicted to be

$$\delta b_{21} \sim \delta \tilde{b}_{21} \left(\frac{m}{M}\right) \frac{1}{\Lambda} [(1 + w_\phi) \rho_\phi]^{1/2} \sim \delta \tilde{b}_{21} \left(\frac{\text{MeV}}{\Lambda}\right) 1.7 \times 10^{-23} \text{GeV} , \tag{25}$$

and

$$\delta b_{32} \sim \delta \tilde{b}_{32} \left(\frac{m}{M}\right) \frac{1}{\Lambda} [(1 + w_\phi) \rho_\phi]^{1/2} \sim \delta \tilde{b}_{32} \left(\frac{\text{MeV}}{\Lambda}\right) 1.7 \times 10^{-23} \text{GeV} . \tag{26}$$

As shown in Eq.(12) the parameters $\delta \tilde{b}_{32}$ and $\delta \tilde{b}_{21}$ are $O(1)$. The equations above indicate that it is quite possible to have CPT violating effects be consistent with the present experimental data and be detectable in the future neutrino factory experiment with Λ no larger

than MeV. It should be noted that if the value of the quintessence field ϕ is at the order of M_{Pl} the mass of the right-handed neutrinos should be shifted greatly by Eq. (2). However, in the dark energy models[19, 60] where the value of ϕ is much lower than M_{Pl} even lower than Λ , our model is consistent.

We now estimate the corresponding CPT violation in the electron sector. For the parameters taken above, we have $c^0 \sim \tilde{c} 3.5 \times 10^{-34}\text{GeV}$. The present experimental limit is $c^0 < 5 \times 10^{-25}\text{GeV}$ [49, 53]. We see the electron CPT violation in our model is well within the present limit.

IV. CONCLUSION

In summary, we have studied the neutrino CPT violation induced from the dark energy sector. Specifically we consider a dark energy model including a scalar boson and right-handed neutrinos, and introduce a derivative coupling between the boson and the neutrinos. This derivative coupling leads to a cosmological CPT violation during the evolution of the dark energy field. We calculate the induced CPT violation in the left-handed neutrinos due to their mixing with the right-handed neutrinos, and also the loop contribution to the electron CPT violation. Our calculations show that the CPT violation in both neutrinos and electrons is well within the present experimental limits. Furthermore, the neutrino CPT violation can be tested in the future neutrino oscillation experiments, such as the neutrino factory. Our study in this paper shows the neutrino oscillations may provide us a non gravitational way to probe the dark energy properties.

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