

The gluon Reggeization in perturbative QCD at NLO *

V.S. Fadin[†]

*Institute for Nuclear Physics, 630090 Novosibirsk, Russia
and Novosibirsk State University, 630090 Novosibirsk, Russia*

Abstract

Compatibility of the Reggeized form of QCD multi-particle amplitudes with the s-channel unitarity requires fulfilment of an infinite number of the "bootstrap" relations. On the other hand, it turns out that fulfilment of all these relations ensures the Reggeized form of energy dependent radiative corrections order by order in perturbation theory. It is extremely nontrivial, that all these relations are fulfilled if the Reggeon vertices and trajectory satisfy several bootstrap conditions. The full set of these conditions in the next-to-leading order was derived in the last year and the ultimate condition was shown to be satisfied recently. It means that the Reggeization hypothesis is proved now in the next-to-leading approximation.

arXiv:hep-ph/0511121v1 9 Nov 2005

* *Work supported in part by the Russian Fund of Basic Researches, grant 04-02-16685.*

[†] *e-mail address:* FADIN@INP.NSK.SU

1 Introduction

One of remarkable properties of Quantum Chromodynamics is the gluon Reggeization. Non vanishing in the high energy limit cross sections are related to gluon exchanges in cross channels. Therefore the gluon Reggeization is extremely important for the description of QCD processes at high energy \sqrt{s} . In particular, this phenomenon appeared as the basis of the BFKL approach [1] to the description of high energy processes. It was proved [2] in the leading logarithmic approximation (LLA), when only the leading terms $(\alpha_S \ln s)^n$ are summed. Owing to this the BFKL approach was grounded in the LLA. Now the approach is intensively developed in the next-to leading approximation (NLA), when the terms $\alpha_S(\alpha_S \ln s)^n$ are also summed. In this approximation the gluon Reggeization remained a hypothesis till now. Evidently, its proof is extremely desirable. The proof is especially necessary because of appearance of statements about existence of contributions violating the Regge ansatz at three loop level [3]. Now the desired proof is completed.

2 The Reggeization hypothesis

The hypothesis determines QCD amplitudes in the multi-Regge kinematics – MRK (at that the Regge kinematics is considered as a particular case of the MRK). MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets. At leading order (LO) only gluons can be produced and each jet is actually a gluon. At next-to-leading order (NLO) a jet can contain a couple of partons (two gluons or quark-antiquark pair). Such kinematics is called also quasi multi-Regge kinematics (QMRK).

The MRK gives dominant contributions to cross sections of QCD processes at high energy \sqrt{s} . In perturbation theory these contributions are related to exchanges of the gluon quantum numbers in cross channels with fixed (not increasing with s) momentum transfers q_i . The hypothesis is based on the calculations of QCD amplitudes. Despite of a great number of contributing Feynman diagrams it turns out that at the Born level in the MRK amplitudes acquire a simple factorized form. Quite uncommonly that radiative corrections to these amplitudes don't destroy this form, and their energy dependence is given by simple Regge factors $s_i^{\omega(q_i)}$, where s_i are invariant masses of couples of neighbouring jets and $\omega(q)$ can be interpreted as a shift of gluon spin from unity, dependent from momentum transfer q . This phenomenon is called gluon Reggeization and $\omega(t)$ is called gluon Regge trajectory (although actually the trajectory is $j(t) = 1 + \omega(t)$). The Reggeization hypothesis affirms that

$$\Re \mathcal{A}_{AB}^{A'B'+n} = 2p_A^+ p_B^- \Gamma_{A'A} \left(\prod_{i=1}^n \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_i^2} \gamma^{J_i}(q_i, q_{i+1}) \right) \frac{e^{\omega(q_{n+1})(y_n-y_{n+1})}}{q_{n+1}^2} \Gamma_{B'B}. \quad (1)$$

Here \Re means a real part; $\mathcal{A}_{AB}^{A'B'+n}$ is the amplitude for production of jets A' , J_1, \dots, J_n , B' , strongly ordered in rapidity space (see Fig.1); $\Gamma_{P'P}$ are the scattering vertices, i.e. the effective vertices for $P \rightarrow P'$ transitions due to interaction with Reggeized gluons; $\gamma^{J_i}(q_i, q_{i+1})$

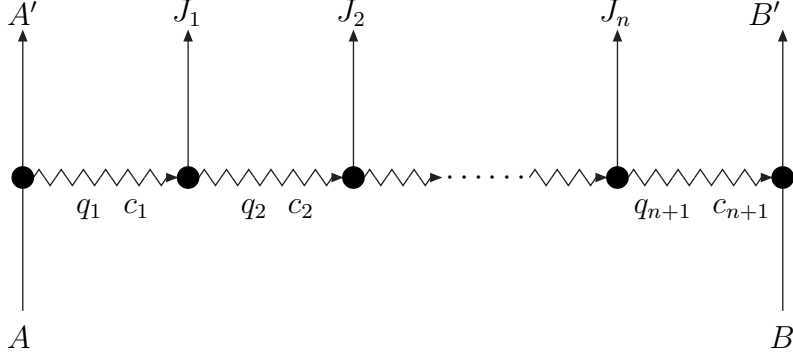


Figure 1: Schematic representation of the process $A + B \rightarrow A' + J_1 + \dots + B'$ in the MRK.

are the production vertices, i.e. the effective vertices for production of jets J_i with momenta $k_i = q_i - q_{i+1}$ in collisions of Reggeons with momenta q_i and $-q_{i+1}$; $q_0 = p_A - p_{A'}$, $q_{n+1} = p_{B'} - p_B$. We use light cone vectors n_1 and n_2 , $n_1^2 = n_2^2 = 0$; $(n_1 n_2) = 2$ and denote $p^\pm = (pn_{2,1})$. It is assumed that initial momenta p_A and p_B have predominant components p_A^+ and p_B^- . For generality we do not assume that transverse to the (n_1, n_2) plane components $p_{A\perp}$, $p_{B\perp}$ are zero. Moreover, A and B , as well as A' and B' , can represent jets. In (1) y_i are jet rapidities, $y_i = \frac{1}{2} \ln(k_i^+/k_i^-)$ for $i = 1, \dots, n$, $y_0 = y_A \equiv \ln(p_A^+/q_{1\perp})$, $y_{n+1} = y_B \equiv \ln(q_{(n+1)\perp}/p_B^-)$. We use positive Euclidean metric for transverse components.

Note that Reggeons, as well as gluons, belong to colour octet, so that the vertices carry colour indices. For simplicity, we omit these indices when they are not necessary for understanding.

The factorized form of QCD amplitudes in the MRK was proved at the Born level using the t -channel unitarity and analyticity. Their Reggeization was firstly derived in the LLA on the basis of the direct calculations at the three-loop level for elastic amplitudes and the one-loop level for one-gluon production amplitudes. Later it was proved [2] in the LLA for all amplitudes at arbitrary number of loops with the help of bootstrap relations. At NLO the Reggeization remained a hypothesis till now.

The hypothesis is extremely powerful since an infinite number of amplitudes is expressed in terms of the gluon Regge trajectory and several Reggeon vertices.

3 Idea of the proof of the hypothesis

A basic idea of the proof is based on use of the s -channel unitarity. In order to realize the idea we need to express real parts of amplitudes in terms of their s -channel discontinuities. It is not difficult to do for elastic amplitudes. Unfortunately, it is quite not so for inelastic amplitudes.

But if in the MRK we confine ourselves to the NLO, the situation changes drastically: [4] the discontinuities (more precisely, real parts of definite combinations of the discontinuities) are related to the derivatives of the amplitudes over jet rapidities :

$$\sum_{l=j+1}^{n+1} \Delta_{jl} - \sum_{l=0}^{j-1} \Delta_{lj} = \frac{\partial}{\partial y_j} \Re \left[e^{y_B - y_A} \mathcal{A}_{AB}^{A'B'+n}(y_i) \right], \quad (2)$$

where

$$\Delta_{jl} = e^{y_B - y_A} \Re \frac{1}{-\pi i} \text{disc}_{s_{jl}} \mathcal{A}_{AB}^{A'B'+n}, \quad (3)$$

$s_{jl} = (k_j + k_l)^2$ and in the R.H.S. of (2) the amplitude is considered as a function of y_i , $i = 0, \dots, n+1$, and transverse momenta. Taking sum of the equations over j from 0 to $n+1$ it is easy to see from (2) that $\Re \mathcal{A}_{AB}^{A'B'+n}(y_i)$ depends only on differences of the rapidities y_i , as it must be.

The important point is that the relations (2), (3) give a possibility to find in the NLA real parts of all MRK amplitudes in all orders of coupling constant, if $\Re \mathcal{A}_{AB}^{A'B'+n}(y_i)$ are known (for all n) in the one-loop approximation. Indeed, these relations express all partial derivatives of the real parts at some number L of loops through the discontinuities, which can be calculated using the s -channel unitarity in terms of amplitudes with smaller number of loops; at that in the NLA only the MRK is important and only real parts of amplitudes do contribute. To find $\Re \mathcal{A}_{AB}^{A'B'+n}(y_i)$ besides the derivatives determined by (2), (3) initial conditions are required; but since they can be taken at fixed y_i , they are necessary only in the one-loop approximation. Thus (2), (3) allows to calculate $\Re \mathcal{A}_{AB}^{A'B'+n}(y_i)$ loop-by-loop using the one-loop approximation as an input. Note that requirement of equality of mixed derivatives taken in different orders imposes strong restrictions on the input. If it is self-consistent, it determines $\Re \mathcal{A}_{AB}^{A'B'+n}(y_i)$ unambiguously.

Therefore in order to prove the Reggeization in the NLA it is sufficient to know that (1) is valid in the one-loop approximation and satisfies (2), (3), where the discontinuities (3) are calculated using (1) in the unitarity relations.

4 Bootstrap relations for the Reggeized amplitudes

Substituting (1) in (2), we obtain the relations

$$\sum_{l=j+1}^{n+1} \Delta_{jl} - \sum_{l=0}^{j-1} \Delta_{lj} = (\omega(t_{j+1}) - \omega(t_j)) \Re \mathcal{A}_{AB}^{A'B'+n},$$

which are called bootstrap relations. Evidently, there is an infinite number of the bootstrap relations, because there is an infinite number of the amplitudes $\mathcal{A}_{AB}^{A'B'+n}$. At the first sight, it seems a miracle to satisfy all of them, since all these amplitudes are expressed through several Reggeon vertices and the gluon Regge trajectory. Moreover, it is quite nontrivial to satisfy even some definite bootstrap relation for a definite amplitude, because it connects two infinite series in powers of y_i , and therefore it leads to an infinite number of equalities between coefficients of these series.

In fact, two miracles must occur in order to satisfy all the bootstrap relations: first, for each particular amplitude $\mathcal{A}_{AB}^{A'B'+n}$ it must be possible to reduce the bootstrap relation to a limited number of restrictions (bootstrap conditions) on the gluon trajectory and the Reggeon vertices, and secondly, starting from some $n = n_0$ these bootstrap conditions must be the same as obtained for amplitudes with $n < n_0$. And finally, all bootstrap conditions must be satisfied by known expressions for the trajectory and the vertices.

5 Representation of the discontinuities

The miracles start from the discontinuities calculated using (1) and the s -channel unitarity. To present them in a compact form let us use operator denotations. Then the values of $(2\pi)^{D-1}\delta(q_{i\perp} - q_{(j+1)\perp} - \sum_{l=i}^{l=j} k_{l\perp}) \frac{\Delta_{ij}}{4N_c}$ can be presented [5] as obtained from the R.H.S. of (1) by the replacement of

$$\gamma^{J_i}(q_i, q_{i+1}) \left(\prod_{l=i+1}^j \frac{e^{\omega(q_i)(y_{l-1}-y_l)}}{q_l^2} \gamma^{J_l}(q_l, q_{l+1}) \right) \quad (4)$$

for

$$\langle J_i, \bar{R}_i | \left(\prod_{l=i+1}^{j-1} e^{\hat{\mathcal{K}}(y_{l-1}-y_l)} \hat{\mathcal{J}}_l \right) e^{\hat{\mathcal{K}}(y_{j-1}-y_j)} | \bar{J}_j, \bar{R}_{j+1} \rangle. \quad (5)$$

Eqs. (4), (5) remain valid for $i = 0$ with the substitutions $\gamma^{J_0}(q_0, q_1) \rightarrow \Gamma_{A'A}$ and $\langle J_0, \bar{R}_0 | \rightarrow \langle A', \bar{A} |$, as well for $j = n+1$, with the substitutions $\gamma^{J_{n+1}}(q_{n+1}, q_{n+2}) \rightarrow \Gamma_{B'B}$ and $| \bar{J}_{n+1}, \bar{R}_{n+2} \rangle \rightarrow | \bar{B}', B \rangle$. Here $\hat{\mathcal{K}}$ is the operator of colour octet BFKL kernel, $\hat{\mathcal{J}}_i$ is the jet J_i production operator; the state $| \bar{B}', B \rangle$ represents the scattering particles (jets) B and B' from the cross-channel point of view, $| \bar{J}_l, \bar{R}_{l+1} \rangle$ represents Reggeon with momentum q_{l+1} and jet J_l ; $\langle A', \bar{A} |$ and $\langle J_l, \bar{R}_l |$ are corresponding conjugate states. The matrix elements in (5) is calculated using the full set of states $| r_{1\perp}, r_{2\perp}, a \rangle$ of two Reggeons with definite transverse momenta in the adjoint representation. If $| r_{\perp}, a \rangle$ is the one-Reggeon state with transverse momentum r_{\perp} and colour index a , then $| r_{1\perp}, r_{2\perp}, a \rangle = i f_{aa_1 a_2} | r_{1\perp}, a_1 \rangle | r_{2\perp}, a_2 \rangle$. We use normalization $\langle r_{1\perp}, a_1 | r_{2\perp}, a_2 \rangle = \delta_{a_1 a_2} r_{1\perp}^2 \delta(r_{1\perp} - r_{2\perp})$. Everywhere in the following symmetrization in Reggeon momenta r_1 and r_2 is assumed.

At that we have

$$\hat{\mathcal{K}} = \omega(\hat{r}_1) + \omega(\hat{r}_2) + \hat{\mathcal{K}}_r, \quad (6)$$

where the subscript r means the contribution coming from real particle production. To escape a double counting in the NLA we introduce an auxiliary parameter $\Delta \gg 1$ dependence from which vanishes at large Δ :

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_r^{\Delta} - \hat{\mathcal{K}}_r^B \hat{\mathcal{K}}_r^B \Delta; \quad (7)$$

the superscript B here and below denotes quantities calculated in the LO; $\hat{\mathcal{K}}_r^{\Delta}$ concerns with production of jets J with intervals of particle rapidities Δ_J in them less than Δ :

$$\langle r'_{1\perp}, r'_{2\perp}, a' | \hat{\mathcal{K}}_r^{\Delta} | r_{1\perp}, r_{2\perp}, a \rangle = \delta_{aa'} \delta(r_{1\perp} + r_{2\perp} - r'_{1\perp} - r'_{2\perp})$$

$$\times \frac{f_{c_1 c_2 c} f_{c'_1 c'_2 c}}{N_c (N_c^2 - 1)} \sum_J \int \gamma_{c_1 c'_1}^J(r_1, r'_1) \left(\gamma_{c_2 c'_2}^J(-r_2, -r'_2) \right)^* \frac{d\phi_J}{2(2\pi)^{D-1}} \theta(\Delta - \Delta_J), \quad (8)$$

where

$$d\phi_J = \frac{dk_J^2}{2\pi} (2\pi)^D \delta^D(k_J - \sum_i l_i) \prod_i \frac{d^{D-1}l_i}{(2\pi)^{D-1} 2\epsilon_i} \quad (9)$$

for a jet J with total momentum k_J consisting of particles with momenta l_i ; \mathcal{K}_r^B is given by (8) in the LO, and the second term in (6) serves for subtraction of contributions already taken into account in the LLA.

The states describing particle (jet) transitions due to interaction with Reggeized gluons are presented as

$$|\bar{B}'B\rangle = |\bar{B}'B^\Delta\rangle - \left(\omega^B(\hat{r}_1) \ln \left| \frac{\hat{r}_{1\perp}}{q_{B\perp}} \right| + \omega^B(\hat{r}_2) \ln \left| \frac{\hat{r}_{2\perp}}{q_{B\perp}} \right| + \hat{\mathcal{K}}_r^B \Delta \right) |\bar{B}'B^B\rangle, \quad (10)$$

$$\langle r_{1\perp}, r_{2\perp}; a | \bar{B}'B^\Delta \rangle = \delta(q_{B\perp} - r_{1\perp} - r_{2\perp}) i \frac{f_{ac_1 c_2}}{N_c} \sum_{\bar{B}} \int \Gamma_{\bar{B}\bar{B}}^{c_1} \Gamma_{\bar{B}'\bar{B}}^{c_2} d\phi_{\bar{B}} \prod_l \theta(\Delta - (z_l - y_B)), \quad (11)$$

and

$$\langle A'\bar{A} | = \langle A'\bar{A}^\Delta | - \langle A'\bar{A}^B | \left(\omega^B(\hat{r}_1) \ln \left| \frac{\hat{r}_{1\perp}}{q_{A\perp}} \right| + \omega^B(\hat{r}_2) \ln \left| \frac{\hat{r}_{2\perp}}{q_{A\perp}} \right| + \hat{\mathcal{K}}_r^B \Delta \right), \quad (12)$$

$$\langle A'\bar{A}^\Delta | r_{1\perp}, r_{2\perp}; a \rangle = \delta(q_{A\perp} + r_{1\perp} + r_{2\perp}) i \frac{f_{ac_1 c_2}}{N_c} \sum_{\bar{A}} \int \Gamma_{\bar{A}\bar{A}}^{c_1} \Gamma_{\bar{A}'\bar{A}}^{c_2} d\phi_{\bar{A}} \prod_l \theta(\Delta - (y_A - z_l)), \quad (13)$$

where $q_A = p_A - p_{A'}$, $q_B = p_B - p_{B'}$ and z_l are rapidities of particles in intermediate jets. Note that when a two-particle jet enters in some state, the second term in corresponding equation can be omitted and the first taken in the Born approximation.

Quite analogously

$$|\bar{J}_i, \bar{R}_{i+1}\rangle = |\bar{J}_i, \bar{R}_{i+1}^\Delta\rangle - \left((\omega(\hat{r}_1) - \omega(q_{i+1})) \ln \left| \frac{k_{i\perp}}{(\hat{r}_{1\perp} + q_{(i+1)\perp})} \right| + \omega(\hat{r}_2) \ln \left| \frac{k_{i\perp}}{\hat{r}_{2\perp}} \right| + \hat{\mathcal{K}}_r^{Born} \Delta \right) |\bar{J}_i, \bar{R}_{i+1}^B\rangle. \quad (14)$$

$$\begin{aligned} \langle r_{1\perp}, r_{2\perp}; a | \bar{J}_i, \bar{R}_{i+1}^\Delta \rangle &= \delta(q_{(i+1)\perp} + k_{i\perp} + r_{1\perp} + r_{2\perp}) \\ &\times i \frac{f_{ac_1 c_2}}{N_c} \sum_J \int \gamma_{c_1 a_{i+1}}^J(-r_1, q_{i+1}) \Gamma_{J_i J}^{c_2} d\phi_J \prod \theta(\Delta - (z_l - y_i)), \end{aligned} \quad (15)$$

and

$$\begin{aligned} \langle J_i, \bar{R}_i | &= \langle J_i, \bar{R}_i^\Delta | - \langle J_i, \bar{R}_i^B | \left((\omega(q_i) - \omega(\hat{r}_1)) \ln \left| \frac{k_{i\perp}}{(q_{i\perp} + \hat{r}_{1\perp})} \right| \right. \\ &\quad \left. - \omega(\hat{r}_2) \ln \left| \frac{k_{i\perp}}{\hat{r}_{2\perp}} \right| + \hat{\mathcal{K}}_r^{Born} \Delta \right), \end{aligned} \quad (16)$$

$$\langle J_i \bar{R}_i^\Delta | r_{1\perp}, r_{2\perp}; a \rangle = \delta(r_{1\perp} + r_{2\perp} + q_{i\perp} - k_{i\perp}) i \frac{f_{ac_1 c_2}}{N_c}$$

$$\times \sum_J \int \gamma_{a_i c_1}^J(q_i, -r_1) \Gamma_{J_i J}^{c_2} d\phi_J \prod \theta(\Delta - (y_i - z_l)), \quad (17)$$

where a_l are colour indices of Reggeons R_l .

At last,

$$\begin{aligned} \hat{\mathcal{J}}_i &= \hat{\mathcal{J}}_i^\Delta - \left(\hat{\mathcal{K}} \hat{\mathcal{J}}_i + \hat{\mathcal{J}}_i \hat{\mathcal{K}} \right) \Delta, \quad \langle r'_{1\perp}, r'_{2\perp}; a' | \hat{\mathcal{J}}_i | r_{1\perp}, r_{2\perp}; a \rangle \\ &= \delta(r_{1\perp} + r_{2\perp} - k_{i\perp} - r'_{1\perp} - r'_{2\perp}) \frac{f^{ac_1 c_2} f^{a' c'_1 c'_2}}{N_c} \left[2 \gamma_{c_1 c'_1}^{J_i}(r_1, r'_1) \delta(r_{2\perp} - r'_{2\perp}) r_{2\perp}^2 \delta_{c_2 c'_2} \right. \\ &\quad \left. + \sum_J \int_{y_i - \Delta}^{y_i + \Delta} \frac{dz_J}{2(2\pi)^{D-1}} \left(\gamma_{c_1 c'_1}^J(r_1, r'_1) \left(\gamma_{c_2 c'_2}^{\bar{J}_i J}(-r_2, -r'_2) \right)^* + \gamma_{c_1 c'_1}^{J_i J}(r_1, r'_1) \left(\gamma_{c_2 c'_2}^J(-r_2, -r'_2) \right)^* \right) \right]. \end{aligned} \quad (18)$$

When J_i is a two-particle jet, in the NLA in the first of these equations the second term can be omitted and the first taken in the Born approximation, so that the last term in (18) must be retained only when both J_i and J are single gluons.

All Reggeon vertices entering in these equations, as well as the gluon trajectory, are known now with required accuracy (see [6] and references therein).

6 Bootstrap conditions

Using representation (5) for the discontinuities it was proved [5] that an infinite number of the bootstrap relations (4) are satisfied if the following bootstrap conditions are fulfilled: the impact factors for scattering particles satisfy equations

$$|\bar{B}' B\rangle = \frac{g}{2} \Gamma_{B' B} |R_\omega(q_B)\rangle, \quad \langle A' \bar{A}| = \frac{g}{2} \Gamma_{A' A} \langle R_\omega(q_A)|, \quad (19)$$

where $|R_\omega(q)\rangle$ is the universal (process independent) eigenstate of the kernel $\hat{\mathcal{K}}$ with the eigenvalue $\omega(t)$,

$$\hat{\mathcal{K}} |R_\omega(q)\rangle = \omega(q) |R_\omega(q)\rangle, \quad (20)$$

and the normalization

$$\frac{g^2 t N_c}{2(2\pi)^{D-1}} \langle R_\omega(q) | R_\omega(q) \rangle = \omega(t); \quad (21)$$

the Reggeon-gluon impact factors and the gluon production vertices satisfy the equations

$$\begin{aligned} |\bar{J}_i \bar{R}_{i+1}\rangle + \frac{g q_{i+1}^2}{2} \hat{\mathcal{J}}_i |R_\omega(q_{i+1})\rangle &= \frac{g}{2} \gamma^{J_i}(q_i, q_{i+1}) |R_\omega(q_i)\rangle, \\ \langle J_i \bar{R}_i| + \frac{g q_i^2}{2} \langle R_\omega(q_i) | \hat{\mathcal{J}}_i &= \frac{g}{2} \gamma^{J_i}(q_i, q_{i+1}) \langle R_\omega(q_{i+1})|. \end{aligned} \quad (22)$$

Actually the second of equations (19), (22) are not independent; they follow from the first ones.

The bootstrap conditions (19) and (20) are known for a long time [7]–[9] and are proved to be satisfied [10]–[13]. The bootstrap relations for elastic amplitudes require only a

weak form of the conditions (19) and (20), namely these conditions projected on R_ω . It was recognized [4] that the bootstrap relations for one-gluon production amplitudes besides (19) and (20) require also a weak form of the condition (22). Thus, the bootstrap relations for one-gluon production amplitudes play a twofold role: they strengthen the conditions imposed by the elastic bootstrap and give a new one. One could expect that the history will repeat itself upon addition of each next gluon in the final state. If it were so, we would have to consider the bootstrap relations for production of arbitrary number of gluons and would obtain an infinite number of bootstrap conditions. Fortunately, history is repeated only partly: it occurs that already the bootstrap relations for two-gluon production only require the strong form of the last condition (i.e. (22)) and don't require new conditions [14]. At last, it was proved [5] that all bootstrap relations (4) are satisfied if the conditions (19)-(22) are fulfilled.

The bootstrap conditions with two-particle jets are required in the NLA only with the Reggeon vertices taken in the Born approximation. They were checked and proved to be satisfied in [15], [6]. After that only (22) remained unproved. Its fulfilment was proved recently [5]. Thus, now it is shown that all bootstrap conditions are fulfilled, that completes the proof of the gluon Reggeization.

7 Summary

The gluon Reggeization is one of remarkable properties of QCD. It is extremely important for description of high energy processes. In particular, it appears as the basis of the BFKL approach to summation of the terms strengthened by powers of $\log(1/x)$. The hypothesis is extremely powerful, since all scattering amplitudes are expressed in terms of the gluon trajectory and several Reggeon vertices. Now the hypothesis is proved in the NLA. The proof is based on the bootstrap relations. It is shown that an infinite number of these relations is reduced to several bootstrap conditions on the gluon trajectory and the Reggeon vertices. It is shown that fulfilment of these conditions means a proof of the Reggeization hypothesis. All bootstrap conditions are formulated explicitly and are proved to be fulfilled.

References

- [1] V.S. Fadin, E.A. Kuraev and L.N. Lipatov, Phys. Lett. B **60**, 50 (1975); E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Zh. Eksp. Teor. Fiz. **71**, 840 (1976) [Sov. Phys. JETP **44**, 443 (1976)]; Zh. Eksp. Teor. Fiz. **72**, 377 (1977) [Sov. Phys. JETP **45**, 199 (1977)]; Ya.Ya. Balitskii and L.N. Lipatov, Yad. Fiz. **28**, 1597 (1978) [Sov. J. Nucl. Phys. **28**, 822 (1978)].
- [2] Ya.Ya. Balitskii, L.N. Lipatov and V.S. Fadin, in *Materials of IV Winter School of LNPI* (Leningrad, 1979) p.109.
- [3] T. Kucs, arXiv:hep-ph/0403023;
- [4] V.S. Fadin, *Talk given at the NATO Advanced Research Workshop "Diffraction 2002", August 31 - September 6, 2002, Alushta, Crimea, Ukraine*, in *Diffraction 2002*, Ed. by R. Fiore *et al.*, NATO Science Series, Vol. 101, p.235.
- [5] V.S. Fadin, M.G. Kozlov and A.V. Reznichenko, to be published.
- [6] V. S. Fadin, Phys. Atom. Nucl. **66** 2017 (2003).
- [7] V.S. Fadin and R. Fiore, Phys. Lett. B **440**, 359 (1998).
- [8] M. Braun and G.P. Vacca, Phys. Lett. B **454**, 319; M. Braun, hep-ph/9901447 (1999).
- [9] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Lett. B **495**, 329 (2000).
- [10] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Rev. D **61**, 094005 (2000); **61**, 094006 (2000).
- [11] M. Braun and G.P. Vacca, Phys. Lett. B **477**, 156 (2000).
- [12] V.S. Fadin, R. Fiore and A. Papa, Phys. Rev. D **60**, 074025 (1999); V.S. Fadin, R. Fiore and M.I. Kotsky, Phys. Lett. B **494**, 100 (2000).
- [13] V. S. Fadin and A. Papa, Nucl. Phys. B **640**, 309 (2002).
- [14] J. Bartels, V. S. Fadin and R. Fiore, Nucl. Phys. B **672** 329 (2003).
- [15] V.S. Fadin, M.G. Kozlov and A.V. Reznichenko, Yad. Fiz. **67** (2004) 377 [Phys. Atom. Nucl. **67** 359 (2004)].