

# 1/m<sub>Q</sub> corrections to B → ρlν decay and |V<sub>ub</sub>|

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In the heavy quark effective field theory of QCD, we analyze the order 1/m<sub>Q</sub> contributions to heavy to light vector decays. Light cone sum rule method is applied with including the effects of 1/m<sub>Q</sub> order corrections. We then extract |V<sub>ub</sub>| from B → ρlν decay up to order of 1/m<sub>Q</sub> corrections.

PACS: 11.55.Hx, 12.39.Hg, 13.20.Fc, 13.20.He

Keywords: B → ρlν, 1/m<sub>Q</sub> correction, heavy quark effective field theory, light cone sum rule

## I. INTRODUCTION

Much effort has been devoted to discuss the heavy to light hadron semileptonic decays. In particular, B → π(ρ)lν decays attracted the most interest [1–9] because they can be used to determine the quark mixing matrix element |V<sub>ub</sub>|, a parameter of significance in particle physics. The heavy quark symmetry and relevant effective theory greatly simplify the study of hadrons each of which containing a single heavy quark and any number of light quarks, and provide relations between different processes. This symmetry is applied to study B(D)<sub>(s)</sub> → π(ρ, K, K\*)lν decays in Refs. [10–12], where the finite heavy quark mass (m<sub>Q</sub>) corrections are not considered. Ref. [13] extends the study on B → πlν decay up to the next to leading order of the heavy quark expansion. For a more complete knowledge of the magnitude of the finite mass corrections to heavy to light meson decays, and to the determination of |V<sub>ub</sub>|, one should also study the 1/m<sub>Q</sub> order corrections to semileptonic B decays to light vector mesons.

In this short letter we will apply the heavy quark effective field theory (HQEFT) developed in Refs. [14–18] to analyze the 1/m<sub>Q</sub> corrections to the B → ρlν decay. And the light cone sum rule method will be adopted to numerically estimate the nonperturbative functions, i.e., the heavy to light vector form factors with including 1/m<sub>Q</sub> order corrections. In section II the 1/m<sub>Q</sub> order corrections are formulated in HQEFT framework. Section III devotes to evaluate wave functions using light cone sum rule method in HQEFT. And section IV is the numerical results and discussion.

## II. B → ρLν DECAY IN HQEFT

The transition matrix element responsible to the B → ρlν decay is generally parameterized by form factors as

$$\begin{aligned} \langle \rho(p, \epsilon^*) | \bar{u} \gamma^\mu (1 - \gamma^5) b | B(p_B) \rangle = & -i(m_B + m_\rho) A_1(q^2) \epsilon^{*\mu} + i \frac{A_2(q^2)}{m_B + m_\rho} (\epsilon^* \cdot (p + q)) (2p + q)^\mu \\ & + i \frac{A_3(q^2)}{m_B + m_\rho} (\epsilon^* \cdot (p + q)) q^\mu + \frac{2V(q^2)}{m_B + m_\rho} \epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha^* (p + q)_\beta p_\gamma, \end{aligned} \quad (2.1)$$

where  $q = p_B - p$  is the momentum carried by the lepton pair.

In the framework of HQEFT [14,15], the QCD quantum field  $Q$  for heavy quark is decomposed into particle field  $Q^+$  and antiparticle field  $Q^-$ , so that the quark and antiquark fields are treated on the same footing in a symmetric way. The effective quark and antiquark fields in HQEFT are defined as

$$Q_v^\pm = e^{i\not{p} m_Q v \cdot x} \hat{Q}_v^\pm = e^{i\not{p} m_Q v \cdot x} P_\pm Q^\pm \quad (2.2)$$

$$R_v^\pm = P_\mp Q^\pm \quad (2.3)$$

with  $v$  being an arbitrary four-velocity satisfying  $v^2 = 1$ , and  $P_\pm \equiv (1 \pm \not{v})/2$  being the projection operators.  $\hat{Q}_v^\pm$  defined above are actually the large components of the heavy quark and antiquark fields respectively.  $R_v^\pm$  are the small components of the heavy quark and antiquark fields respectively. The quantum field in QCD Lagrangian can

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be written as  $Q \equiv Q^+ + Q^- \equiv \hat{Q}_v^+ + \hat{Q}_v^- + R_v^+ + R_v^-$ , which contains all large and small components of particle and antiparticle. The decomposition of  $Q$  is presented in detail in Refs. [14,15].

After the small components of particle and antiparticle fields being integrated out, QCD Lagrangian turns into

$$\mathcal{L}_{Q,v} = \mathcal{L}_{Q,v}^{(++)} + \mathcal{L}_{Q,v}^{(--) } + \mathcal{L}_{Q,v}^{(+-)} + \mathcal{L}_{Q,v}^{(-+)} \quad (2.4)$$

with

$$\mathcal{L}_{Q,v}^{(\pm\pm)} = \bar{Q}_v^\pm i \not{D}_v Q_v^\pm, \quad (2.5)$$

$$\begin{aligned} \mathcal{L}_{Q,v}^{(\pm\mp)} &= \frac{1}{2m_Q} \bar{Q}_v^\pm (i \overleftarrow{\not{D}}_v) e^{2i\phi m_Q v \cdot x} \left(1 - \frac{i\not{v} \cdot D}{2m_Q}\right)^{-1} (i \not{D}_\perp) Q_v^\mp \\ &= \frac{1}{2m_Q} \bar{Q}_v^\pm (-i \overleftarrow{\not{D}}_\perp) \left(1 - \frac{-i \not{v} \cdot \overleftarrow{D}}{2m_Q}\right)^{-1} e^{-2i\phi m_Q v \cdot x} (i \not{D}_v) Q_v^\mp, \end{aligned} \quad (2.6)$$

where

$$\begin{aligned} i \not{D}_v &= i\not{v} \cdot D + \frac{1}{2m_Q} i \not{D}_\perp \left(1 - \frac{i\not{v} \cdot D}{2m_Q}\right)^{-1} i \not{D}_\perp, \\ i \overleftarrow{\not{D}}_v &= -i \not{v} \cdot \overleftarrow{D} + \frac{1}{2m_Q} (-i \overleftarrow{\not{D}}_\perp) \left(1 - \frac{-i \not{v} \cdot \overleftarrow{D}}{2m_Q}\right)^{-1} (-i \overleftarrow{\not{D}}_\perp), \\ i \not{D}_\perp &= i \not{D} - i\not{v} \cdot D, \quad -i \overleftarrow{\not{D}}_\perp = -i \overleftarrow{\not{D}} + i \not{v} \cdot \overleftarrow{D}, \end{aligned} \quad (2.7)$$

which is treated as HQEFT in the case that the longitudinal and transverse residual momenta, i.e. the operators  $i\not{v} \cdot D$  and  $D_\perp$  are at the same order of power counting in  $1/m_Q$  expansions.

The heavy-light quark current  $\bar{q}\Gamma Q$  with  $\Gamma$  being arbitrary Dirac matrices can be expanded in powers of  $1/m_Q$  as

$$\bar{q}\Gamma Q \rightarrow e^{-im_Q v \cdot x} \bar{q} \left\{ \Gamma + \frac{1}{2m_Q} \Gamma \frac{1}{i\not{v} \cdot D} (i \not{D}_\perp)^2 + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \right\} Q_v^+. \quad (2.8)$$

Here the contributions from both heavy quark and antiquark fields have been considered.

According to above expansions for effective Lagrangian and effective current, one can write the matrix element in Eq. (2.1) as the following form in powers of  $1/m_Q$ ,

$$\begin{aligned} \langle \rho | \bar{u} \gamma^\mu (1 - \gamma^5) b | B \rangle &= \sqrt{\frac{m_B}{\bar{\Lambda}_B}} \left\{ \langle \rho | \bar{u} \gamma^\mu (1 - \gamma^5) Q_v^+ | M_v \rangle + \frac{1}{2m_Q} \langle \rho | \bar{u} \gamma^\mu (1 - \gamma^5) \frac{1}{i\not{v} \cdot D} P_+ \left( D_\perp^2 \right. \right. \\ &\quad \left. \left. + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} \right) Q_v^+ | M_v \rangle + \mathcal{O}(1/m_Q^2) \right\}, \end{aligned} \quad (2.9)$$

where  $\bar{\Lambda}_B = m_B - m_b$ , and  $F^{\alpha\beta}$  is the gluon field strength tensor. The effective heavy meson state  $|M_v\rangle$  satisfies the heavy quark spin-flavor symmetry. Its normalization is

$$\langle M_v | \bar{Q}_v^+ \gamma^\mu Q_v^+ | M_v \rangle = 2\bar{\Lambda} v^\mu \quad (2.10)$$

with the binding energy  $\bar{\Lambda} \equiv \lim_{m_Q \rightarrow \infty} \bar{\Lambda}_M$  being heavy flavor independent.

It should be noted that the  $1/m_Q$  corrections in Eq.(2.9) include both contributions from the current expansion (2.8) and from the insertion of the effective Lagrangian (2.4). In Eqs.(2.8) and (2.9) the operator  $1/(i\not{v} \cdot D)$  arises from the contraction of effective heavy quark and antiquark fields [15,18]. In the  $v \cdot A = 0$  gauge to be used in our calculation, this operator is tantamount to the heavy quark propagator.

As can be seen in Eq.(2.9) that the  $1/m_Q$  order corrections to  $B \rightarrow \rho l \nu$  transition are only attributed to one kinematic operator and one chromomagnetic operator. The heavy quark symmetry enables us to parameterize the matrix elements in HQEFT as

$$\langle \rho(p, \epsilon^*) | \bar{u} \Gamma Q_v^+ | M_v \rangle = -Tr[\Omega(v, p) \Gamma \mathcal{M}_v], \quad (2.11)$$

$$\langle \rho(p, \epsilon^*) | \bar{u} \Gamma \frac{P_+}{i\not{v} \cdot D} D_\perp^2 Q_v^+ | M_v \rangle = -Tr[\Omega_1(v, p) \Gamma \mathcal{M}_v], \quad (2.12)$$

$$\langle \rho(p, \epsilon^*) | \bar{u} \Gamma \frac{P_+}{i\not{v} \cdot D} \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q_v^+ | M_v \rangle = -Tr[\Omega_1^{\alpha\beta}(v, p) \Gamma P_+ \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}_v], \quad (2.13)$$

where the pseudoscalar heavy meson spin wave function  $\mathcal{M}_v = -\sqrt{\Lambda}P_+\gamma^5$  is independent of the heavy quark flavor.  $\Omega(v, p)$ ,  $\Omega_1(v, p)$  and  $\Omega_1^{\alpha\beta}(v, p)$  can be decomposed into Lorentz scalar functions,

$$\begin{aligned}
\Omega(v, p) &= L_1(v \cdot p)\not{\epsilon}^* + L_2(v \cdot p)(v \cdot \epsilon^*) + [L_3(v \cdot p)\not{\epsilon}^* + L_4(v \cdot p)(v \cdot \epsilon^*)]\not{p}, \\
\Omega_1(v, p) &= \delta L_1(v \cdot p)\not{\epsilon}^* + \delta L_2(v \cdot p)(v \cdot \epsilon^*) + [\delta L_3(v \cdot p)\not{\epsilon}^* + \delta L_4(v \cdot p)(v \cdot \epsilon^*)]\not{p}, \\
\Omega_1^{\alpha\beta}(v, p) &= (\hat{p}^\alpha \gamma^\beta - \hat{p}^\beta \gamma^\alpha) \left[ \not{\epsilon}^* (R_1 + R_2 \not{p}) + (v \cdot \epsilon^*) (R_3 + R_4 \not{p}) \right] \\
&\quad + (\epsilon^{*\alpha} \gamma^\beta - \epsilon^{*\beta} \gamma^\alpha) \left[ R_5 + R_6 \not{p} \right] + (\epsilon^{*\alpha} \hat{p}^\beta - \epsilon^{*\beta} \hat{p}^\alpha) \left[ R_7 + R_8 \not{p} \right] \\
&\quad + i\sigma^{\alpha\beta} \left[ \not{\epsilon}^* (R_9 + R_{10} \not{p}) + (v \cdot \epsilon^*) (R_{11} + R_{12} \not{p}) \right]
\end{aligned} \tag{2.14}$$

with  $\hat{p}^\mu = p^\mu / (v \cdot p)$  and  $\xi \equiv v \cdot p = (m_B^2 + m_\rho^2 - q^2) / 2m_B$ . Eqs.(2.9)-(2.14) yield

$$\begin{aligned}
\langle \rho(p, \epsilon^*) | \bar{u} \gamma^\mu (1 - \gamma^5) b | B(p_B) \rangle &= -2i \sqrt{\frac{m_B \bar{\Lambda}}{\Lambda_B}} \left\{ i L'_3 \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu \hat{p}_\alpha v_\beta + (L'_1 + L'_3) \epsilon^{*\mu} \right. \\
&\quad \left. - (L'_3 - L'_4) (v \cdot \epsilon^*) \hat{p}^\mu - L'_2 (v \cdot \epsilon^*) v^\mu + \mathcal{O}(1/m_Q^2) \right\}
\end{aligned} \tag{2.15}$$

with

$$\begin{aligned}
L'_i &= L_i + \frac{1}{2m_Q} \delta L'_i = L_i + \frac{1}{2m_Q} (\delta L_i + R'_i), \\
R'_1 &= -2R_1 - 2R_5 + R_7 - 3R_9 + (2R_2 - R_8) \hat{p}^2, \\
R'_2 &= -2R_3 - 2R_5 - 3R_{11} - (2R_4 + R_8) \hat{p}^2, \\
R'_3 &= 2R_1 - 2R_2 - 2R_6 - R_7 + R_8 - 3R_{10}, \\
R'_4 &= -2R_6 - R_7 - 2R_3 - 2R_4 - 3R_{12}.
\end{aligned}$$

Comparison between Eqs.(2.1) and (2.15) gives relations between the form factors  $A_i$  ( $i = 1, 2, 3$ ),  $V$  and the universal wave functions,

$$\begin{aligned}
A_1(q^2) &= \frac{2}{m_B + m_\rho} \sqrt{\frac{m_B \bar{\Lambda}}{\Lambda_B}} \{ L'_1(v \cdot p) + L'_3(v \cdot p) \} + \dots; \\
A_2(q^2) &= 2(m_B + m_\rho) \sqrt{\frac{m_B \bar{\Lambda}}{\Lambda_B}} \left\{ \frac{L'_2(v \cdot p)}{2m_B^2} + \frac{L'_3(v \cdot p) - L'_4(v \cdot p)}{2m_B(v \cdot p)} \right\} + \dots; \\
A_3(q^2) &= 2(m_B + m_\rho) \sqrt{\frac{m_B \bar{\Lambda}}{\Lambda_B}} \left\{ \frac{L'_2(v \cdot p)}{2m_B^2} - \frac{L'_3(v \cdot p) - L'_4(v \cdot p)}{2m_B(v \cdot p)} \right\} + \dots; \\
V(q^2) &= \sqrt{\frac{m_B \bar{\Lambda}}{\Lambda_B}} \frac{m_B + m_\rho}{m_B(v \cdot p)} L'_3(v \cdot p) + \dots,
\end{aligned} \tag{2.16}$$

where the dots denote higher order  $1/m_Q$  contributions not to be taken into account in the following calculations.

### III. LIGHT CONE SUM RULES IN HQEFT

For the derivation of the  $1/m_Q$  order corrections to  $B \rightarrow \rho l \nu$  decay, we consider the two-point correlation function

$$\begin{aligned}
F^\mu &= i \int d^4x e^{-i(p_B - m_Q v) \cdot x} \langle \rho(p, \epsilon^*) | T \left\{ \bar{u}(0) \gamma^\mu (1 - \gamma^5) \frac{1}{i v \cdot D} P_+ \left( D_\perp^2 + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} \right) Q_v^+(0), \right. \\
&\quad \left. \bar{Q}_v^+(x) i \gamma^5 d(x) \right\} | 0 \rangle
\end{aligned} \tag{3.1}$$

where  $\bar{Q}_v^+(x) i \gamma^5 d(x)$  is the interpolating current for  $B$  meson. Inserting between the two currents in Eq.(3.1) a complete set of intermediate states with the  $B$  meson quantum number, one gets

$$\begin{aligned} & \frac{m_B \bar{\Lambda}}{m_Q \Lambda_B} \frac{2iF}{2\Lambda_B - \omega} \left\{ \delta L'_3 \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* \hat{p}_\alpha v_\beta + i(\delta L'_3 - \delta L'_4)(v \cdot \epsilon^*) \hat{p}^\mu - i(\delta L'_1 + \delta L'_3) \epsilon^{*\mu} + i\delta L'_2 (v \cdot \epsilon^*) v^\mu \right\} \\ & + \int_{s_0}^{\infty} ds \frac{\rho(\xi, s)}{s - \omega} + \text{subtraction} \end{aligned} \quad (3.2)$$

with  $\xi \equiv v \cdot p$  and  $\omega \equiv 2v \cdot k$ , where  $k = p_B - m_Q v$  is the residual momentum of the bottom quark. The second term in (3.2) represents the higher resonance contributions.  $F$  is the decay constant of  $B$  meson at the leading order of  $1/m_Q$  expansion, defined by [17]

$$\langle 0 | \bar{q} \Gamma Q_v^+ | B_v \rangle = \frac{F}{2} \text{Tr}[\Gamma \mathcal{M}_v]. \quad (3.3)$$

In deep Euclidean region the correlator (3.1) can be calculated in effective field theory. The result can be written also as an integral over a theoretic spectral density,

$$\int_0^{\infty} ds \frac{\rho_{th}(\xi, s)}{s - \omega} + \text{subtraction}. \quad (3.4)$$

The standard treatment of sum rule is to assume the quark-hadron duality, and to equal the hadronic representation (3.2) and the theoretic one (3.4), which provides an equation:

$$\begin{aligned} & \frac{m_B \bar{\Lambda}}{m_Q \Lambda_B} \frac{2iF}{2\Lambda_B - \omega} \left\{ \delta L'_3 \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* \hat{p}_\alpha v_\beta + i(\delta L'_3 - \delta L'_4)(v \cdot \epsilon^*) \hat{p}^\mu - i(\delta L'_1 + \delta L'_3) \epsilon^{*\mu} + i\delta L'_2 (v \cdot \epsilon^*) v^\mu \right\} \\ & + \int_{s_0}^{\infty} ds \frac{\rho(\xi, s)}{s - \omega} = \int_0^{\infty} ds \frac{\rho_{th}(\xi, s)}{s - \omega} + \text{subtraction}. \end{aligned} \quad (3.5)$$

To ensure the reliability of sum rule estimates, one should enhance the importance of the ground state contribution, suppress higher order nonperturbative contributions and remove the *subtraction*. These can be achieved by performing the Borel transformation

$$\hat{B}_T^{(\omega)} \equiv \lim_{\substack{-\omega, n \rightarrow \infty \\ -\omega/n = \bar{T}}} \frac{(-\omega)^{n+1}}{n!} \left( \frac{d}{d\omega} \right)^n$$

to both sides of the equation (3.5). With using the formulae

$$\hat{B}_T^{(\omega)} \frac{1}{s - \omega} = e^{-s/T}, \quad \hat{B}_T^{(\omega)} e^{\lambda\omega} = \delta\left(\lambda - \frac{1}{T}\right), \quad (3.6)$$

one gets

$$\begin{aligned} & 2iF \left\{ \delta L'_3 \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* \hat{p}_\alpha v_\beta - i(\delta L'_1 + \delta L'_3) \epsilon^{*\mu} + i(\delta L'_3 - \delta L'_4)(v \cdot \epsilon^*) \hat{p}^\mu + i\delta L'_2 (v \cdot \epsilon^*) v^\mu \right\} e^{-2\bar{\Lambda}_B/T} \\ & = \int_0^{s_0} ds e^{-s/T} \rho(\xi, s), \end{aligned} \quad (3.7)$$

where the spectral density  $\rho(\xi, s)$  can be derived via double Borel transformations,

$$\rho(\xi, s) = \hat{B}_{1/s}^{(-1/T)} \hat{B}_T^{(\omega)} F^\mu(\xi, \omega). \quad (3.8)$$

In calculating the three-point function (3.1), one may represent the nonperturbative contributions embedded in the hadronic matrix element in terms of light cone wave functions. Among them are the two-particle distribution functions and the three-particle ones. However, if we restrict our calculation to the lowest twist (twist 2) level, the three-particle functions do not contribute. As a result, the chromomagnetic operator in Eq.(3.1) could be neglected in the lowest twist approximation. The leading twist distribution functions are defined by [2,4-6]

$$\begin{aligned} \langle \rho(p, \epsilon^*) | \bar{u}(0) \sigma_{\mu\nu} d(x) | 0 \rangle &= -i f_\rho^\perp (\epsilon_\mu^* p_\nu - \epsilon_\nu^* p_\mu) \int_0^1 du e^{iup \cdot x} \phi_\perp(u), \\ \langle \rho(p, \epsilon^*) | \bar{u}(0) \gamma_\mu d(x) | 0 \rangle &= f_\rho m_\rho p_\mu \frac{\epsilon^* \cdot x}{p \cdot x} \int_0^1 du e^{iup \cdot x} \phi_\parallel(u) \\ &\quad + f_\rho m_\rho (\epsilon_\mu^* - p_\mu \frac{\epsilon^* \cdot x}{p \cdot x}) \int_0^1 du e^{iup \cdot x} g_\perp^{(v)}(u), \\ \langle \rho(p, \epsilon^*) | \bar{u}(0) \gamma_\mu \gamma_5 d(x) | 0 \rangle &= \frac{1}{4} f_\rho m_\rho \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha x^\beta \int_0^1 du e^{iup \cdot x} g_\perp^{(a)}(u) \end{aligned} \quad (3.9)$$

with  $\phi_{\perp,\parallel}$ , and  $g_{\perp}^{(v,a)}$  being functions with nonperturbative nature.

Then the effective heavy quark fields  $Q_v^+(x_1)\bar{Q}_v^+(x_2)$  can be contracted into a propagator of heavy quark,  $P_+ \int_0^\infty dt \delta(x_1 - x_2 - vt)$ . In the lowest twist approximation, only the kinematic operator contributes to the  $1/m_Q$  order corrections to  $B \rightarrow \rho l \nu$  decay. At  $v \cdot A = 0$  gauge we used, the correlation function (3.1) simplifies as

$$\begin{aligned}
& i \int d^4x \int d^4y \int_0^\infty dl \int_0^\infty dt e^{-ik \cdot x} \langle \rho(p, \epsilon^*) | \bar{u}(0) \gamma^\mu (1 - \gamma^5) P_+ \delta(-y - vl) \\
& \quad \times \left[ \partial_{(y)}^2 - v_\alpha v_\beta \partial_{(y)}^\alpha \partial_{(y)}^\beta - \partial_{(y)}^\alpha A_\alpha(y) - A_\alpha(y) \partial_{(y)}^\alpha \right. \\
& \quad \left. + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta}(y) + A_\alpha(y) A^\alpha(y) \right] P_+ \delta(y - x - vt) \gamma^5 d(x) | 0 \rangle \\
& \rightarrow i \int d^4x \int d^4y \int_0^\infty dl \int_0^\infty dt e^{-ik \cdot x} \delta(-y - vl) \left[ \left( \frac{\partial^2}{\partial y^2} - v^\alpha v^\beta \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \right) \delta(y - x - vt) \right] \\
& \quad \langle \rho(p, \epsilon^*) | \bar{u}(0) \frac{1}{2} \left[ \gamma^\mu \gamma^5 - \gamma^\mu - \sigma^{\alpha\beta} (i v_\beta g_{\alpha\mu} + \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} v_\nu) \right] d(x) | 0 \rangle
\end{aligned} \tag{3.10}$$

with  $\partial_{(y)}^\alpha \equiv \partial / (\partial y_\alpha)$ . The final formula in (3.10) includes only the terms related to the two-particle distribution functions (3.9).

Now the transition matrix element can be evaluated through the distribution functions defined in (3.9). Using Eqs. (3.6) and (3.9), the spectral function is found to be

$$\begin{aligned}
\rho(\xi, s) = & \hat{B}_{1/s}^{(-1/T)} \hat{B}_T^{(\omega)} F^\mu = \frac{1}{2\xi} \left\{ i \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* p_\alpha v_\beta \left[ -\frac{1}{4\xi^2} f_\rho m_\rho^3 (u^2 g_\perp^{(a)})'' + f_\rho^\perp m_\rho^2 \frac{1}{\xi} (u^2 \phi_\perp)' \right. \right. \\
& + \frac{1}{4} f_\rho m_\rho (u^2 g_\perp^{(a)})'' - \frac{3}{4} f_\rho m_\rho^2 (u g_\perp^{(a)})' \left. \right] + \epsilon^{*\mu} \left[ f_\rho m_\rho^3 \frac{1}{\xi} (u^2 g_\perp^{(v)})' - 2 f_\rho^\perp m_\rho^2 u \phi_\perp \right. \\
& - f_\rho m_\rho \xi (u^2 g_\perp^{(v)})' - 2 f_\rho^\perp \xi^2 u \phi_\perp + 4 f_\rho^\perp \xi^2 u \phi_\perp \left. \right] + (v \cdot \epsilon^*) p^\mu \left[ -\frac{1}{\xi^2} f_\rho m_\rho^3 (u^2 g_\perp^{(v)})' \right. \\
& + \frac{1}{\xi^2} f_\rho m_\rho^3 (u^2 \phi_\parallel)' - \frac{2}{\xi^2} f_\rho m_\rho^3 u g_\perp^{(v)} + \frac{2}{\xi^2} f_\rho m_\rho^3 u \phi_\parallel - \frac{2}{\xi^3} f_\rho m_\rho^3 \xi G_\perp^{(v)} + \frac{2}{\xi^3} f_\rho m_\rho^3 \xi \Phi_\parallel \\
& + f_\rho m_\rho (u^2 g_\perp^{(v)})' - f_\rho m_\rho (u^2 \phi_\parallel)' \left. \right] + (v \cdot \epsilon^*) v^\mu \left[ 2 f_\rho^\perp m_\rho^2 u \phi_\perp - 2 f_\rho^\perp m_\rho^2 (u^2 \phi_\perp)' \right. \\
& \left. - 4 f_\rho^\perp \xi^2 u \phi_\perp + 2 f_\rho^\perp \xi^2 (u^2 \phi_\perp)' \right] \left. \right\}_{u \rightarrow \frac{s}{2\xi}}, \tag{3.11}
\end{aligned}$$

where  $'$  denotes derivative with respect to the variable  $u$ , while  $G_\perp^{(v)}(u)$  and  $\Phi_\parallel(u)$  are functions related to  $g_\perp^{(v)}$  and  $\phi_\parallel$  by  $\frac{\partial}{\partial u} G_\perp^{(v)}(u) = g_\perp^{(v)}(u)$ ,  $\frac{\partial}{\partial u} \Phi_\parallel(u) = \phi_\parallel(u)$ . The detailed procedure in deriving Eq.(3.11) are similar to those in Refs. [10,11,13].

#### IV. NUMERICAL ANALYSIS

$\phi_\perp$  and  $\phi_\parallel$  are the lowest twist distributions in the fraction of total momentum carried by the quark in transversely and longitudinally polarized mesons. They can be expanded in Gegenbauer polynomials  $C_n^{3/2}(x)$  whose coefficients are renormalized multiplicatively. With the scale dependence explicitly, one has [4]

$$\begin{aligned}
\phi_{\perp(\parallel)}(u, \mu) = & 6u(1-u) \left[ 1 + \sum_{n=2,4,\dots} a_n^{\perp(\parallel)}(\mu) C_n^{3/2}(2u-1) \right], \\
a_n^{\perp(\parallel)}(\mu) = & a_n^{\perp(\parallel)}(\mu_0) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{(\gamma_n^{\perp(\parallel)} - \gamma_0^{\perp(\parallel)}) / (2\beta_0)}, \tag{4.1}
\end{aligned}$$

where  $\beta_0 = 11 - (2/3)n_f$ , and  $\gamma_n^\parallel, \gamma_n^\perp$  are the one loop anomalous dimensions [19,20]. The nonperturbative parameters  $a_n^\perp$  and  $a_n^\parallel$  have been obtained in [4] with the values

$$a_2^\perp(1\text{GeV}) = 0.2 \pm 0.1, \quad a_2^\parallel(1\text{GeV}) = 0.18 \pm 0.10 \tag{4.2}$$

and  $a_n^{\perp(\parallel)} = 0$  for  $n \neq 2$ .

The functions  $g_{\perp}^{(v)}$  and  $g_{\perp}^{(a)}$  describe transverse polarizations of quarks in the longitudinally polarized mesons. As in Ref. [6] they are parameterized as

$$\begin{aligned}
g_{\perp}^{(v)}(u, \mu) &= \frac{3}{4} \left(1 + (2u - 1)^2\right) + \frac{3}{2} a_1^{\parallel}(\mu) (2u - 1)^3 + \left(\frac{3}{7} a_2^{\parallel}(\mu) + 5\xi_3(\mu)\right) \left(3(2u - 1)^2 - 1\right) \\
&\quad + \left[\frac{9}{112} a_2^{\parallel}(\mu) + \frac{15}{64} \xi_3(\mu) (3\omega_3^V(\mu) - \omega_3^A(\mu))\right] \left(3 - 30(2u - 1)^2 + 35(2u - 1)^4\right), \\
g_{\perp}^{(a)}(u, \mu) &= 6u(1 - u) \left[1 + a_1^{\parallel}(\mu) (2u - 1) + \left(\frac{1}{4} a_2^{\parallel}(\mu) + \frac{5}{3} \xi_3(\mu) \left(1 - \frac{3}{16} \omega_3^A(\mu) + \frac{9}{16} \omega_3^V(\mu)\right)\right)\right. \\
&\quad \left. \times \left(5(2u - 1)^2 - 1\right)\right]
\end{aligned} \tag{4.3}$$

All the nonperturbative parameters in Eqs.(4.3) have been estimated in Ref. [6]. The asymptotic form of these factors and the renormalization scale dependence are given by perturbative QCD [21,22]. As in Refs. [11], the typical virtuality of the bottom quark

$$\mu_b \sim \sqrt{m_B^2 - m_b^2} \approx 2.4\text{GeV}, \tag{4.4}$$

is used for the energy scale for the current calculation.

The values of the hadron quantities  $f_{\rho}$ ,  $f_{\rho}^{\perp}$ ,  $\bar{\Lambda}_B$ ,  $\bar{\Lambda}$  and  $F$  have been extracted in the previous work (see, e.g., [4,17,23,24]). For consistency, here we use for them the same values as in Ref. [11], i.e.,

$$\begin{aligned}
f_{\rho^{\pm}} &= (195 \pm 7)\text{MeV}, \quad f_{\rho^0} = (216 \pm 5)\text{MeV}, \quad f_{\rho}^{\perp} = (160 \pm 10)\text{MeV}, \\
\bar{\Lambda}_B &\approx \bar{\Lambda} = 0.53\text{GeV}, \quad F = (0.30 \pm 0.06)\text{GeV}^{3/2}.
\end{aligned} \tag{4.5}$$

$\delta L_i$  as functions of  $\xi$ ,  $T$  and  $s_0$  can be derived from Eqs.(3.7) and (3.11). Fig.1 shows the variation of  $\delta L_i$  as functions of the Borel parameter  $T$  at  $v \cdot p = 2.5$  GeV. The curves in each figure correspond to different values adopted for the threshold  $s_0$ .

The rule of LCSR method is to determine  $s_0$  from the stability of relevant curves in the reliable region of  $T$ , where both the higher nonperturbative corrections and the contributions from excited and continuum states should not be large. In the current case, we focus on the region around  $T = 1.5 - 2\text{GeV}$ . As shown in Fig.1,  $\delta L_i$  are found to be stable with respect to the Borel parameter  $T$ . In Ref. [11] the threshold  $s_0 = 2.1 \pm 0.6\text{GeV}$  is adopted in evaluating the leading order wave functions  $L_i$ . In calculating the decay width we will use for  $\delta L_i$  the same threshold values as those for the leading order wave function  $L_i$ , i.e.,  $s_0 \approx 2.1\text{GeV}$ .

With (2.16), the form factors  $A_1$ ,  $A_2$ ,  $A_3$  and  $V$  with including  $1/m_Q$  order corrections can be calculated. It is convenient to represent each of these form factors in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F q^2/m_B^2 + b_F (q^2/m_B^2)^2}, \tag{4.6}$$

where  $F(q^2)$  can be any one of  $A_1(q^2)$ ,  $A_2(q^2)$ ,  $A_3(q^2)$  and  $V(q^2)$ . The parameters  $F(0)$ ,  $a_F$  and  $b_F$  presented in table 1 are fitted from the the LCSR results at  $s_0 = 2.1\text{GeV}$ . Fig.2 shows the form factors as functions of the momentum transfer, where the dashed curves are for the leading order results while the solid ones for the results with the  $1/m_Q$  order corrections included.

	$F(0)$	$a_F$	$b_F$	
$A_1$	0.26	0.37	-0.19	LO
	0.27	0.32	-0.19	NLO
$A_2$	0.26	1.11	0.26	LO
	0.26	1.15	0.30	NLO
$A_3$	-0.26	1.12	0.26	LO
	-0.26	1.11	0.25	NLO
$V$	0.32	1.24	0.29	LO
	0.31	1.26	0.32	NLO

Table 1. Results of LCSR calculations up to leading (LO) and next leading order (NLO) in HQEFT. The leading order results are obtained in Ref. [10].

The differential decay width of  $B \rightarrow \rho l \nu$  with the lepton mass neglected is

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{1/2} q^2 (H_0^2 + H_+^2 + H_-^2) \quad (4.7)$$

with the helicity amplitudes

$$\begin{aligned} H_{\pm} &= (m_B + m_{\rho}) A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B + m_{\rho}} V(q^2), \\ H_0 &= \frac{1}{2m_{\rho} \sqrt{q^2}} \{ (m_B^2 - m_{\rho}^2 - q^2)(m_B + m_{\rho}) A_1(q^2) - \frac{\lambda}{m_B + m_{\rho}} A_2(q^2) \} \end{aligned} \quad (4.8)$$

and

$$\lambda \equiv (m_B^2 + m_{\rho}^2 - q^2)^2 - 4m_B^2 m_{\rho}^2. \quad (4.9)$$

The total width of  $B \rightarrow \rho l \nu$  can be obtained by integrating (4.7) over the whole accessible region of  $q^2$ . We get

$$\Gamma(B \rightarrow \rho l \nu) = (13.6 \pm 4.0) |V_{ub}|^2 \text{ps}^{-1}, \quad (4.10)$$

where the error results from the variation of the threshold energy  $s_0 = 1.5 - 2.7 \text{GeV}$ .

The branching fraction of  $B^0 \rightarrow \rho^- l^+ \nu$  is measured to be  $\text{Br}(B^0 \rightarrow \rho^- l^+ \nu) = (2.6 \pm 0.7) \times 10^{-4}$  [25]. This and the world average of the  $B^0$  lifetime [25]  $\tau_{B^0} = 1.536 \pm 0.014 \text{ps}$  yields

$$\Gamma(B^0 \rightarrow \rho^- l^+ \nu) = (1.69 \pm 0.47) \times 10^{-4} \text{ps}^{-1}. \quad (4.11)$$

$|V_{ub}|$  is then extracted from Eqs.(4.10) and (4.11). It is

$$|V_{ub}| = (3.53 \pm 0.49 \pm 0.52) \times 10^{-3}, \quad (4.12)$$

where the first and second errors correspond to the experimental and theoretical uncertainties, respectively. This value may be compared with the ones previously obtained [10,11,13,16,26]. From the exclusive semileptonic decays  $B \rightarrow \pi(\rho) l \nu$ , we then have

$$\begin{aligned} |V_{ub}| &= (3.4 \pm 0.5 \pm 0.5) \times 10^{-3} \quad (B \rightarrow \pi l \nu, \text{LO}) \\ |V_{ub}| &= (3.2 \pm 0.5 \pm 0.4) \times 10^{-3} \quad (B \rightarrow \pi l \nu, \text{to NLO}) \\ |V_{ub}| &= (3.7 \pm 0.6 \pm 0.7) \times 10^{-3} \quad (B \rightarrow \rho l \nu, \text{LO}) \\ |V_{ub}| &= (3.5 \pm 0.5 \pm 0.5) \times 10^{-3} \quad (B \rightarrow \rho l \nu, \text{to NLO}) \\ |V_{ub}| &= (3.5 \pm 0.6 \pm 0.1) \times 10^{-3} \quad (B \text{ inclusive semileptonic decays}) \end{aligned}$$

As a summary, we have studied  $B \rightarrow \rho l \nu$  decay up to the  $1/m_Q$  order corrections in HQEFT. In HQEFT,  $1/m_Q$  order corrections from the effective current and from effective Lagrangian are given by the same operator forms, which simplifies the structure of transition matrix elements. These  $1/m_Q$  order contributions have been calculated using light cone sum rules with considering the lowest twist distribution functions. Numerically, the  $1/m_Q$  order wave functions give only corrections lower than 10% to the transition form factors. Similar to the  $B \rightarrow \pi l \nu$  case, the correction indicates a slightly smaller value of the CKM matrix element  $|V_{ub}|$ . The discussion concerning  $1/m_Q$  order corrections to  $B \rightarrow \rho l \nu$  decay in this paper is also applicable to other heavy to light vector meson decays.

## ACKNOWLEDGMENTS

This work was supported in part by the key projects of National Science Foundation of China (NSFC) and Chinese Academy of Sciences, and by the BEPC National Lab Opening Project.

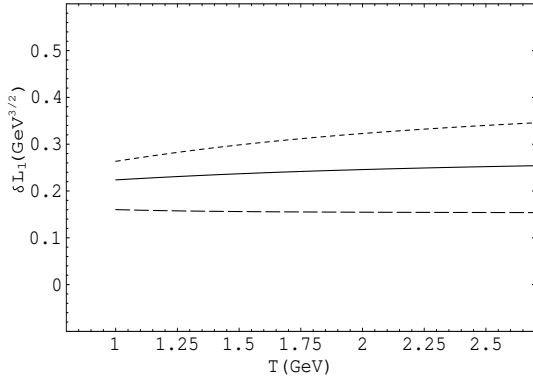
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[1] V. M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. C **60**, 349 (1993).

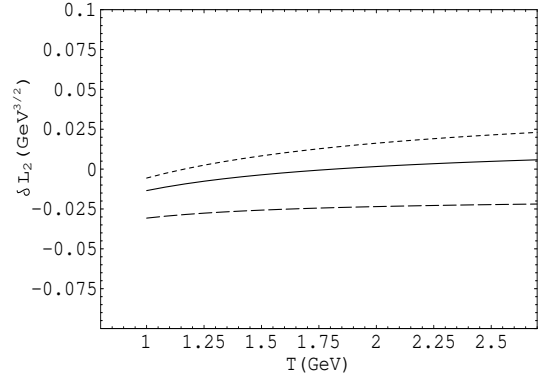
- [2] A. Khodjamirian and R. Rückl, WUE-ITP-97-049, MPI-PhT/97-85, hep-ph/9801443.
- [3] A. Khodjamirian, R. Rückl, S. Weinzierl, C. W. Winhart and O. Yakovlev, Phys. Rev. D **62**, 114002 (2000).
- [4] P. Ball and V. M. Braun, Phys. Rev. D **54**, 2182 (1996).
- [5] P. Ball and V. M. Braun, Phys. Rev. D **55**, 5561 (1997).
- [6] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).
- [7] M. A. Ivanov, Y. L. Kalinovsky and C. D. Roberts, Phys. Rev. D **60**, 034018 (1999).
- [8] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000).
- [9] M. A. Ivanov, Y. L. Kalinovsky and C. D. Roberts, Preprint ANL-PHY-9594-TH-2000, hep-ph/0006189.
- [10] W. Y. Wang, Y. L. Wu, Phys. Lett. B **515**, 57 (2001).
- [11] W. Y. Wang, Y. L. Wu, Phys. Lett. B **519**, 219 (2001).
- [12] W. Y. Wang, Y. L. Wu, M. Zhong, Phys. Rev. D **67**, 014024 (2003).
- [13] W. Y. Wang, Y. L. Wu, M. Zhong, J. Phys. G **29** 2743 (2003).
- [14] Y. L. Wu, Mod. Phys. Lett. A **8**, 819 (1993).
- [15] W. Y. Wang, Y. L. Wu and Y. A. Yan, Int. J. Mod. Phys. A **15**, 1817 (2000).
- [16] Y. A. Yan, Y. L. Wu, W. Y. Wang, Int. J. Mod. Phys. A **15**, 2735 (2000).
- [17] W. Y. Wang and Y. L. Wu, Int. J. Mod. Phys. A **16**, 377 (2001).
- [18] W. Y. Wang, Y. L. Wu, Int. J. Mod. Phys. A **16**, 2505 (2001).
- [19] D. J. Gross and F. Wilczek, Phys. Rev. D **9**, 980 (1974).
- [20] M. A. Shifman and M. I. Vysotsky, Nucl. Phys. B **186**, 475 (1981).
- [21] V. M. Braun and I. B. Filyanov, Z. Phys. C **48**, 239 (1990).
- [22] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
- [23] M. A. Benitez et. al., Phys. Lett. B **239**, 1 (1990).
- [24] L. Montanet et. al., Phys. Rev. D **50**, 1173 (1994).
- [25] S. Eidelman, et al., Phys. Lett. B **592**, 1 (2004).
- [26] Y. L. Wu and Y. A. Yan, Int. J. Mod. Phys. A **16**, 285 (2001)



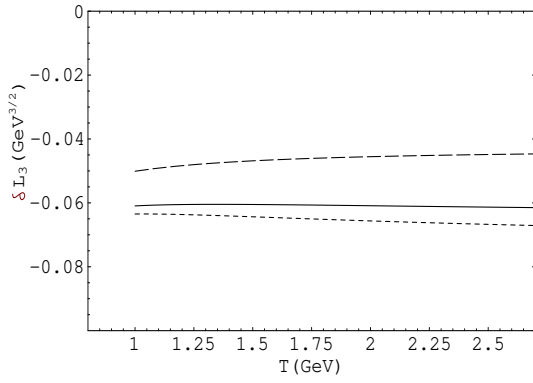
# FIGURES



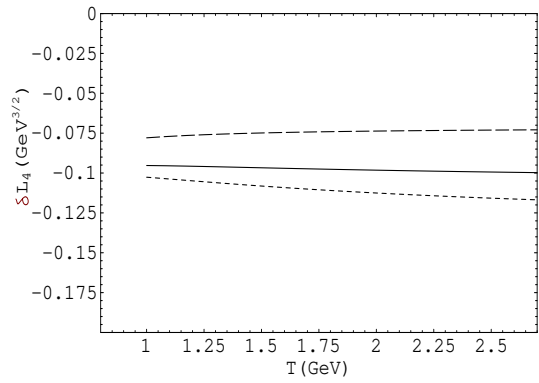
(a)



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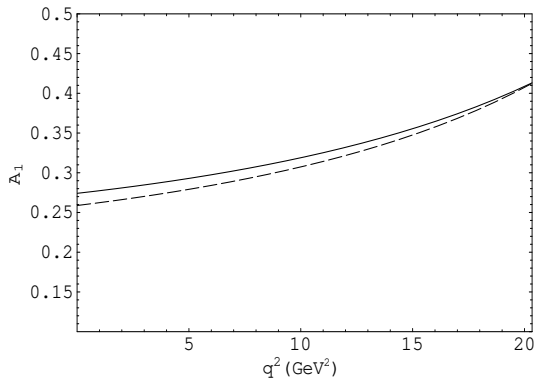


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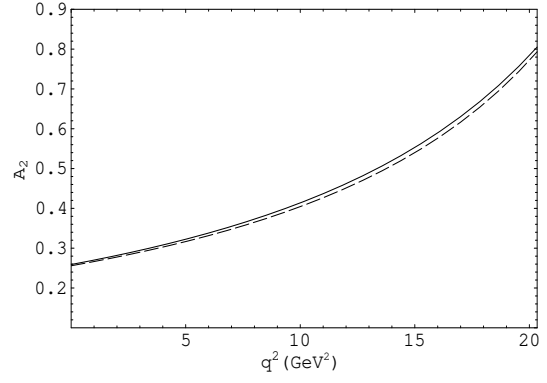


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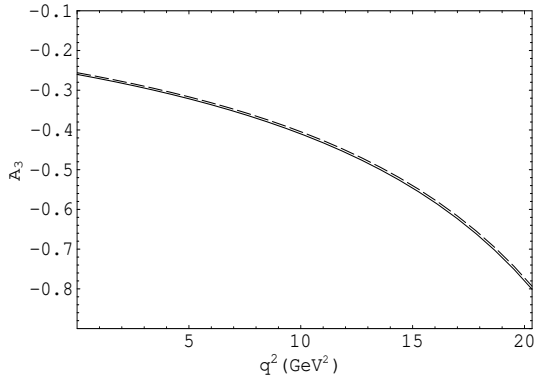
Fig.1. Variation of  $1/m_Q$  order wave functions  $\delta L_i (i = 1, 2, 3, 4)$  with respect to the Borel parameter  $T$  at  $\xi = v \cdot p = 2.5$  GeV. The dashed, solid and dotted curves correspond to the thresholds  $s_0 = 1.5, 2.1$  and  $2.7$  GeV respectively.



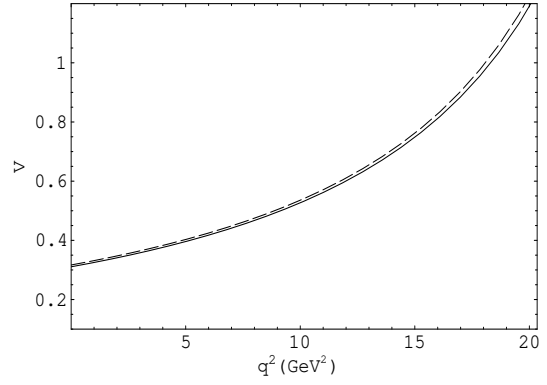
(a)



(b)



(c)



(d)

Fig.2. Form factors  $A_i (i = 1, 2, 3)$  and  $V$  obtained from light cone sum rules in HQEFT. The dashed curves are the leading order results in HQEFT [10,12], while the solid curves are the results with including  $1/m_Q$  order correction.