

THE EXISTENCE OF A MASS GAP IN QUANTUM YANG-MILLS THEORY

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(Dated: February 28, 2006)

The skeleton loop integrals which contribute into the gluon self-energy have been iterated (skeleton loops expansion) within the Schwinger-Dyson equation for the full gluon propagator. No any truncations/approximations as well as no special gauge choice have been made. It is explicitly shown that such obtained general iteration solution for the full gluon propagator can be exactly and uniquely decomposed as a sum of the two principally different terms. The first term is the Laurent expansion in integer powers of severe (i.e., more singular than $1/q^2$) infrared singularities accompanied by the corresponding powers of the mass gap and multiplied by the corresponding residues. The second (perturbative) term is always as much singular as $1/q^2$ and otherwise remaining undetermined. We have explicitly demonstrated that the mass gap is hidden in the above-mentioned skeleton loop integrals due to the nonlinear interaction of massless gluon modes. It shows explicitly up when the gluon momentum goes to zero. The appropriate regularization scheme has been applied in order to make a gauge-invariant existence of the mass gap perfectly clear. Moreover, it survives an infinite series summation of the relevant skeleton loop contributions into the gluon self-energy. The physical meaning of the mass gap is to be responsible for the large scale structure of the true QCD vacuum.

PACS numbers: 11.15.Tk, 12.38.Lg

I. INTRODUCTION

The Lagrangian of QCD [1, 2] does not contain explicitly any of the mass scale parameters which could have a physical meaning even after the corresponding renormalization program is performed. However, if QCD itself is a confining theory then a characteristic scale has to exist. It should be directly responsible for the large scale structure of the true QCD vacuum in the same way as Λ_{QCD} is responsible for the nontrivial perturbative dynamics there (scale violation, asymptotic freedom (AF) [1, 2]). On one hand, the color confinement problem is not yet solved [3]. On the other hand, today there is no doubt that color confinement is closely related to the above-mentioned structure of the true QCD ground state [4, 5] and vice-versa. The perturbation theory (PT) technics fail to investigate them.

The main goal of this paper is to show how the above-mentioned characteristic scale (the mass gap in what follows, for simplicity) responsible for the nonperturbative (NP) dynamics in the infrared (IR) region may explicitly appear in QCD. This especially becomes imperative after Jaffe and Witten have formulated their theorem "Yang-Mills Existence And Mass Gap" [6]. Moreover, we will show that the mass gap may not only appear, but it may also survive after an infinite series summation of the relevant skeleton loop contributions into the gluon self-energy. Thus the paper is devoted to a possible solution of one of the important problems in theoretical particle/nuclear physics, namely to the dynamical generation of a mass gap in quantum field gauge theories.

The propagation of gluons is one of the main dynamical effects in the true QCD vacuum. The gluon Green's function is (Euclidean signature here and everywhere below)

$$D_{\mu\nu}(q) = i \{ T_{\mu\nu}(q) d(q^2, \xi) + \xi L_{\mu\nu}(q) \} \frac{1}{q^2}, \quad (1.1)$$

where ξ is the gauge-fixing parameter and $T_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu / q^2 = \delta_{\mu\nu} - L_{\mu\nu}(q)$. Evidently, $T_{\mu\nu}(q)$ is the transverse ("physical") component of the full gluon propagator, while $L_{\mu\nu}(q)$ is its longitudinal (unphysical) one. The free gluon propagator is obtained by setting simply the full gluon form factor to $d(q^2, \xi) = 1$ in Eq. (1.1), i.e.,

$$D_{\mu\nu}^0(q) = i \{ T_{\mu\nu}(q) + \xi L_{\mu\nu}(q) \} \frac{1}{q^2}. \quad (1.2)$$

The main tool of our investigation is the Schwinger-Dyson (SD) equation of motion [1, 7, 8] (and references therein) for the full gluon propagator (1.1), since its solutions reflect the quantum-dynamical structure of the true QCD ground

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state. Some results of the present investigation have been already briefly announced in our preliminary publications [9, 10]. Here we present the full (without any estimates, approximations/truncations, specific gauge choice, etc.) investigation of the problem of the dynamical origin of a mass gap in quantum Yang-Mills (YM) theory.

II. GLUON SD EQUATION

The general structure of the gluon SD equation [1, 7, 9] can be written down symbolically as follows (for our purposes it is more convenient to consider the SD equation for the full gluon propagator and not for its inverse):

$$D(q) = D^0(q) + D^0(q)T_g[D](q)D(q). \quad (2.1)$$

Here and in some places below, we omit the dependence on the Dirac indices, for simplicity, as well as the quark and ghost skeleton loop contributions into the gluon self-energy (as it is required by the YM character of our consideration). The nonlinear (NL) pure gluon contribution $T_g[D](q)$ into the gluon self-energy is the sum of the four topologically independent skeleton loop integrals, namely

$$T_g[D](q) = \frac{1}{2}T_t + \frac{1}{2}T_1(q) + \frac{1}{2}T_2(q) + \frac{1}{6}T'_2(q), \quad (2.2)$$

where the so-called constant tadpole term is

$$T_t = g^2 \int \frac{id^4q_1}{(2\pi)^4} T_4^0 D(q_1), \quad (2.3)$$

and all other skeleton loop integrals are given explicitly below as follows:

$$T_1(q) = g^2 \int \frac{id^4q_1}{(2\pi)^4} T_3^0(q, -q_1, q_1 - q) T_3(-q, q_1, q - q_1) D(q_1) D(q - q_1), \quad (2.4)$$

$$T_2(q) = g^4 \int \frac{id^4q_1}{(2\pi)^4} \int \frac{id^n q_2}{(2\pi)^4} T_4^0 T_3(-q_2, q_3, q_2 - q_3) T_3(-q, q_1, q_3 - q_2) D(q_1) D(-q_2) D(q_3) D(q_3 - q_2), \quad (2.5)$$

$$T'_2(q) = g^4 \int \frac{id^4q_1}{(2\pi)^4} \int \frac{id^4q_2}{(2\pi)^4} T_4^0 T_4(-q, q_1, -q_2, q_3) D(q_1) D(-q_2) D(q_3), \quad (2.6)$$

where in the last two skeleton loop integrals $q - q_1 + q_2 - q_3 = 0$ as usual. Evidently, neither the color group factors nor the Dirac indices play any role in tracking down the mass gap, which can only be of dynamical origin. That is the reason why they are not explicitly shown throughout this paper.

The general iteration solution (i.e., when the above-mentioned skeleton loop integrals are to be iterated) of the gluon SD equation (2.1) looks like

$$\begin{aligned} D(q) &= D^{(0)}(q) + D^{(1)}(q) + D^{(2)}(q) + D^{(3)}(q) + \dots \\ &= D^0(q) + D^0(q)T_g[D^0 + D^{(1)} + D^{(2)} + D^{(3)} + \dots](q)[D^0(q) + D^{(1)}(q) + D^{(2)}(q) + D^{(3)}(q) + \dots], \end{aligned} \quad (2.7)$$

and $D^{(0)}(q) = D^0(q)$. It is nothing but the skeleton loops expansion, and it is not the PT series. First of all, the magnitude of the coupling constant squared cannot be fixed to be small, i.e., it is arbitrary. Secondly, the dependence of the skeleton loop integrals on the coupling constant squared is also completely arbitrary, i.e., it cannot be explicitly fixed on general ground. However, this expansion is rather formal, since the corresponding skeleton loop integrals are not yet regularized. In the deep IR limit $q^2 \rightarrow 0$ the skeleton loop integrals (2.4)-(2.6) tend to their corresponding divergent constant values (in Euclidean metrics $q^2 \rightarrow 0$ implies $q_i \rightarrow 0$). Just these constants having the dimensions of a mass squared are the main objects of our investigation. The only problem is as how to extract them, i.e., to make their existence and important role perfectly clear. Fortunately, this can be explicitly done through the necessary regularization of the initial skeleton loop integrals.

III. REGULARIZATION

Due to AF [1, 2] all the skeleton loop integrals as well as those which will appear in the formal iteration solution (2.7) are divergent, so the general problem of their regularization arises. For our future purpose it is convenient to regularize the above-mentioned skeleton loop integrals by subtracting as usual their values at a safe (slightly different from zero) space-like point $q^2 = \mu^2$ (Euclidean signature is already chosen). Thus, one obtains

$$\begin{aligned} T_1^R(q) &= T_1(q) - T_1(q^2 = \mu^2), \\ T_2^R(q) &= T_2(q) - T_2(q^2 = \mu^2), \\ T_2'^R(q) &= T_2'(q) - T_2'(q^2 = \mu^2), \end{aligned} \quad (3.1)$$

where all the divergent constants having the dimensions of a mass squared are

$$\begin{aligned} T_1(q^2 = \mu^2) &= g^2 \int \frac{id^4 q_1}{(2\pi)^4} T_3^0(q, -q_1, q_1 - q) T_3(-q, q_1, q - q_1) D(q_1) D(q - q_1)|_{q^2 = \mu^2}, \\ T_2(q^2 = \mu^2) &= g^4 \int \frac{id^4 q_1}{(2\pi)^4} \int \frac{id^n q_2}{(2\pi)^4} T_4^0 T_3(-q_2, q_3, q_2 - q_3) T_3(-q, q_1, q_3 - q_2) \\ &\quad D(q_1) D(-q_2) D(q_3) D(q_3 - q_2)|_{q^2 = \mu^2}, \\ T_2'(q^2 = \mu^2) &= g^4 \int \frac{id^4 q_1}{(2\pi)^4} \int \frac{id^4 q_2}{(2\pi)^4} T_4^0 T_4(-q, q_1, -q_2, q_3) D(q_1) D(-q_2) D(q_3)|_{q^2 = \mu^2}. \end{aligned} \quad (3.2)$$

At the same time, the introduction of the ultraviolet (UV) cutoff Λ^2 in these integrals is assumed. All this makes it possible first of all to explicitly release the above-mentioned divergent constants, and put all the divergent skeleton loop integrals under firm control. The UV cutoff should go to infinity at the final stage only. The formal subtraction procedure for the constant tadpole term (2.3) implies $T_t^R = T_t - T_t = 0$. This is in agreement with the dimensional regularization method [1, 2] where $T_t(D^0) = 0$, indeed. So, we can discard this term within the formal iteration solution (2.7) without losing generality.

The subtractions in Eq. (3.1) mean that the decomposition of the regularized quantities into the independent tensor structures can be written down as follows:

$$\begin{aligned} T_1^R(q) &\equiv T_{(1)\mu\nu}^R(q) = \delta_{\mu\nu} q^2 T_1^{(1)}(q^2) + q_\mu q_\nu T_1^{(2)}(q^2), \\ T_2^R(q) &\equiv T_{(2)\mu\nu}^R(q) = \delta_{\mu\nu} q^2 T_2^{(1)}(q^2) + q_\mu q_\nu T_2^{(2)}(q^2), \\ T_2'^R(q) &\equiv T_{(2)\mu\nu}'^R(q) = \delta_{\mu\nu} q^2 T_2'^{(1)}(q^2) + q_\mu q_\nu T_2'^{(2)}(q^2). \end{aligned} \quad (3.3)$$

where all invariant functions $T_1^{(n)}(q^2), T_2^{(n)}(q^2), T_2'^{(n)}(q^2)$ at $n = 1, 2$ are dimensionless ones. In the region of small q^2 they are represented in the form of the corresponding Taylor expansions and remain arbitrary otherwise. Due to the definition $q_\mu q_\nu = q^2 L_{\mu\nu}$, instead of the independent structures $\delta_{\mu\nu}$ and $q_\mu q_\nu$ in Eqs. (3.3) and below, one can use $T_{\mu\nu}$ and $L_{\mu\nu}$ as the independent structures with their own invariant functions. From these relations it follows that the regularized skeleton loop contributions are always of the order q^2 , i.e., $T_1^R(q) = O(q^2)$, $T_2^R(q) = O(q^2)$ and $T_2'^R(q) = O(q^2)$.

The NL pure gluon part (2.2) thus can be exactly decomposed as the sum of the two terms:

$$T_g[D](q) = T_g[D] + T_g^R[D](q), \quad (3.4)$$

where

$$T_g[D] = \frac{1}{2} T_1(D^2) + \frac{1}{2} T_2(D^4) + \frac{1}{6} T_2'(D^3), \quad (3.5)$$

and

$$T_g^R[D](q) = \frac{1}{2} T_1^R(q) + \frac{1}{2} T_2^R(q) + \frac{1}{6} T_2'^R(q) = O(q^2; D). \quad (3.6)$$

In Eq. (3.5) we introduce the following notations: $T_1(q^2 = \mu^2) = T_1(D^2)$, $T_2(q^2 = \mu^2) = T_2(D^4)$ and $T'_2(q^2 = \mu^2) = T'_2(D^3)$, showing their dependence on the corresponding number of the gluon propagators only. In Eq. (3.6) the last equality shows explicitly that its left-hand-side is always of the order q^2 , depending again on D in general. Thus the gluon SD equation (2.1) becomes divided into the two principally different terms, namely

$$D(q) = D^0(q) + D^0(q)T_g[D]D(q) + D^0(q)O(q^2; D)D(q). \quad (3.7)$$

Let us note that both blocks, $T_g[D]$ and $O(q^2; D)$ (which should be iterated with respect to D), depend on D themselves. This means that there is no way to sum up in general these iteration series into the corresponding geometric progression. The latter possibility appears only when the blocks to be iterated do not depend on D like in quantum electrodynamics (QED), where the electron skeleton loop is only present. The difference between these two cases lies, of course, in the NL dynamics of QCD in comparison with the linear one in QED.

Let us emphasize once more that the constant block $T_g[D]$ having the dimensions of a mass squared is just the object we have worried about to demonstrate explicitly its crucial role within our approach. In this connection a few additional remarks are in order. As it follows from the standard gluon SD equation (3.7), the corresponding equation for the gluon self-energy looks like

$$D^{-1}(q) = D_0^{-1}(q) - T_g[D] - O(q^2; D), \quad (3.8)$$

where we put $D^0(q) \equiv D_0(q)$. In order to unravel overlapping UV divergence problems in YM theory, the necessary number of the differentiation with respect to the external momentum should be done first (in order to lower divergences). Then the point-like vertices, which are present in the skeleton loop integrals, should be replaced by their full counterparts via the corresponding integral equations. Finally, one obtains the corresponding SD equations which are much more complicated than the standard ones, containing different scattering amplitudes, which skeleton expansions are, however, free from the above-mentioned overlapping divergences. Of course, the real procedure [11] (and references therein) is much more tedious than briefly described above. However, even at this level, it is clear that by taking derivatives with respect to the external momentum q in Eq. (3.8), the main initial information due to the constant block $T_g[D]$ will be totally lost. Whether it will be restored somehow or not at the later stages of the renormalization program is not clear at all. Thus in order to remove overlapping UV divergences ("the water") from the SD equations and skeleton expansions, we are in danger to completely lose the information on the constant block $T_g[D]$ which is the dynamical source of the mass gap ("the baby") within our approach. In order to avoid this danger and to be guaranteed that no any dynamical information are lost, we are just using the standard gluon SD equation (3.7). The presence of any kind of UV divergences (overlapping and usual (overall)) in the skeleton expansions will not cause any problems in order to detect the mass gap responsible for the IR structure of the true QCD vacuum. In other words, the direct iteration solution of the standard gluon SD equation (3.7), complemented by the proposed regularization scheme, is reliable to release a mass gap, and thus to make its existence perfectly clear. The problem of convergence of such regularized skeleton series is completely irrelevant in the context of the present investigation. Anyway, we keep any kind of UV divergences under control within our method. At the same time, any kind of UV divergences play no any role in the existence of a mass gap responsible for the IR structure of the full gluon propagator, i.e., its existence does not depend on whether overlapping divergences are present or not in the SD equations and corresponding skeleton expansions. All this is the main reason why our starting point is the standard gluon SD equation (2.1) for the unrenormalized Green's functions (this also simplifies notations).

IV. NONLINEAR ITERATION

The formal iteration solution of the gluon SD equation (3.7) now looks like

$$\begin{aligned} D(q) &= D^{(0)}(q) + D^{(1)}(q) + D^{(2)}(q) + D^{(3)}(q) + \dots = D^0(q) \\ &+ D^0(q)T_g[D^0 + D^{(1)} + D^{(2)} + D^{(3)}(q) + \dots][D^0(q) + D^{(1)}(q) + D^{(2)}(q) + D^{(3)}(q) + \dots] \\ &+ D^0(q)O(q^2; D^0 + D^{(1)} + D^{(2)} + D^{(3)}(q) + \dots)[D^0(q) + D^{(1)}(q) + D^{(2)}(q) + D^{(3)}(q) + \dots], \end{aligned} \quad (4.1)$$

where

$$\begin{aligned}
D^{(0)}(q) &= D^0(q), \\
D^{(1)}(q) &= D^0(q)F_1[D^0]D^0(q) + D^0(q)O(q^2; D^0)D^0(q), \\
D^{(2)}(q) &= D^0(q)F_2[D^0, D^{(1)}][D^0(q) + D^{(1)}(q)] + D^0(q)O(q^2; D^0 + D^{(1)})[D^0(q) + D^{(1)}(q)], \\
D^{(3)}(q) &= D^0(q)F_3[D^0, D^{(1)}, D^{(2)}][D^0(q) + D^{(1)}(q) + D^{(2)}(q)] \\
&\quad + D^0(q)O(q^2; D^0 + D^{(1)} + D^{(2)})[D^0(q) + D^{(1)}(q) + D^{(2)}(q)],
\end{aligned} \tag{4.2}$$

and so on. Let us consider the divergent constants $F_1[D^0]$, $F_2[D^0, D^{(1)}]$, $F_3[D^0, D^{(1)}, D^{(2)}]$, ..., introducing the following notations:

$$\begin{aligned}
F_1[D^0] &= T_g[D^0] = \frac{1}{2}T_1((D^0)^2) + \frac{1}{2}T_2((D^0)^4) + \frac{1}{6}T_2'((D^0)^3), \\
F_2[D^0, D^{(1)}] &= T_g[D^0 + D^{(1)}] = \frac{1}{2}T_1((D^0 + D^{(1)})^2) + \frac{1}{2}T_2((D^0 + D^{(1)})^4) + \frac{1}{6}T_2'((D^0 + D^{(1)})^3), \\
F_3[D^0, D^{(1)}, D^{(2)}] &= T_g[D^0 + D^{(1)} + D^{(2)}] = \frac{1}{2}T_1((D^0 + D^{(1)} + D^{(2)})^2) \\
&\quad + \frac{1}{2}T_2((D^0 + D^{(1)} + D^{(2)})^4) + \frac{1}{6}T_2'((D^0 + D^{(1)} + D^{(2)})^3),
\end{aligned} \tag{4.3}$$

and so on. As underlined above, each of them has the dimensions of a mass squared, so on general ground one can represent them as follows:

$$\begin{aligned}
F_1 &\equiv F_1[D^0] = \Delta^2 C_1(\lambda, \nu, \xi, g^2), \\
F_2 &\equiv F_2[D^0, D^{(1)}] = \Delta^2 C_2(\lambda, \nu, \xi, g^2), \\
F_3 &\equiv F_3[D^0, D^{(1)}, D^{(2)}] = \Delta^2 C_3(\lambda, \nu, \xi, g^2),
\end{aligned} \tag{4.4}$$

and so on. In these relations Δ^2 is the above-mentioned mass gap. The dimensionless constants C_1, C_2, C_3, \dots depend on the dimensionless UV cutoff λ introduced as follows: $\Lambda^2 = \lambda\Delta^2$. They depend on the dimensionless renormalization point ν introduced as follows: $\mu^2 = \nu\Delta^2$. Evidently, via the corresponding subscripts these constants depend on which iteration for the gluon propagator D is actually done, which in its turn gives the dependence on the gauge-fixing parameter ξ . They also depend on the dimensionless coupling constant squared g^2 (see Eqs. (3.2)). The parameters λ and ν may depend on ξ and g^2 (and hence vice-versa). The dependence of the mass gap Δ^2 on all these parameters is not shown explicitly, for convenience, but can be restored any time, if necessary.

In the relations (4.3) and (4.4) we use the short-hand notation $D^0 \equiv D^0(q)$, however, it is more appropriate to introduce the short-hand notations as follows:

$$\begin{aligned}
D^0(q) &\equiv D_0(q) \equiv D_0, \quad O_1(q^2) \equiv O(q^2; D^0), \\
O_2(q^2) &\equiv O(q^2; D^0 + D^{(1)}), \quad O_3(q^2) \equiv O(q^2; D^0 + D^{(1)} + D^{(2)}),
\end{aligned} \tag{4.5}$$

and so on. Then the formal iteration solution (4.1) becomes

$$\begin{aligned}
D(q) &= D^{(0)}(q) + D^{(1)}(q) + D^{(2)}(q) + D^{(3)}(q) + \dots = D_0 + [D_0 F_1 D_0 + D_0 O_1(q^2) D_0] \\
&\quad + [D_0 F_2 D_0 + D_0 F_2 D_0 F_1 D_0 + D_0 F_2 D_0 O_1(q^2) D_0 \\
&\quad + D_0 O_2(q^2) D_0 + D_0 O_2(q^2) D_0 F_1 D_0 + D_0 O_2(q^2) D_0 O_1(q^2) D_0] \\
&\quad + [D_0 F_3 D_0 + D_0 F_3 D_0 F_1 D_0 + D_0 F_3 D_0 O_1(q^2) D_0 \\
&\quad + D_0 F_3 D_0 F_2 D_0 + D_0 F_3 D_0 F_2 D_0 F_1 D_0 + D_0 F_3 D_0 F_2 D_0 O_1(q^2) D_0 \\
&\quad + D_0 F_3 D_0 O_2(q^2) D_0 + D_0 F_3 D_0 O_2(q^2) D_0 F_1 D_0 + D_0 F_3 D_0 O_2(q^2) D_0 O_1(q^2) D_0 \\
&\quad + D_0 O_3(q^2) D_0 + D_0 O_3(q^2) D_0 F_1 D_0 + D_0 O_3(q^2) D_0 O_1(q^2) D_0 \\
&\quad + D_0 O_3(q^2) D_0 F_2 D_0 + D_0 O_3(q^2) D_0 F_2 D_0 F_1 D_0 + D_0 O_3(q^2) D_0 F_2 D_0 O_1(q^2) D_0 \\
&\quad + D_0 O_3(q^2) D_0 O_2(q^2) D_0 + D_0 O_3(q^2) D_0 O_2(q^2) D_0 F_1 D_0 + D_0 O_3(q^2) D_0 O_2(q^2) D_0 O_1(q^2) D_0] + \dots .
\end{aligned} \tag{4.6}$$

V. SHIFTING PROCEDURE

The formal iteration series (4.6), however, much more convenient to equivalently rewrite as follows:

$$\begin{aligned}
D(q) = & [D_0^2(F_1 + F_2 + F_3 + \dots) + D_0^3(F_1F_2 + F_3F_1 + F_2F_3 + \dots) + D_0^4(F_1F_2F_3 + \dots) + \dots] \\
& + [D_0F_2D_0O_1(q^2)D_0 + D_0O_2(q^2)D_0F_1D_0 + D_0F_3D_0O_1(q^2)D_0 + D_0F_3D_0F_2D_0O_1(q^2)D_0 \\
& + D_0F_3D_0O_2(q^2)D_0 + D_0F_3D_0O_2(q^2)D_0F_1D_0 + D_0F_3D_0O_2(q^2)D_0O_1(q^2)D_0 \\
& + D_0O_3(q^2)D_0F_1D_0 + D_0O_3(q^2)D_0F_2D_0 + D_0O_3(q^2)D_0F_2D_0F_1D_0 \\
& + D_0O_3(q^2)D_0F_2D_0O_1(q^2)D_0 + D_0O_3(q^2)D_0O_2(q^2)D_0F_1D_0 + \dots] \\
& + [D_0 + D_0O_1(q^2)D_0 + D_0O_2(q^2)D_0 + D_0O_3(q^2)D_0 + D_0O_2(q^2)D_0O_1(q^2)D_0 \\
& + D_0O_3(q^2)D_0O_1(q^2)D_0 + D_0O_3(q^2)D_0O_2(q^2)D_0 + D_0O_3(q^2)D_0O_2(q^2)D_0O_1(q^2)D_0 + \dots].
\end{aligned} \tag{5.1}$$

Since $D_0 \equiv D_0(q) \sim (q^2)^{-1}$, there are three formally different types of terms. The first type of terms is singular as much as $D_0^{2+k} \sim (q^2)^{-2-k}$, $k = 0, 1, 2, 3, \dots$. Evidently, only the divergent constants F_1, F_2, F_3, \dots and their combinations enter into these terms. The third type of terms is singular as much as the free gluon propagator, i.e., they are of the order $D_0 \sim (q^2)^{-1}$, since all functions $O_n(q^2)$, $n = 1, 2, 3, \dots$ are of the order q^2 . Evidently, only the functions $O_n(q^2)$, $n = 1, 2, 3, \dots$ and their products enter into these terms. The second type of terms is the so-called mixed up terms which contain the divergent constants F_1, F_2, F_3, \dots and the functions $O_n(q^2)$, $n = 1, 2, 3, \dots$ in the different combinations.

Let us show that any mixed up term is simply the exact sum of the first and third types of terms. In order to show this explicitly, let us recall that all functions $O_n(q^2)$, $n = 1, 2, 3, \dots$ are regular functions of q^2 . So, we can equivalently represent them in the form of the corresponding Taylor expansions. It is convenient to present such kind of expansions in terms of $D_0^{-1} \sim q^2$ and making an exact decomposition as follows:

$$\begin{aligned}
O_n(q^2) &= O_n(q^2) - D_0^{-1} \sum_{m=0}^{k-1} (D_0\Delta^2)^{-m} O_n^{(m)}(1) + D_0^{-1} \sum_{m=0}^{k-1} (D_0\Delta^2)^{-m} O_n^{(m)}(1) \\
&= D_0^{-1} \sum_{m=0}^{k-1} (D_0\Delta^2)^{-m} O_n^{(m)}(1) + D_0^{-1} (D_0\Delta^2)^{-k} f_n(q^2), \quad n = 1, 2, 3, \dots
\end{aligned} \tag{5.2}$$

Here $f_n(q^2)$ are dimensionless functions with constant behavior at zero momentum and otherwise remaining arbitrary. The last term always has the corresponding order in powers of D_0^{-1} , so that

$$\begin{aligned}
D_0^{-1} (D_0\Delta^2)^{-k} f_n(q^2) &= O_n(q^2) - D_0^{-1} \sum_{m=0}^{k-1} (D_0\Delta^2)^{-m} O_n^{(m)}(1) \\
&= D_0^{-1} \sum_{m=0}^{\infty} (D_0\Delta^2)^{-m} O_n^{(m)}(1) - D_0^{-1} \sum_{m=0}^{k-1} (D_0\Delta^2)^{-m} O_n^{(m)}(1) \sim D_0^{-1} (D_0\Delta^2)^{-k},
\end{aligned} \tag{5.3}$$

indeed. The same is true for the products of two, three, and more different $O_n(q^2)$ functions (see below).

For the first mixed up term, $D_0F_2D_0O_1(q^2)D_0$, one then gets

$$\begin{aligned}
D_0F_2D_0O_1(q^2)D_0 &= D_0^{2+1}F_2[D_0^{-1}O_1^{(0)}(1) + D_0^{-1}(D_0\Delta^2)^{-1}f_1(q^2)] \\
&= D_0^2F_2O_1^{(0)}(1) + D_0F_2\Delta^{-2}f_1(q^2) = D_0^2F_2O_1^{(0)}(1) + O(D_0),
\end{aligned} \tag{5.4}$$

since for this term we should put $k = 1$ in Eq. (5.2). The combination of the mass squared parameters $F_2\Delta^{-2}$ is the dimensionless one (see relations (4.4)), and the last term is obviously denoted as $O(D_0)$. Thus, one concludes that this mixed up term becomes the exact sum of the two different terms. The first one should be included into the terms of the order D_0^2 shown as the first term in the iteration solution (5.1), while the second term should be combined with the terms which always are of the order D_0 .

For the mixed up term, $D_0 F_3 D_0 F_2 D_0 O_1(q^2) D_0$, we obtain

$$\begin{aligned}
D_0 F_3 D_0 F_2 D_0 O_1(q^2) D_0 &= D_0^{2+2} F_2 F_3 [D_0^{-1} O_1^{(0)}(1) + D_0^{-2} \Delta^{-2} O_1^{(1)}(1) + D_0^{-1} (D_0 \Delta^2)^{-2} f_1(q^2)] \\
&= D_0^3 F_2 F_3 O_1^{(0)}(1) + D_0^2 F_2 F_3 \Delta^{-2} O_1^{(1)}(1) + D_0 F_3 F_2 \Delta^{-4} f_1(q^2) \\
&= D_0^3 F_2 F_3 O_1^{(0)}(1) + D_0^2 F_2 F_3 \Delta^{-2} O_1^{(1)}(1) + O(D_0),
\end{aligned} \tag{5.5}$$

since for this term we should put $k = 2$ in Eq. (5.2). The combination $F_2 F_3 \Delta^{-2}$ has the dimensions of a mass squared (see Eqs. (4.4)), while the combination $F_2 F_3 \Delta^{-4}$ is dimensionless. Again the last term is denoted as $O(D^0)$. So this term also becomes the exact sum of the three terms. The first term from this sum has to be shifted into the second term of Eq. (5.1). The second term from this sum has to be shifted into the first term of Eq. (5.1), and the last one has to be shifted into the last term of the general iteration solution (5.1).

It is instructive to consider in more details the terms which contain two and more $O(q^2)$ functions. The corresponding Taylor expansions for the product of any two and three functions are as follows:

$$\begin{aligned}
O_n(q^2) O_l(q^2) &= D_0^{-2} \sum_{m=0}^{k-2} (D_0 \Delta^2)^{-m} O_{nl}^{(m)}(1) + D_0^{-2} (D_0 \Delta^2)^{-k+1} f_{nl}(q^2), \\
O_n(q^2) O_l(q^2) O_j(q^2) &= D_0^{-3} \sum_{m=0}^{k-3} (D_0 \Delta^2)^{-m} O_{nlj}^{(m)}(1) + D_0^{-3} (D_0 \Delta^2)^{-k+2} f_{nlj}(q^2),
\end{aligned} \tag{5.6}$$

and so on. Here $f_{nl}(q^2)$ and $f_{nlj}(q^2)$ are dimensionless functions with constant behavior at small momentum q^2 and otherwise remaining arbitrary. Similarly to the previous case, the last terms are the terms of the corresponding orders in powers of D_0^{-1} , so that

$$\begin{aligned}
D_0^{-2} (D_0 \Delta^2)^{-k+1} f_{nl}(q^2) &\sim O_{nl}(D_0^{-k-1}) \sim D_0^{-2} (D_0 \Delta^2)^{-k+1}, \\
D_0^{-3} (D_0 \Delta^2)^{-k+2} f_{nlj}(q^2) &\sim O_{nlj}(D_0^{-k-1}) \sim D_0^{-3} (D_0 \Delta^2)^{-k+2},
\end{aligned} \tag{5.7}$$

indeed, and so on. Then, for example, for the mixed up term, $D_0 O_3(q^2) D_0 F_2 D_0 O_1(q^2) D_0$, one gets

$$\begin{aligned}
D_0 O_3(q^2) D_0 F_2 D_0 O_1(q^2) D_0 &= D_0^{2+2} F_2 [D_0^{-2} O_{31}^{(0)}(1) + D_0^{-2} (D_0 \Delta^2)^{-1} f_{31}(q^2)] \\
&= D_0^2 F_2 O_{31}^{(0)}(1) + D_0 F_2 \Delta^{-2} f_{31}(q^2) \\
&= D_0^2 F_2 O_{31}^{(0)}(1) + O(D_0),
\end{aligned} \tag{5.8}$$

since for this term we should put $k = 2$ in the first of Eqs. (5.6). The combination $F_2 \Delta^{-2}$ is dimensionless, and the last term is denoted as $O(D_0)$. Again this mixed up term becomes the exact sum of two terms. The first term should be shifted into the first, while the second one should be shifted into the last term of the general iteration solution (5.1).

Moreover, all other mixed up terms, which are explicitly present and omitted in formal series (5.1), should be treated in the same way. Let us underline that the exact and unique separation between the two kind of terms ($\sim D_0^{2+k}(q)$, $k = 0, 1, 2, 3, \dots$ and $\sim D_0(q)$) is achieved by keeping the necessary number of terms in the corresponding Taylor expansions. This "shifting" method in its general form has been formulated and applied in our previous publications [7, 10]. It is worth emphasizing that by shifting we do not change the functional dependence (and hence the dependence on the mass gap) in the terms $\sim D_0^{2+k}(q)$, only the accompanied q^2 -independent factors will be changed. At the same time, the terms $\sim D_0(q)$ will be changed (many new terms of the same order will appear). Let us also note that all the combinations of different masses squared (like the above-mentioned $F_2 \Delta^{-2}$, $F_2 F_3 \Delta^{-2}$, etc.) can be reduced to the corresponding powers of the mass gap Δ^2 and dimensionless coefficients C_1, C_2, C_3, \dots via the relations (4.4). They are to be multiplied by the different dimensionless constants $O_n^{(m)}(1)$, $O_{nl}^{(m)}(1)$, $O_{nlj}^{(m)}(1)$, etc., which appear through the shifting procedure, and which in general may depend on the same quantities as in Eq. (4.4).

Rearranging all the terms, one gets that the general iteration solution (5.1) for full gluon propagator becomes the exact sum of the two different terms, namely

$$D(q) = D_0^2(q) \Delta^2 \sum_{k=0}^{\infty} [D_0(q) \Delta^2]^k \sum_{m=0}^{\infty} a_{k,m}(\lambda, \nu, \xi, g^2) + O(D_0(q)). \tag{5.9}$$

It is worth emphasizing once more that Eq. (5.9) obtained by the shifting procedure is equivalent to the initial general iteration series (5.1). In other words, the shifting method is not an approximation, but the exact method of the corresponding rearrangement of the terms. It makes it possible to represent initial series (5.1) in the form much more convenient for our purpose (to track down the mass gap), while shifting functional ambiguity of the initial series (5.1) (which is due to the functions $O_n(q^2)$) into the term $O(D_0(q))$ in Eq. (5.9). Let us recall that the terms $O(D_0(q))$ is the sum of the terms which are of the order $D_0(q)$ from the very beginning (the third type of terms in the expansion (5.1)) and the terms of the same order having appeared due to the above-described shifting procedure.

VI. EXACT STRUCTURE OF THE FULL GLUON PROPAGATOR

Restoring the tensor structure, omitting the tedious algebra and again taking into account that $D_0(q) \sim (q^2)^{-1}$, the general iteration solution of the gluon SD equation (5.9) for the full gluon propagator can be algebraically (i.e., exactly) decomposed as the sum of the two principally different terms as follows:

$$\begin{aligned} D_{\mu\nu}(q) &= D_{\mu\nu}^{INP}(q, \Delta^2) + D_{\mu\nu}^{PT}(q) \\ &= iT_{\mu\nu}(q) \frac{\Delta^2}{(q^2)^2} \sum_{k=0}^{\infty} (\Delta^2/q^2)^k \sum_{m=0}^{\infty} \phi_{k,m}(\lambda, \nu, \xi, g^2) + i \left[T_{\mu\nu}(q) \sum_{m=0}^{\infty} a_m(q^2; \xi) + \xi L_{\mu\nu}(q) \right] \frac{1}{q^2}, \end{aligned} \quad (6.1)$$

where the superscript "INP" stands for the intrinsically NP part of the full gluon propagator. Its exact structure inevitably stems from the general iteration solution of the standard gluon SD equation. The important feature of our method is that the skeleton loop integrals have been iterated (skeleton loops expansion), so no any assumptions and approximations have been made. We distinguish between two terms in Eq. (6.1) by the character of the corresponding IR singularities and the explicit presence of the mass gap (see below). It is worth emphasizing that both terms are valid in the whole energy/momentum range, i.e., they are not asymptotics. At the same time, we achieved the exact separation between two terms responsible for the NP (dominating in the IR ($q^2 \rightarrow 0$)) and PT (dominating in the UV ($q^2 \rightarrow \infty$)) dynamics in the true YM vacuum.

The PT part of the full gluon propagator remains undetermined. The exact dependence of the PT gluon form factor $d^{PT}(q^2, \xi) = \sum_{m=0}^{\infty} a_m(q^2, \xi)$ on q^2 cannot be fixed on general ground like it has been done in its INP counterpart (what we only know about $a_m(q^2, \xi)$ -functions is that all of them are regular at small q^2 , and the sum over them produces the PT logarithm improvements at large q^2 due to AF). Evidently, the presence of overlapping and overall UV divergences in the PT part of the full gluon propagator cannot change the structure of its INP part, i.e., its functional dependence on q^2 and hence its dependence on the mass gap. They may only affect the q^2 -independent factors $\phi_{k,m}(\lambda, \nu, \xi, g^2)$, which concrete values, however, are not important. In the PT part the sum over m indicates that all iterations contribute into the PT IR singularity only, which is defined as always being as much singular as the power-type IR singularity of the free gluon propagator $(q^2)^{-1}$. That is why the longitudinal component of the full gluon propagator should be included into its PT part. Anyway, we are not responsible for this part. It is the prize we have paid to fix exactly the functional dependence of the INP part of the full gluon propagator. In Refs. [7, 9, 10] we came to the same structure (6.1) but in a rather different ways.

A. The INP phase in QCD

The exact decomposition of the full gluon propagator into the two principally different terms in Eq. (6.1) is only possible on the basis of the corresponding decomposition of the full gluon form factor $d(q^2, \xi)$ in Eq. (1.1), namely

$$d(q^2, \xi) = d(q^2, \xi) - d^{PT}(q^2, \xi) + d^{PT}(q^2, \xi) = d^{NP}(q^2, \xi) + d^{PT}(q^2, \xi). \quad (6.2)$$

Let us note that in principle the full gluon form factor can be defined as the effective charge of QCD, i.e., $d(q^2) = \alpha_s(q^2)$, where the dependence on ξ is omitted, for simplicity. This algebraic decomposition makes it possible to define correctly the NP phase in comparison with the PT one. The full gluon form factor $d(q^2, \xi)$ being the NP effective charge, nevertheless, is "contaminated" by the PT contributions, while $d^{NP}(q^2, \xi)$ is the truly NP one, since it is free of them, by construction [7, 10]. Substituting the exact decomposition (6.2) into the full gluon propagator (1.1), one obtains

$$D_{\mu\nu}(q) = i \{ T_{\mu\nu}(q) d(q^2, \xi) + \xi L_{\mu\nu}(q) \} \frac{1}{q^2} = D_{\mu\nu}^{INP}(q) + D_{\mu\nu}^{PT}(q), \quad (6.3)$$

where

$$D_{\mu\nu}^{INP}(q) = iT_{\mu\nu}(q)d^{NP}(q^2, \xi) \frac{1}{q^2} = iT_{\mu\nu}(q)d^{INP}(q^2, \xi), \quad (6.4)$$

and

$$D_{\mu\nu}^{PT}(q) = i \left[T_{\mu\nu}(q)d^{PT}(q^2, \xi) + \xi L_{\mu\nu}(q) \right] \frac{1}{q^2}, \quad (6.5)$$

in complete agreement with Eq. (6.1), indeed.

As it follows from Eq. (6.1), the INP part of the full gluon propagator in Eq. (6.4) is nothing else, but the corresponding Laurent expansion in integer powers of q^2 accompanied by the corresponding powers of the mass gap squared and multiplied by the sum over the q^2 -independent factors, namely

$$D_{\mu\nu}^{INP}(q, \Delta^2) = iT_{\mu\nu}(q) \sum_{k=0}^{\infty} (q^2)^{-2-k} (\Delta^2)^{1+k} \phi_k(\lambda, \nu, \xi, g^2), \quad (6.6)$$

where

$$\phi_k(\lambda, \nu, \xi, g^2) = \sum_{m=0}^{\infty} \phi_{k,m}(\lambda, \nu, \xi, g^2) \quad (6.7)$$

are the so-called residues at poles. The sum over m indicates that an infinite number of iterations (all iterations) of the corresponding skeleton loop integrals invokes each severe (for definition see below) IR singularity labelled by k .

Using the exact decomposition of the full gluon propagator described above, we can define in general terms the INP phase in QCD as follows:

(i). The INP phase is characterized by the presence of the power-type severe (or equivalently NP) IR singularities $(q^2)^{-2-k}$, $k = 0, 1, 2, 3, \dots$. So these IR singularities are defined as more singular than the power-type IR singularity of the free gluon propagator $(q^2)^{-1}$. The Laurent expansion (6.6) necessarily starts from the simplest NP IR singularity $(q^2)^{-2}$ possible in four-dimensional QCD, indeed [7].

(ii). It depends only on the transverse ("physical") degrees of freedom of gauge bosons.

(iii). It is gauge-invariant. Though the coefficients $\phi_{k,m}(\lambda, \nu, \xi, g^2)$ of the Laurent expansion (6.6) may explicitly depend on the gauge-fixing parameter ξ , the structure of this expansion itself does not depend on it (see discussion below as well).

(iv). The INP part of the full gluon propagator vanishes as the mass gap goes to zero, while the PT part survives.

Within our approach the mass gap determines the power-type deviation of the full gluon propagator from the free one in the IR limit ($q^2 \rightarrow 0$), while Λ_{QCD} determines the logarithmic deviation of the full gluon propagator from the free one in the UV limit ($q^2 \rightarrow \infty$). So, we distinguish between the two different phases in QCD not by the strength of the coupling constant squared (which is arbitrary in our approach), but rather by the explicit presence of the mass gap, in which case the coupling constant plays no any role. The INP phase disappears when it goes to zero even if the NP IR singularities are not explicitly present. So the subtraction (6.2) can be equivalently written down as follows:

$$d^{NP}(q^2, \Delta^2) = d(q^2, \Delta^2) - d(q^2, \Delta^2 = 0) = d(q^2, \Delta^2) - d^{PT}(q^2), \quad (6.8)$$

i.e., it goes to zero as $\Delta^2 \rightarrow 0$, indeed. This once more emphasizes the important role of the mass gap in the definition of the truly NP phase as a particular case of the INP one when the NP IR singularities are not explicitly present as stated above. In other words, the existence of the truly NP phase in any approach based on the gluon propagator assumes the regular dependence on the mass scale parameter, which is chosen to play the role of the mass gap. Otherwise, in the absence of the mass gap and in order to recover the truly NP phase the UV asymptotic of the full gluon propagator should be subtracted in agreement with the relation (6.8). It is worth emphasizing once more that the existence of the mass gap and the presence of the NP IR singularities in the Laurent expansion (6.6) (and hence in the full gluon propagator (6.1)) is absolutely general phenomenon. It does not depend on the concrete values of the parameters: λ , ν , ξ , g^2 . It is only due to the NL interaction of massless gluons.

VII. DISCUSSION

The unavoidable presence of the first term in Eq. (6.1) makes the principal difference between non-abelian QCD and abelian QED, where such kind of term in the full photon propagator is certainly absent (in the former theory there is direct coupling between massless gluons, while in the latter one there is no direct coupling between massless photons). Precisely this term may violate the cluster properties of the Wightman functions [12], and thus validates the Strocchi theorem [13], which allows for such IR singular behavior of the full gluon propagator.

The INP part of the full gluon propagator in the form of the corresponding Laurent expansion describes the so-called zero momentum modes enhancement (ZMME or simply ZME which means zero momentum enhancement) effect in the true QCD vacuum due to the NL dynamics of massless gluon modes there. As underlined above, we do not specify explicitly the value of the gauge-fixing parameter ξ . So the ZMME effect takes place at its any value. In this sense this effect is gauge-invariant. This is very similar to AF. It is well known that the exponent which determines the logarithmic deviation of the full gluon propagator from the free one in the UV region ($q^2 \gg \Lambda_{QCD}^2$) explicitly depends on the gauge-fixing parameter. At the same time, AF itself does not depend on it, i.e., it takes place at any ξ .

Also, due to the arbitrariness of the above-mentioned residues $\phi_k(\lambda, \nu, \xi, g^2)$ there is no way to sum up these Laurent series into the function with regular behavior at small q^2 . However, the smooth in the IR gluon propagator is also possible depending on different truncations/approximations used, since the gluon SD equation is highly nonlinear one. The number of solutions for such kind of systems is not fixed *a priori*. The singular and smooth in the IR solutions for the gluon propagator are independent from each other, and thus should be considered on equal footing. Anyway, in order to find the smooth gluon propagator completely different (from direct iteration) method of the solution of the gluon SD equation should be used [8, 14] (and references therein as well), since we have explicitly shown here in a gauge-invariant way and making no any approximations/truncations that the general iteration solution is inevitably severely singular at small gluon momentum.

The QCD Lagrangian does not contain a mass gap. However, we discovered that the mass scale parameter responsible for the NP dynamics in the IR region exists in the true QCD ground state. At the level of the gluon SD equation it is hidden in the skeleton loop contributions into the gluon self-energy. It explicitly shows up (and hence the corresponding severe IR singularities) when the gluon momentum goes to zero. At the fundamental quark-gluon (i.e., Lagrangian) level the dynamical source of the mass gap is the triple and quartic gluon vertices, i.e., the NL dynamics of QCD. The former vanishes when all the gluon momenta involved go to zero ($T_3(0,0) = 0$), while the latter survives in the same limit ($T_4(0,0,0) \neq 0$). Because of these features it would be tempting to think that the quartic potential $(A \wedge A)^2$ in the action plays much more important role than its triple counterpart in the arising of severe IR singularities in quantum YM theory. However, since we are dealing with the skeleton loops expansion, we are unable (at least at this stage) to exactly establish that the mass gap could arise from the quartic potential rather than from its triple counterpart [6] (see also Ref. [15]). So this dilemma remains an open but an interesting problem to be solved.

In this connection, it is necessary to discuss an interesting feature of the INP part of the full gluon propagator. Its functional dependence on q^2 and hence on the mass gap (i.e., the Laurent structure of the expansion in Eq. (6.6)) is exactly established up to the corresponding residues $\phi_k(\lambda, \nu, \xi, g^2)$. In these residues the contributions from both the triple and quartic gluon vertices have been taken into account. However, the Laurent structure of the INP part does not depend on whether we will take into account only three- or only four-gluon vertices or both of them in the above-mentioned skeleton loop integrals. We cannot omit both, so one of the NL interactions should be always present. On the other hand, the residues themselves will depend, of course, on the character of the NL interaction taken into account. However, obviously, their concrete values are not important, only the general dependence of the residues $\phi_k(\lambda, \nu, \xi, g^2)$ on their arguments is all that matters (in the subsequent paper we will show this explicitly). At the same time, the dependence of the functions $a_m(q^2; \xi)$ and hence of the PT form factor $d^{PT}(q^2; \xi)$ in Eq. (6.1) on q^2 will heavily depend on the character of the NL interactions taken into account in the corresponding skeleton loop integrals but this is not important for us as underlined above.

Thus the true QCD vacuum is really beset with severe IR singularities. They should be summarized (accumulated) into the full gluon propagator and effectively correctly described by its structure in the deep IR domain, exactly represented by its INP part. The second step is to assign a mathematical meaning to the integrals, where such kind of severe IR singularities will explicitly appear, i.e., to define them correctly in the IR region [7, 9, 10]. This can be done by the use of the dimensional regularization method [16] correctly implemented into the distribution theory [17] (see subsequent paper). Just this IR violent behavior makes QCD as a whole an IR unstable theory, and therefore it has no IR stable fixed point, indeed [1, 18]. This means that QCD itself might be a confining theory without involving some extra degrees of freedom [1, 18, 19] (and references therein).

There is no doubt that our solution for the full gluon propagator, obtained at the expense of remaining unknown its PT part, nevertheless, satisfies the gluon SD equation (3.7), since it has been obtained by the direct iteration solution of this equation. To show this explicitly by substituting it back into the initial gluon SD equation is not a simple

task, however, and this is to be done elsewhere. The problem is that the decomposition of the full gluon propagator into the INP and PT parts by regrouping the so-called mixed up terms in section V was a well defined procedure (there was an exact criterion introduced in section VI how to distinguish between these two terms in a single D). However, to do the same at the level of the gluon SD equation itself, which is nonlinear in D , is not so obvious. Also, the corresponding severe IR singularities should be put under control at first within the distribution theory (all this will be explicitly demonstrated in a forthcoming paper).

VIII. CONCLUSIONS

A few years ago Jaffe and Witten (JW) have formulated the following theorem [6]:

Yang-Mills Existence And Mass Gap: Prove that for any compact simple gauge group G , quantum Yang-Mills theory on \mathbf{R}^4 exists and has a mass gap $\Delta > 0$.

Of course, at present to prove the existence of the YM theory with compact simple gauge group G is a formidable task yet. It is rather mathematical than physical problem. However, one of the main results of our investigation here can be formulated similar to the above-mentioned JW theorem as follows:

Mass Gap Existence: If quantum Yang-Mills theory with compact simple gauge group $G = SU(3)$ exists on \mathbf{R}^4 , then it exhibits a mass gap.

Our mass gap Δ^2 remains neither IR nor UV renormalized yet, since at this stage it has been only regularized, i.e., $\Delta^2 \equiv \Delta^2(\lambda, \nu, \xi, g^2)$. However, there is no doubt that it will survive both renormalization programs (see subsequent paper). So denoting its IR and UV renormalized version in advance as Λ_{NP} , then a symbolic relation between it, the JW mass gap ($\Delta \equiv \Delta_{JW}$) and $\Lambda_{QCD} \equiv \Lambda_{PT}$ could be written as

$$\Lambda_{NP} \xleftarrow[0 \leftarrow M_{IR}]{\infty \leftarrow \alpha_s} \Delta_{JW} \xrightarrow[M_{UV} \rightarrow \infty]{\alpha_s \rightarrow 0} \Lambda_{PT}. \quad (8.1)$$

Here α_s is obviously the fine structure coupling constant of strong interactions, while M_{UV} and M_{IR} are the UV and IR cut-offs, respectively. The right-hand-side limit is well known as the weak coupling regime, while the left-hand-side can be regarded as the strong coupling regime. We know how to take the former [1, 2, 18], and we hope that we have explained here how to deal with the latter one, not solving the gluon SD equation directly, which is a formidable task, anyway. However, there is no doubt that the final goal of this limit, namely, the mass gap Λ_{NP} exists, and should be renormalization group invariant in the same way as Λ_{QCD} . It is solely responsible for the large scale structure of the true QCD ground state, while Λ_{PT} is responsible for the nontrivial PT dynamics there.

It is important to emphasize once more that the mass gap has not been introduced by hand. We have explicitly demonstrated that it is hidden in the skeleton loop integrals contributing into the gluon self-energy due to the NL interaction of massless gluon modes (Eqs. (2.4)-(2.6)). The mass gap shows explicitly up when the gluon momentum goes to zero. An appropriate regularization scheme has been applied to make the existence of the mass gap perfectly clear. Moreover, it survives an infinite series summation of the corresponding skeleton loop contributions (skeleton loop expansion). In other words, an infinite number of iterations of the relevant skeleton loops has to be made in order to exhibit a mass gap. No any truncations/approximations have been made as well as no special gauge choice, i.e., the result of the summation is gauge-invariant. In the presence of the mass gap the QCD coupling constant plays no any role. All its orders contribute into the mass gap (skeleton loop expansion). This explains why the interaction in our picture can be considered as a strong one. It is worth emphasizing that our mass gap and the JW mass gap cannot be interpreted as the gluon mass, i.e., they always remain massless. These features point out on a possible similarity between our mass gap and the JW one. Moreover, in a next paper we will explicitly show (by investigating the IR renormalization properties of the mass gap) that the interaction in our picture is not only strong but short-ranged as well. This will allow us to analytically formulate the gluon confinement criterion in a gauge-invariant way.

Our second important result, a byproduct of the proof of the existence of a mass gap, is that the general iteration solution of the gluon SD equation is unavoidably severely IR singular at small gluon momentum. The exactly established Laurent structure of the INP part of the full gluon propagator (6.1) clearly shows that an infinite number of iterations of the relevant skeleton loops (skeleton loops expansion) invokes each NP IR singularity. Again no special gauge choice and no any truncations/approximations have been made as well in such obtained general iteration solution of the gluon SD equation for the full gluon propagator. So, our gluon propagator (more precisely its INP part) takes into account the importance of the quantum excitations of severely singular IR degrees of freedom in the true QCD vacuum. They lead to the formation of the purely transverse quantum virtual field configurations with the enhanced

low-frequency components/large scale amplitudes due to the NL dynamics of the massless gluon modes. We will call them the purely transverse singular gluon fields, for simplicity. In the presence of such severe IR singularities the IR multiplicative renormalization (IRMR) program is needed to perform in order to remove them in a self-consistent way from the theory (see subsequent paper). In another forthcoming paper we will show that the quark and ghost degrees of freedom play no any significant role in the dynamical generation of a mass gap. The NL interaction of massless gluon modes is only important within our approach.

In summary, the existence of a mass gap in quantum YM theory has been proved in a gauge-invariant way. We have explicitly shown (again in a gauge-invariant way) that the direct iteration solution of the gluon SD equation is inevitably severely singular at small gluon momentum. Our general conclusion is that the behavior of QCD at large distances is governed by the mass gap, and therefore it should play a crucial role in the NL realization of the quantum-dynamical mechanism of color confinement.

Acknowledgments

Support in part by HAS-JINR and Hungarian OTKA-T043455 grants is to be acknowledged. I would like to thank P. Forgacs for bringing my attention to the JW presentation of the Millennium Prize Problem in Ref. [6]. It is a great pleasure to thank A. Jaffe for bringing my attention to the revised version of the above-mentioned presentation. The author is also grateful to D.V. Shirkov, A.N. Tavkhelidze, G.V. Efimov, D. Kazakov, J. Nyiri, Gy. Pocsik, P. Levai, T. Biro, P. Vecsernyes and especially V.A. Rubakov for useful discussions, remarks, suggestions and help at the preliminary stages of this investigation.

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