

# Limiting energy dependence of spin parameters in small-angle elastic pp-scattering

S.M. Troshin, N.E. Tyurin

*Institute for High Energy Physics,*

*Protvino, Moscow Region, 142281 Russia*

## Abstract

We consider limiting behavior with energy of spin parameters at small values of  $t$  close to zero which corresponds to the saturation of the Froissart-Martin bound for the total cross sections.

## Introduction

The most global characteristic of the hadronic collision is the total cross-section and the nature of the total-cross section rising energy dependence is an open problem in hadron physics at large distances. There is a widespread conjecture that the Froissart-Martin bound should be saturated at high energies and there are several model parameterizations for the total cross-sections using  $\ln^2 s$  dependence for  $\sigma_{tot}(s)$  even at available energies. Saturation of the Froissart-Martin bound results from the corresponding saturation of unitarity for the partial amplitudes  $|f_l(s)| = 1$  at  $l \leq \sqrt{s} \ln s$  and it leads to the prediction [1] of limiting behaviour of other observables, e.g.  $\sigma_{el}(s) \sim \ln^2 s$ , slope of diffraction cone  $B(s) \sim \ln^2 s$ , ratio of real to the imaginary part of the forward scattering amplitude  $\rho(s) \sim \ln^{-1}(s)$ . The above limiting energy dependencies are related to observables averaged over spin degrees of freedom. It is evident that the mentioned regularities should be related somehow to the nonperturbative QCD.

It is important nowadays to have corresponding limiting energy dependencies for spin parameters; at RHIC the experiments are performed with polarized proton beams of high energies in the region  $\sqrt{s} = 200 - 500$  GeV and those dependencies can be useful for estimation of the magnitude of spin parameters and the likelihood of the assumptions on the role of the helicity amplitudes and possibility to neglect some of them.

Discussion of a role and magnitude of helicity-flip amplitudes in small-angle elastic scattering has a long history and is an important issue in the studies of the spin properties of high-energy diffraction. Recent results of the  $pp2pp$  experiment at RHIC on  $A_N$  measurements at  $|t|$  in the region of 0.01 to 0.03  $(GeV/c)^2$  have indicated that hadronic spin-flip amplitude is non-zero and the dominating high energy scattering mechanism, Pomeron, can flip the helicity [2]. Mechanism of Pomeron helicity flip based on the two-pion contribution to the scattering amplitude has been proposed in [3] and in [4] it was shown that unitarity generates phase difference between helicity flip and nonflip Pomeron amplitudes, so there is no need to introduce C-odd exchange called Odderon to generate nonzero asymmetry  $A_N$ . It was predicted [4] that  $A_N$  at small  $t$  values decreases like  $1/\ln s$ . The bounds for the helicity amplitudes at  $t = 0$  have been discussed in [5] and recent discussion of these bounds and models for spin dependence in high energy proton scattering has been given in [6].

This brief note is devoted to the discussion of the same problem: it considers limiting energy dependencies of spin parameters which result from the saturation of unitarity for the helicity amplitudes.

We need to consider all helicity amplitudes and do not neglect some of them from the very beginning. For instance the double helicity-flip amplitudes can also contribute into  $A_N$  and their behavior at high energies is also important for the spin correlation parameters and the total cross-section differences in experiments with two polarized beams available at RHIC nowadays. It could also affect the small  $t$  dependence of the unpolarized differential cross-sections at the LHC.

## 1 Unitarity bounds and limiting energy dependence

Unitarity bounds for the helicity amplitudes have been obtained in [7]. One should recall that saturation of unitarity for the helicity amplitudes leads to a peripheral dependence of the amplitudes  $f_i(s, b)$  ( $i = 2, 4, 5$ ) on the impact parameter  $b$  at high energy, i.e.

$$|f_i(s, b = 0)| \rightarrow 0$$

at  $s \rightarrow \infty$ . At small impact parameters only helicity non-flip amplitudes survive. This is a consequence of the explicit unitarity representation for the helicity amplitudes in the rational form. This fact allows one to get better bounds for the helicity-flip amplitudes at  $t = 0$  compared to those obtained in [5, 6].

In what follows we will consider limit  $s \rightarrow \infty$  and  $-t$  is small. Assuming saturation of the unitarity bounds obtained in [7], we will have the following limiting behaviour of the helicity amplitudes

$$F_1(s, t), F_3(s, t), \sim s \ln^2 s, \quad F_5(s, t) \sim \sqrt{-t} s \ln^2 s,$$

$$F_2(s, t) \sim s \ln s, \quad F_4(s, t) \sim -ts \ln^3 s. \quad (1)$$

The expressions for spin observables in terms of helicity amplitudes in  $pp$  elastic scattering have the following form. The analyzing power  $A_N$  looks like

$$\sigma A_N = -\text{Im}[(F_1 + F_2 + F_3 - F_4)F_5^*], \quad (2)$$

where

$$\sigma = \frac{1}{2}(|F_1|^2 + |F_2|^2 + |F_3|^2 + |F_4|^2 + 4|F_5|^2)$$

stands for the differential cross-section (up to the normalization factor). The expressions for the initial state spin correlation parameters are given in terms of the  $pp$ -scattering helicity amplitudes as follows:

$$\begin{aligned} \sigma A_{LL} &= \frac{1}{2}(-|F_1|^2 - |F_2|^2 + |F_3|^2 + |F_4|^2), \\ \sigma A_{NN} &= \text{Re}(F_1 F_2^* - F_3 F_4^*) + 2|F_5|^2, \\ \sigma A_{SS} &= \text{Re}(F_1 F_2^* + F_3 F_4^*), \\ \sigma A_{SL} &= \text{Re}[(-F_1 + F_3 + F_2 + F_4)F_5^*]. \end{aligned} \quad (3)$$

In the experiments with polarized beams, besides the spin correlation parameters which are differential characteristics of the scattering processes there are possibilities to study global characteristics such as differences of the total cross sections for pure spin states of the initial particles. These quantities  $\Delta\sigma_L(s)$  and  $\Delta\sigma_T(s)$  correspond to definite orientations of spins of the initial particles in the longitudinal and transverse directions respectively. They are defined as follows (where arrows indicate the spin directions for particles  $a$  and  $b$  in the process  $a + b \rightarrow X$ ):

$$\begin{aligned} \Delta\sigma_L(s) &= \sigma_{\vec{\uparrow}\vec{\uparrow}}^{\text{tot}}(s) - \sigma_{\vec{\uparrow}\vec{\downarrow}}^{\text{tot}}(s), \\ \Delta\sigma_T(s) &= \sigma_{\uparrow\downarrow}^{\text{tot}}(s) - \sigma_{\uparrow\uparrow}^{\text{tot}}(s). \end{aligned} \quad (4)$$

Here the first or top arrow refers to particle  $a$ . For the case of  $pp$  scattering according to optical theorem the quantities  $\Delta\sigma_L(s)$  and  $\Delta\sigma_T(s)$  are determined by the values of the helicity amplitudes at  $t = 0$ :  $\text{Im}[F_1(s, t = 0) - F_3(s, t = 0)]$  and  $-\text{Im}F_2(s, t = 0)$  respectively.

Using limiting dependencies Eqs. (1) and above formulas for observables in terms of helicity amplitudes we arrive to the following energy dependencies of spin observables in the limit of high energies and small and fixed values of  $t$ :

$$A_N(s, t), A_{NN}(s, t), A_{SS}(s, t), A_{SL}(s, t) \sim 1/\ln s, \quad (5)$$

while

$$A_{LL}(s, t) \rightarrow 1. \quad (6)$$

It should be noted that the limiting behavior of  $A_N$  coincides with the model result of [4] and decreases with energy as  $1/\ln s$  while the ratio of the single flip amplitude  $F_5$  to non-spin flip amplitudes  $F_1$  and  $F_3$  is constant in the high energy limit.

In the high energy limit the following relations should take place in the above kinematical limit:

$$A_{NN}(s, t) \simeq -A_{SS}(s, t) \quad (7)$$

and

$$A_{LL}(s, t) \gg A_{NN}(s, t), A_{SS}(s, t), A_{SL}. \quad (8)$$

To obtain conclusive result on the difference of the total cross-sections with longitudinal spin orientations the additional model assumptions are needed, since it is the difference of the two amplitudes with the same energy dependence. Otherwise, the difference of total cross-sections with transverse spin orientations has an unambiguous increasing limiting energy dependence

$$\Delta\sigma_T(s) \sim \ln s. \quad (9)$$

The magnitude of the helicity amplitude  $F_2$  at small  $-t$  could be not small, it increases with energy and it possibly can be measured directly at RHIC through the measurements of  $\Delta\sigma_T$ . This measurement would provide an important contribution to the studies of the spin properties of diffraction.

The knowledge of limiting dependence  $\Delta\sigma_T(s)$ , Eq. (9), allows one to obtain transverse spin structure function  $h_1(x)$  in the limit of small Bjorken  $x$  since in the low- $x$  region structure function  $h_1$  is related to discontinuity of the helicity amplitude  $F_2$  of the quark-hadron forward scattering, i.e.

$$xh_1(x) \sim \ln \frac{1}{x}. \quad (10)$$

This is an important estimate which show that the structure function  $h_1$  can be large at small  $x$  and it enhances the case of the transversity experimental studies in Drell-Yan processes in polarized proton-proton scattering.

Finally, we would like to note that at the LHC energies one could expect noticeable effects in the small  $t$  differential cross-section due to the contribution of the  $F_4$  amplitude which is proportional to  $s \ln^3 s$ . Due to very high energy of the LHC this effect could take place in the region of small  $t$  despite the fact that  $F_4$  goes to zero as  $t$  in the limit  $-t \rightarrow 0$ . It might happen that contributions of the helicity flip amplitudes  $F_2$ ,  $F_4$  and  $F_5$  would affect the results on the total cross-sections extraction performed by the extrapolation to the optical point.

The limiting energy dependencies of spin parameters have been derived as a result of the bounds saturation which follows from explicit solution of unitarity for all helicity flip amplitudes and have the similar footing as the saturation of the Froissart-Martin bound for the total cross-section averaged over spins of the initial particles. Thus, to be selfconsistent one should assume saturation of the above unitary bounds for spin parameters too if the Froissart-Martin bound is supposed to be saturated at high energies since the underlying mechanism in both cases is the same, i.e. saturation of unitarity bounds for partial amplitudes (helicity flip and non flip ones). The respective dependencies are given by Eqs. (5,7-10) and they can be relevant for the experimental studies of elastic scattering at RHIC and LHC.

## References

- [1] G. Auberson, T. Kinoshita, A. Martin, Phys. Rev. D 3, 3185 (1971).
- [2] S. Bültman et al., <http://www.rhic.bnl.gov/pp2pp/documents/PhysLettBfinal.pdf>.
- [3] J. Pumplin, G.L. Kane, Phys. Rev. D 11, 1183 (1975).
- [4] S.M. Troshin, N.E. Tyurin, JETP Lett. 23, 660 (1976).
- [5] G. Mahoux, A. Martin, Phys. Rev. 174, 2140 (1968).
- [6] N.H. Buttimore, B.Z. Kopeliovich, E. Leader, J. Soffer, T.L. Trueman, Phys. Rev. D 59, 114010 (1999).
- [7] S.M. Troshin, N.E. Tyurin, Phys. Lett. B 459, 641 (1999), *ibid.* B 580, 54 (2004).