Spontaneous generation of the Nambu –Jona-Lazinio interaction in quantum chromodynamics with two light quarks

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In QCD with two light quarks with application of Bogolubov quasi-averages approach a possibility of spontaneous generation of an effective interaction, leading to the Nambu – Jona-Lazinio model, is studied. Compensation equations for form-factor of the interaction is shown to have the non-trivial solution leading to theory with two parameters: average low-energy value of α_s and dimensional parameter f_π. All other parameters: the current and the constituent quark masses, the quark condensate, mass of π meson, mass of σ meson and its width are expressed in terms of the two initial parameters in satisfactory correspondence to experimental phenomenology. The results being obtained allow to state an applicability of the approach in the low-energy hadron physics and promising possibilities of its applications to other problems.

1 Introduction

In work [1] based on quasi-averages approach by N.N. Bogolubov [2, 3] a method is proposed aimed to obtain an effective interaction in a renormalizable quantum field theory. In particular such an effective interaction is necessary for construction of the well-known Nambu – Jona-Lazinio model [4], which describes the low-energy physics of strong interactions. In view of this it is of interest to apply the developed method [1] to a study of possibility of generation of effective four-fermion interaction, which is intrinsic to the Nambu – Jona-Lazinio model. Following the results of work [1] one is to expect the essential contraction of number of initial parameters of the model.

An application of the method [1] to a rather extensively studied low-energy region of hadron physics may show to what extent the use of the method is justified. In the present work the first non-perturbative approximation of the method will be developed in application to the problem of a spontaneous generation of Nambu – Jona-Lazinio Lagrangian in the genuine strong interaction theory, namely in QCD with doublet of light quarks.

2 Compensation equation for effective form-factor

Now we start with QCD Lagrangian with two light quarks (u and d) with number of colours $N = 3$

$$L = \sum_{k=1}^{2}\Bigl(\frac{\imath}{2}\Bigl(\bar\psi_k\gamma_\mu\partial_\mu\psi_k - \partial_\mu\bar\psi_k\gamma_\mu\psi_k\Bigr) - m_0\bar\psi_k\psi_k + g\bar\psi_k\gamma_\mu t^a A^a_\mu\psi_k\Bigr) - \frac{1}{4}\Bigl(F^a_{\mu\nu}F^a_{\mu\nu}\Bigr); \quad (1)$$

where we use the standard QCD notations.

In accordance to the Bogolubov approach, application of which to such problems being described in details in work [1], we look for a non-trivial solution of a compensation equation, which is formulated on the basis of the Bogolubov procedure "add – subtract". Namely let us write down the initial expression (1) in the following form

$$\begin{aligned}
&L = L_0 + L_{int}\,;\\
&L_0 = \frac{\imath}{2}\Bigl(\bar\psi\gamma_\mu\partial_\mu\psi - \partial_\mu\bar\psi\gamma_\mu\psi\Bigr) - \frac{1}{4}\,F^a_{0\,\mu\nu}F^a_{0\,\mu\nu} - m_0\bar\psi\,\psi + \frac{G_1}{2}\cdot\Bigl(\bar\psi\tau^b\gamma_5\psi\,\bar\psi\tau^b\gamma_5\psi -\\
&-\bar\psi\,\psi\,\bar\psi\,\psi\Bigr) + \frac{G_2}{2}\cdot\Bigl(\bar\psi\tau^b\gamma_\mu\psi\,\bar\psi\tau^b\gamma_\mu\psi + \bar\psi\tau^b\gamma_5\gamma_\mu\psi\bar\psi\tau^b\gamma_5\gamma_\mu\psi\Bigr)\,; \qquad (2)\\
&L_{int} = g_s\,\bar\psi\gamma_\mu t^a A^a_\mu\psi - \frac{1}{4}\Bigl(F^a_{\mu\nu}F^a_{\mu\nu} - F^a_{0\,\mu\nu}F^a_{0\,\mu\nu}\Bigr) - \frac{G_1}{2}\cdot\Bigl(\bar\psi\tau^b\gamma_5\psi\,\bar\psi\tau^b\gamma_5\psi - \bar\psi\,\psi\,\bar\psi\,\psi\Bigr) -\\
&-\frac{G_2}{2}\cdot\Bigl(\bar\psi\tau^b\gamma_\mu\psi\,\bar\psi\tau^b\gamma_\mu\psi + \bar\psi\tau^b\gamma_5\gamma_\mu\psi\bar\psi\tau^b\gamma_5\gamma_\mu\psi\Bigr)\,. \qquad (3)
\end{aligned}$$

Here ψ is the isotopic doublet, colour summation is performed inside of each fermion bilinear combination, $F_{0\,\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and notation $G_1\cdot\bar\psi\psi\bar\psi\psi$ corresponds to non-local vertex in the momentum space

$$\imath\,(2\pi)^4\,G_1\,F_1(p1,p2,p3,p4)\,\delta(p1+p2+p3+p4)\,; \quad (4)$$

where $F_1(p1,p2,p3,p4)$ is a form-factor and $p1,\ p2,\ p3,\ p4$ are incoming momenta. In the same way we define vertices, containing Dirac and isotopic matrices. We comment the composition of the vector sector, being proportional to G_2, in what follows.

Let us consider expression (2) as the new **free** Lagrangian L_0, whereas expression (3) as the new **interaction** Lagrangian L_{int}. Then compensation conditions (see again [1]) will consist in demand of full connected four-fermion vertices, following from Lagrangian L_0, to be zero. This demand gives a set of non-linear equations for form-factors F_i.

These equation according to terminology of works [2, 3] are called *compensation equations.* In a study of these equations it is always evident the existence of a perturbative trivial solution (in our case $G_i = 0$), but, in general, a non-perturbative non-trivial solution may also exist. Just the quest of a non-trivial solution inspires the main interest in such problems. One can not succeed in finding an exact non-trivial solution in a realistic theory, therefore the goal of a study is a quest of an adequate approach, the first non-perturbative approximation of which describes the main features of the problem. Improvement of a precision of results is to be achieved by corrections to the initial first approximation.

Thus our task is to formulate the first approximation. Here the experience acquired in the course of performing of work [1] could be helpful. Now in view of obtaining the first approximation we assume:

1) In compensation equation we restrict ourselves by terms with loop numbers 0, 1, 2.

2) In expressions thus obtained we perform a procedure of linearizing, which leads to linear integral equations. It means that in loop terms only one vertex contains the form-factor, being defined above, while other vertices are considered to be point-like. In diagram form equation for form-factor F_1 is presented in Fig 1.

3) While evaluating diagrams with point-like vertices diverging integrals appear. Bearing in mind that as a result of the study we obtain form-factors decreasing at momenta infinity, we introduce a cut-off Λ in the diverging integrals. It will be shown that results do not depend on the value of this cut-off.

4) We can obtain analytic expressions for massless quarks only. We take masses into account by introducing the cut-off in the lower limit of integration by momentum squared q^2 at a value, which equals to a value of the corresponding propagator denominator at $q = 0$. Namely with the denominator $(q^2 + m^2)$ the cut-off parameter equals m^2. In doing this we keep at nominators only the leading terms in m expansions because taking into account of the next terms evidently means supererogation of accuracy.

5) We shall take into account at most the first two terms of the $1/N$ expansion.

Let us formulate compensation equations in this approximation. For *free* Lagrangian L_0 full connected four-fermion vertices are to vanish. One can succeed in obtaining analytic solutions for the following set of momentum variables (see Fig. 1): left-hand legs have momenta p and $-p$, and right-hand legs have zero momenta. In particular this kinematics suits for description of zero-mass bound states. Under some assumptions solutions obtained under these conditions may be generalized to momentum set $(p, -p, q, -q)$. The construction of expressions with an arbitrary set of momenta is the problem for the subsequent approximations.

Now following the rules being stated above we obtain the following equation for form-factor $F_1(p)$ in scalar colour singlet channel

$$G_1 F_1(p^2) = \frac{G_1^2 N \Lambda^2}{2\pi^2}\Bigl(1 + \frac{1}{4N} - \frac{G_1 N}{2\pi^4}\Bigl(1 + \frac{1}{2N}\Bigr)\int \frac{F_1(q^2)\,dq}{q^2}\Bigr) +$$
$$+ \frac{3G_1\,G_2}{8\pi^2}\Bigl(2\,\Lambda^2 + p^2 \log\frac{p^2}{\Lambda^2} - \frac{3}{2}\,p^2 - \frac{\mu^2}{2\,p^2}\Bigr) - \frac{(G_1^2 + 6G_1G_2)N}{32\,\pi^6} \times$$

$$\times \int \Big(2\Lambda^2 + (p-q)^2 \log \frac{(p-q)^2}{\Lambda^2} - \frac{3}{2}(p-q)^2 - \frac{\mu^2}{2(p-q)^2}\Big) \frac{G_1 F_1(q^2) dq}{q^2}\,; \qquad (5)$$

Here integration is performed in the four-dimensional Euclidean momentum space, $\mu = m_0^2$. One-loop terms contains terms proportional to N and 1 while two-loop terms correspond to N^2 and N. The leading terms are the same for scalar and pseudo-scalar cases. We begins the study with the scalar channel, because it defines chiral symmetry breaking effect. Equation (5) evidently has trivial solution $G_1 = 0$. Bearing in mind our goal to look for non-trivial solutions we divide the equation by G_1. As a result of four-dimensional angle integration (see Appendix) we have

$$\begin{aligned}
&F_1(x) = A + \frac{3\,G_2}{8\pi^2}\Big(2\Lambda^2 + x \log \frac{x}{\Lambda^2} - \frac{3}{2}\,x - \frac{\mu^2}{2x}\Big) - \frac{(G_1^2 + 6G_1G_2)N}{32\,\pi^4} \times \\
&\times \Big(\frac{1}{6\,x} \int_\mu^x (y^2 - 3\mu^2) F_1(y)\, dy \,+\, \frac{3}{2} \int_\mu^x y F_1(y)\, dy + \log\, x \int_\mu^x y F_1(y)\, dy + \\
&+ x\, \log\, x \int_\mu^x F_1(y)\, dy + \int_x^\infty y \log\, y\, F_1(y)\, dy + \, x \int_x^\infty \Big(\log\, y\, + \frac{3}{2}\Big) F_1(y)\, dy \,+ \\
&+ \frac{x^2 - 3\mu^2}{6} \int_x^\infty \frac{F_1(y)}{y}\, dy + \Big(2\Lambda^2 - \frac{3}{2}\, x\Big) \int_\mu^\infty F_1(y)\, dy - \frac{3}{2} \int_\mu^\infty y F_1(y)\, dy \,- \\
&- \,\log\, \Lambda^2 \Big(\int_\mu^\infty y F_1(y)\, dy \,+\, x \int_\mu^\infty F_1(y)\, dy \Big) \Big)\,; \qquad (6) \\
&A \,=\, \frac{G_1^2 N \Lambda^2}{2\pi^2} \Big(1 + \frac{1}{4N} - \frac{G_1 N}{2\pi^2} \Big(1 + \frac{1}{2N} \Big) \int_\mu^\infty F_1(y)\, dy \Big)\,; \\
&x \,=\, p^2\,; \qquad y \,=\, q^2\,.
\end{aligned}$$

In view of taking into account of quark mass the lower limit of the momentum integration $\mu = m_0^2$ is introduced. Equation (6) by a sequential six-fold differentiation reduces to the following differential equation

$$\begin{aligned}
&\frac{d^2}{dx^2}\Big(x\, \frac{d^2}{dx^2}\Big(x\, \frac{d^2}{dx^2}\Big(x\, F_1(x)\Big) + \frac{\beta\, \mu^2}{4}\, F_1(x)\Big)\Big) \,=\, \beta\, \frac{F_1(x)}{x}\,. \qquad (7) \\
&\beta \,=\, \frac{(G_1^2 + 6\, G_1 G_2)\, N}{16\, \pi^4}\,;
\end{aligned}$$

with boundary conditions to be formulated below.

Equation (7) reduces to Meijer equation [6], [7]. Namely with the simple substitution we have

$$\begin{aligned}
&\Big(\Big(z\, \frac{d}{dz} - b \Big) \Big(z\, \frac{d}{dz} - a \Big)\, z\, \frac{d}{dz} \Big(z\, \frac{d}{dz} - \frac{1}{2} \Big) \Big(z\, \frac{d}{dz} - \frac{1}{2} \Big) \Big(z\, \frac{d}{dz} - 1 \Big) - z \Big) F_1(z) =\, 0\,; \; (8) \\
&z \,=\, \frac{\beta\, x^2}{2^6}\,; \quad a \,=\, -\,\frac{1 - \sqrt{1 - 64u}}{4}\,; \quad b \,=\, -\,\frac{1 + \sqrt{1 - 64u}}{4}\,; \quad u \,=\, \frac{\beta\, \mu^2}{64}\,.
\end{aligned}$$

Boundary conditions for equation (8) are formulated in the same way as in works [1], [5]. At first we have to choose solutions decreasing at infinity, that is combination of the following three solutions

$$
\begin{aligned}
F_1(x) \;=\; & C_1\, G^{40}_{06}\Bigl(z\,|1,\;\frac{1}{2},\;\frac{1}{2},\;0,\;a,\;b\Bigr) \;+\; C_2\, G^{40}_{06}\Bigl(z\,|1,\;\frac{1}{2},\;b,\;a,\;\frac{1}{2},\;0,\;\Bigr) \;+ \\
& +C_3\, G^{40}_{06}\Bigl(z\,|1,\;0,\;b,\;a,\;\frac{1}{2},\;\frac{1}{2}\Bigr)\,; \qquad z \;=\; \frac{\beta\, x^2}{2^6}\,.
\end{aligned}
\tag{9}
$$

where

$$
G^{mn}_{pq}\Bigl(z\,\bigl|^{a_1,\,\dots\,,\,a_p}_{b_1,\,\dots\,,\,b_q}\Bigr)\,;
$$

is the Meijer function [6], [7] with sets of upper indices a_i and of lower ones b_j. In case only one line of parameters is written this means the presence of lower indices only, n and p in the case being equal to zero.

Constants C_i are defined by conditions

$$
\begin{aligned}
& \frac{3\,G_2}{8\pi^2} - \frac{\beta}{2}\int_\mu^\infty F_1(y)\,dy \;=\; 0\,; \\
& \int_\mu^\infty y\,F_1(y)\,dy \;=\; 0\,; \\
& \int_\mu^\infty y^2\,F_1(y)\,dy \;=\; 0\,.
\end{aligned}
\tag{10}
$$

These conditions and condition $A = 0$ as well provide cancellation of all terms in Eq. (6) being proportional to Λ^2 and $\log\,\Lambda^2$. Thus the result does not depend on a value of parameter Λ. By solving linear set (10), in which solution (9) is substituted, we obtain the unique solution. Value of parameter u_0, which is connected with current quark mass, and ratio of two constants G_i we obtain from conditions $F_1(\mu) = 1$ and

$$
\begin{aligned}
A \;=\; & \frac{G_1 N \Lambda^2}{2\pi^2}\Bigl(1 + \frac{1}{4N} - \frac{G_1 N}{2\pi^2}\Bigl(1+\frac{1}{2N}\Bigr)\int_\mu^\infty F_1(y)\,dy\Bigr) \;= \\
=\; & \Bigl(1+\frac{1}{4N}\Bigr)\frac{G_1 N \Lambda^2}{2\pi^2}\Bigl(1 - \frac{6G_2(4N+2)}{(G_1+6G_2)(4N+1)}\Bigr) \;=\; 0\,;
\end{aligned}
\tag{11}
$$

this gives for $N = 3$ with the account of the first of conditions (10)

$$
u_0 \;=\; 1.6\cdot 10^{-8}\,; \quad G_1 \;=\; \frac{6}{13}\,G_2\,. \tag{12}
$$

The form-factor now reads as (9) with

$$
C_1 \;=\; 0.28322\,; \quad C_2 \;=\; -\,3.655\cdot 10^{-8}\,; \quad C_3 \;=\; -\,7.794\cdot 10^{-8}\,; \tag{13}
$$

$F_1(u_0) = 1$ and $F_1(z)$ decreases with z increasing. It is important, that the solution exists only for positive G_2 and due to (12) for positive G_1 as well.

Let us comment an essential point, connected with very small value of parameter u_0. Note, that for $u = 0$ solution satisfying all conditions excluding (11) is given by the following expression

$$\frac{1}{2\sqrt{\pi}}\, G_{06}^{40}\Big(z\,|1,\, \frac{1}{2},\, \frac{1}{2},\, 0,\, 0,\, -\frac{1}{2}\Big)\,;$$

with ratio

$$r(u)_{u=0} \;=\; G_1/G_2 \;=\; -\,3\,+\sqrt{12}\;=\;0.464102\,. \tag{14}$$

At the same time value (12), which follows from (11) is equal to 0.461538. For $u \to 0$ this ratio $x(u)$ tends to value (14) from below. Because of these two numbers being quite close the intersection of curve $x(u)$ with (11) occurs at so small u. In case we use in expression (11) the most leading terms in $1/N$, the ratio is $1/2$ and there is no intersection of $x(u)$ with this value at all, i.e. the solution does not exist. So a value u_0 is sharply dependent on approximations and quite small change of coefficients in expression (11) influences it strongly. We take into account these considerations while commenting values of current quark mass in what follows.

Solution $F_1(p,-p,0,0)$ can be extended for more general kinematics. Namely, let us consider form-factor $F_1(p,-p,q,-q)$. Assuming factorization property and bearing in mind the previous note on small contribution of u_0 the form-factor reads as follows

$$F_1(p,\,-p,\,q,\,-q) = \frac{1}{4\,\pi}\,G_{06}^{40}\Big(z\,|1,\,\frac{1}{2},\,\frac{1}{2},\,0,\,0,\,-\frac{1}{2}\Big)\,G_{06}^{40}\Big(z'\,|1,\,\frac{1}{2},\,\frac{1}{2},\,0,\,0,\,-\frac{1}{2}\Big)\,;$$
$$z\;=\;\frac{\beta\,x^2}{64}\,;\quad z'\;=\;\frac{\beta\,y^2}{64}\,;\quad x\;=\;p^2\,;\quad y\;=\;q^2\,. \tag{15}$$

A study of form-factor $F_2(p^2)$, which enters into four-fermion vector and pseudo-vector terms in (2), leads to result, that conditions of an existence of the corresponding solution does not provide additional restrictions for parameters being introduced above. Vector form-factors will be considered in details elsewhere. The explicit form of $F_2(p^2)$ do not influence results of the present work.

3 Wave function for scalar and pseudo-scalar excitations

Now with the non-trivial solution of the compensation equation we arrive at an effective theory in which there are already no undesirable four-fermion terms in **free** Lagrangian (2) while they are evidently present in **interaction** Lagrangian (3). Indeed four-fermion terms in these two parts of the full Lagrangian differ in sign and the existence of the non-trivial solution of compensation equation for Lagrangian (2) means non-existence of the would be analogous equation, formulated for signs of four-fermion terms in **interaction** Lagrangian (3).

In other words the fact, that sum of a series $\sum G^n a_n = 0$, by no means leads to a conclusion, that sum of the same series with $G \to -G$ vanishes as well.

So provided the non-trivial solution is realized the compensated terms go out from Lagrangian (2) and we obtain the following Lagrangian

$$L = \frac{\imath}{2}\Bigl(\bar\psi\gamma_\mu\partial_\mu\psi - \partial_\mu\bar\psi\gamma_\mu\psi\Bigr) - \frac{1}{4}\,F^a_{0\,\mu\nu}F^a_{0\,\mu\nu} - m_0\bar\psi\,\psi +$$
$$+ g_s\,\bar\psi\gamma_\mu t^a A^a_\mu\psi - \frac{1}{4}\left(F^a_{\mu\nu}F^a_{\mu\nu} - F^a_{0\,\mu\nu}F^a_{0\,\mu\nu}\right) - \frac{G_1}{2}\cdot\Bigl(\bar\psi\tau^b\gamma_5\psi\,\bar\psi\tau^b\gamma_5\psi - \bar\psi\,\psi\,\bar\psi\,\psi\Bigr) -$$
$$- \frac{G_2}{2}\cdot\Bigl(\bar\psi\tau^b\gamma_\mu\psi\,\bar\psi\tau^b\gamma_\mu\psi + \bar\psi\tau^b\gamma_5\gamma_\mu\psi\bar\psi\tau^b\gamma_5\gamma_\mu\psi\Bigr)\,. \qquad (16)$$

Thus, bound state problems in the present approach are formulated starting from Lagrangian (16).

In case of realization of the nontrivial solution, let us consider zero-mass scalar and pseudo-scalar excitations . Let us write down Bethe-Salpeter equation for scalar zero-mass channel in the same approximation as was used in equation (6)

$$\Psi(x) = \frac{G_1\,N}{2\pi^2}\int_\mu^\infty \Psi(y)\,dy + \frac{(G_1^2 + 6G_1G_2)N}{32\,\pi^4}\Bigl(\frac{1}{6\,x}\int_\mu^x (y^2 - 3\mu^2)\Psi(y)\,dy\; +$$
$$+\frac{3}{2}\int_\mu^x y\Psi(y)\,dy + \log\,x\int_\mu^x y\Psi(y)\,dy + x\,\log\,x\int_\mu^x \Psi(y)\,dy + \int_x^\infty y\log\,y\,\Psi(y)\,dy +$$
$$+\,x\int_x^\infty \Bigl(\log\,y\,+\frac{3}{2}\Bigr)\Psi(y)\,dy\;+\;\frac{x^2 - 3\mu^2}{6}\int_x^\infty \frac{\Psi(y)}{y}\,dy + \Bigl(2\bar\Lambda^2 - \frac{3}{2}\,x\Bigr)\int_\mu^\infty \Psi(y)\,dy -$$
$$\frac{3}{2}\int_\mu^\infty y\Psi(y)\,dy\;-\;\log\,\Lambda^2\Bigl(\int_\mu^\infty y\Psi(y)\,dy\;+\;x\int_\mu^\infty \Psi(y)\,dy\Bigr)\Bigr)\,;$$

The corresponding differential equation for $\Psi(x)$ is almost the same, as the previous one (7) with one essential difference. Namely the sign afore β is opposite.

$$\left(\Bigl(z\frac{d}{dz} - b\Bigr)\Bigl(z\frac{d}{dz} - a\Bigr)\,z\frac{d}{dz}\,\Bigl(z\frac{d}{dz} - \frac{1}{2}\Bigr)\Bigl(z\frac{d}{dz} - \frac{1}{2}\Bigr)\Bigl(z\frac{d}{dz} - 1\Bigr) + \frac{\beta\,z}{2^6}\right)F(z)\;=\;0\,; \qquad (17)$$

$$z\;=\;x^2\,;\quad a\;=\;\frac{-1+\sqrt{1-64u}}{4}\,;\quad b\;=\;\frac{-1-\sqrt{1-64u}}{4}\,;\quad u\;=\;\frac{\beta m^4}{64}\,. \qquad (18)$$

Boundary conditions read

$$\int_\mu^\infty \Psi(y)\,dy\;=\;0\,;\quad \int_\mu^\infty y\,\Psi(y)\,dy\;=\;0\,;\quad \int_\mu^\infty y^2\,\Psi(y)\,dy\;=\;0\,. \qquad (19)$$

Now we have four independent solutions decreasing at infinity, which we use for general solution

$$\Psi(x)\;=\;C^*\,G^{30}_{06}\Bigl(\frac{\beta\,x^2}{2^6}|1,\,\frac{1}{2},\,0,\,\frac{1}{2},\,a,\,b\Bigr)\;+\;C^*_1\,G^{30}_{06}\Bigl(\frac{\beta\,x^2}{2^6}|1,\,\frac{1}{2},\,\frac{1}{2},\,0,\,a,\,b\Bigr)\;+$$
$$+\,C^*_2\,G^{30}_{06}\Bigl(\frac{\beta\,x^2}{2^6}|1,\,a,\,b,\,\frac{1}{2},\,\frac{1}{2},\,0\Bigr)\;+\;C^*_3\,G^{30}_{06}\Bigl(\frac{\beta\,x^2}{2^6}|\frac{1}{2},\,a,\,b,\,1,\,\frac{1}{2},\,0\Bigr)\,. \qquad (20)$$

Boundary conditions (19) allows to express C_i^*, $i = 1, 2, 3$ in terms of C^* and condition of wave function to be unity on the mass shell $\Psi(m^2) = 1$ fixes C^* as well. The normalization condition of the Bethe-Salpeter wave function fixes constant of interaction of the bound state with quark-anti-quark pair

$$g \left(\phi \, \bar\psi \, \psi + \imath \, \pi_a \, \bar\psi \, \gamma_5 \, \tau_a \psi \right) . \tag{21}$$

Namely this normalization condition, which corresponds to a correct coefficient afore a kinetic part of scalar and pseudo-scalar particles, gives

$$\frac{g^2 \, N}{4 \, \pi^2} \, I_1 \; = \; 1 \, ; \quad I_1 \; = \; \int_\mu^\infty \frac{\Psi(x)^2 \, dx}{x} \, ; \tag{22}$$

The loop integral of the fourth order in interaction (21) gives four-fold interaction

$$\lambda \, \frac{(\phi^2 \; + \; \pi^a \pi^a)^2}{4!} \, ; \tag{23}$$

where

$$- \, \lambda \; = \; \frac{g^4 \, 3 \, N}{\pi^2} \, I_2 \, ; \quad I_2 \; = \; \int_\mu^\infty \frac{\Psi(x)^4 \, dx}{x} \, . \tag{24}$$

The result, that equation (17) has unique solution, which satisfies all boundary conditions, is the confirmation of the evident fact, that in the same approximation, in which the compensation equation has a non-trivial solution, fields ϕ and π_a are to have zero masses in accordance with Bogolubov-Goldstone theorem [2, 3, 8]. However an account of chromodynamic interaction leads to additional contribution to the masses. Let us calculate a mass correction term due to QCD interaction. For the purpose let us take into account terms of the first order in P^2, where P is the momentum of a scalar (and pseudo-scalar) meson and one-loop QCD term. We have

$$\Psi(p^2) \; = \; \frac{G_1 \, N}{2 \, \pi^4} \int \frac{\Psi(q^2) \, dq}{q^2} \Bigl(1 - \frac{3 \, P^2}{4 \, q^2} + \frac{(qP)^2}{(q^2)^2}\Bigr) \; + \; \frac{(G_1^2 + 6 G_1 G_2) N}{32 \, \pi^6} \; \times$$
$$\int \Bigl(2\Lambda^2 + (p-q)^2 \log \frac{(p-q)^2}{\Lambda^2} - \frac{3}{2} (p-q)^2 \Bigr) \Bigl(1 - \frac{3 \, P^2}{4 \, q^2} + \frac{(qP)^2}{(q^2)^2}\Bigr) \frac{\Psi(q^2) \, dq}{q^2} \; +$$
$$+ \, \frac{g_s^2}{4 \, \pi^4} \int \frac{\Psi(q^2) \, dq}{q^2 (q-p)^2} \, . \tag{25}$$

In the course of QCD term calculation we use transverse Landau gauge [1] . Let us multiply equation (25) by $\Psi(p^2)/p^2$ at $P = 0$ and integrate by p. Due to equation (17) be satisfied we have

$$- \, \frac{P^2}{2} \int \frac{\Psi(q^2)^2 \, dq}{(q^2)^2} \; + \; \frac{g_s^2}{4 \, \pi^4} \int \frac{\Psi(p^2) \, dp}{p^2} \int \frac{\Psi(q^2) \, dq}{q^2 (q-p)^2} \; = \; 0 \, ; \tag{26}$$

[1] In the approximation used the transverse gauge leads to absence of renormalization of both vertex and spinor field

After angle integration we get

$$\frac{P^2\,\pi^2}{2}\,I_1\;=\;\frac{g_s^2}{4}\int_{m^2}^{\infty}\Psi(x)\,dx\left(\frac{1}{x}\int_{m^2}^{x}\Psi(y)\,dy\;+\;\int_{x}^{\infty}\frac{\Psi(y)\,dy}{y}\right)\;=$$
$$=\;\frac{g_s^2}{\sqrt{\beta}}\int_u^{\infty}\frac{\Psi(z)\,dz}{z}\int_u^{z}\frac{\Psi(t)\,dt}{\sqrt{t}}\;=\;\frac{g_s^2\,I_3}{\sqrt{\beta}};\quad z\;=\;\frac{\beta\,x^2}{64}\,;\quad t\;=\;\frac{\beta\,y^2}{64}\,. \tag{27}$$

The integral entering into (27) looks like

$$\int_u^{z}\frac{\Psi(t)\,dt}{\sqrt{t}}\;=\;\Biggl(C^*\,G_{06}^{30}\Bigl(t\,|_{\frac{1}{2},\,1,\,\frac{3}{2},\,0,\,a',\,b'}\Bigr)\;+\;C_1^*\,G_{17}^{31}\Bigl(t\,|^{1}_{1,\,1,\,\frac{3}{2},\,0,\,\frac{1}{2},\,a',\,b'}\Bigr)\;+$$
$$+\,C_2^*\,G_{06}^{30}\Bigl(t\,|_{\frac{3}{2},\,a',\,b',\,0,\,\frac{1}{2},\,1}\Bigr)\;+\;C_3^*\,G_{06}^{30}\Bigl(t\,|_{1,\,a',\,b',\,0,\,\frac{1}{2},\,\frac{3}{2}}\Bigr)\Biggr)_u^z\,; \tag{28}$$
$$a'\;=\;\frac{1-\sqrt{1-64u}}{4},\quad b'\;=\;\frac{1+\sqrt{1-64u}}{4}\,.$$

Relation (27) gives us mass of scalar (and pseudo-scalar) in terms of average low-energy QCD constant $\bar{\alpha}_s$. There are a number of considerations concerning possible values of the parameter. For example, in approach [9] with low-energy freezing of the strong constant value $\bar{\alpha}_s = 0.414$ is quoted. Smooth matching of perturbative and non-perturbative regions in QCD gives value $\bar{\alpha}_s = 0.4354$ [10]. There is also a sum rule definition of this parameter from experimental data $\bar{\alpha}_s = 0.47 \pm 0.07$ [11]. there are also considerations on behalf of larger values of $\bar{\alpha}_s$ [12, 13]. Taking into account these remarks we consider range of values of the effective low-energy constant $\bar{\alpha}_s = 0.4 - 0.75$. In other words we can define the same quantity as value of running strong constant at a characteristic momentum, e.g. 600 MeV. Different variants of low energy behaviour of α_s lead to the same interval. Thus we have

$$m_t^2\;=\;-\,\frac{8\,\bar{\alpha}_s\,I_3}{\pi\sqrt{\beta}\,I_1}\,;\quad I_3\;=\;\int_u^{\infty}\frac{\Psi(z)\,\Psi_1(z)\,dz}{z}\,; \tag{29}$$

where I_1 is defined in (22). Due to both integrals being positive we evidently have tachyon mass and so a scalar condensate appears in the minimum of effective potential

$$m_t^2\,\frac{\phi^2}{2}\;+\;\lambda\,\frac{\phi^4}{24}\,.$$

For the present approach it is highly important, that the account of the QCD interaction leads to tachyon mass of a scalar and pseudo-scalars. The appearance of tachyons and thus the appearance of the scalar condensate in the minimum of the effective potential results in stability of the non-trivial solution and consequently one may conclude, that Lagrangian (16) with the conditions being obtained are valid.

Bearing in mind definitions of quantities entering in the effective potential, we obtain for value of the scalar condensate η

$$\eta^2\;=\;\frac{-\,6\,m_t^2}{\lambda}\,;\quad \eta\;=<\phi>\;. \tag{30}$$

Mass of ϕ after symmetry breaking is $\sqrt{2(-m_t^2)}$, and π mass equals to zero in the present approximation. The constituent quark mass m_q is defined by relation

$$m_q \,-\, m_0 \,=\, g\,\eta \,=\, g\,\sqrt{\frac{6\,(-\,m_t^2)}{\lambda}} \,=\, \frac{m_\phi}{2}\,\sqrt{\frac{I_1}{I_2}}\,. \tag{31}$$

Integrals in this as well as in other relations of this section including boundary conditions are calculated with the lower limits defined not by the current mass m_0, but by the constituent mass m. Now parameter u is defined by relation

$$u \,=\, \frac{\beta\, m^4}{2^6}\,. \tag{32}$$

Thus after the appearance of the scalar condensate the quark propagator takes form

$$G(p) \,=\, \frac{1}{(\gamma\, p) \,-\, \Sigma(p)}\,; \quad \Sigma(p) \,=\, (m - m_0)\,\Psi(p^2) \,+\, m_0\,. \tag{33}$$

Relations (31, 27, 28) give for $\bar{\alpha}_s \,=\, 0.46$

$$u \,=\, 0.001\,; \quad C^* \,=\, -\,0.919\,; \quad C_1^* \,=\, -\,0.0255\,; \quad C_2^* \,=\, -\,1.895\,;$$
$$C_3^* \,=\, 0.228\,; \quad I_1 \,=\, 1.285\,; \quad I_2 \,=\, 0.801\,; \quad I_3 \,=\, 0.302\,. \tag{34}$$

We take $\bar{\alpha}_s$ and current mass m_0 as the initial parameters and express in their terms all other parameters including π-decay constant f_π. We use for the latter the Goldberger – Treiman relation. In the framework of the present approach we obtain the relation by considering the expression for transition $\pi^+ \to \mu^+\,\nu_\mu$. We have

$$f_\pi \,=\, \frac{g\,N}{4\,\pi^2}\int_{m^2}^{\infty}\Bigl((m - m_0)\,\Psi(y)^2 \,+\, m_0\,\Psi(y)\Bigr)\frac{dy}{y} \,=$$
$$=\, \frac{g\,N}{4\,\pi^2}\Bigl((m - m_0)\,I_1 + \,m_0\,I_7\Bigr)\,; \quad I_7 \,=\, \int_u^\infty \frac{\Psi(z)\,dz}{2\,z}\,. \tag{35}$$

Provided either $m_0 = 0$ or $I_1 = I_7$ we get with account of normalization condition (22) just the original Goldberger – Treiman relation $m = g\,f_\pi$. We use full relation (35). However let us note, that values of the two integrals are close $I_1 \simeq I_7$ and the simple original relation works with sufficient accuracy.

At this place it is worth-while to explain that we have a solution for any value of m_0 and the problem is, which value is to be chosen. While m_0 is not measured directly, the accuracy of its definition is not very high. For comparison with experimental data we propose to fix value of $f_\pi \,=\, 93\,\mathrm{MeV}$, which is quite certain, while other parameters including m_0 being subjects for calculation.

As a result we obtain e.g. for $\bar{\alpha}_s \,=\, 0.46$ ($u = 0.001$)

$$g \,=\, 3.2\,;\; G_1 \,=\, \frac{1}{(240.5\,\mathrm{MeV})^2}\,;\; m \,=\, 298.5\,\mathrm{MeV}\,;\; m_0 \,=\, 19.8\,\mathrm{MeV}\,. \tag{36}$$

Mass of σ is defined by above results (34, 36)

$$m_\sigma \;=\; 2\,(m - m_0)\,\sqrt{\frac{I_2}{I_1}} \;=\; 440.1\,\mathrm{MeV}\,. \tag{37}$$

At this stage mass of π-meson is zero due to appearance of vacuum average of scalar field ϕ.

Coupling constant $g_{\sigma\,\pi\,\pi}$ in interaction

$$g_{\sigma\,\pi\,\pi}\,\sigma\,\pi^a\,\pi^a\,; \tag{38}$$

is calculated with the use of wave function (20) with parameters (34, 36). The triangle diagram gives

$$g_{\sigma\,\pi\,\pi} \;=\; \frac{g^3\,N}{\pi^2}\,\int_{m^2}^{\infty}\frac{\Psi(y)^3\,dy}{y}((m - m_0)\Psi(y) + m_0) \;=\; 8.04\,m\,. \tag{39}$$

According to (39) the σ width reads

$$\Gamma_\sigma \;=\; \frac{3\,g^2_{\sigma\,\pi\,\pi}}{16\,\pi\,m_\sigma^2}\,\sqrt{m_\sigma^2 - 4m_\pi^2} \;=\; 605.8\,\mathrm{MeV}\,; \quad m_\pi = 139\,\mathrm{MeV}\,. \tag{40}$$

Values (37, 40) do not contradict the existing data [14], which, as a matter of fact, are characterized by a wide spread of results.

The results on the wave function of a scalar particle and on the form-factor of the effective four-fermion interaction allow to estimate the quark condensate in the next approximation. Indeed vertex $\sigma\bar\psi\,\psi$ form-factor is just $g\Psi(q^2)$. The non-perturbative part of the quark propagator reads $g\,\eta\,\Psi(q^2) \;=\; (m - m_0)\Psi(q^2)$. According to definition of the quark condensate we have

$$< \bar q\, q >= \,-\,\frac{4\,N\,g\eta}{(2\,\pi)^4}\int_{m^2}^{\infty}\,\Psi(q^2)\,dq^2\,. \tag{41}$$

Here the integral equals zero due to boundary conditions (19). However one may try to calculate subsequent approximations. Let us calculate loop corrections: firstly with vertex corresponding to four-fermion interaction in Lagrangian (16) and secondly with gluon exchange in transverse gauge (see Fig. 3). In loop diagrams with the form-factor $F_1(p)$ we sum up infinite loop chain (the senior orders in $1/N$), that gives factor $(1-B)$ in the denominator of the first term in the following expression

$$\begin{aligned}
&< \bar q\, q >= \,-\frac{12\sqrt{3}(m - m_0)}{\pi^2\sqrt{14}\,G_1\,(1 - B)}\,I_4\,I_5 \,-\, \frac{3\,\bar\alpha_s(m - m_0)}{4\,\pi^3}\left(-\,m^2\,I_7 \,+\, \frac{16\,\pi^2}{G_1\,\sqrt{42}}\,I_6\right);\\
&I_4 \;=\; \int_u^\infty \frac{F_1(z)\,dz}{2\sqrt{z}}\,; \quad I_5 \;=\; \int_u^\infty \frac{F_1(z)\,\Psi(z)\,dz}{2\sqrt{z}}\,; \quad I_6 \;=\; \int_u^\infty \frac{\Psi_1(z)\,dz}{2z}\,; \\
&B \;=\; \frac{16\,\sqrt{3}}{\sqrt{14}}\int_u^\infty \frac{(F_1(z))^2\,dz}{2\,\sqrt{z}}\,;
\end{aligned} \tag{42}$$

where function $\Psi_1(z)$ is defined by relation (28). We have e.g. again for $u = 0.001$, $\bar{\alpha}_s = 0.46$

$$< \bar{q}\, q > = \, - \, (167\,\mathrm{MeV})^3\,; \tag{43}$$

that is sufficiently lower than the traditional value (240 MeV)^3. Nevertheless we draw attention to the correct sign of the condensate. It is connected with the sign afore G_1 in the interaction Lagrangian (16),which is defined by conditions of the existence of the non-trivial solution of the compensation equation.

4 Discussion of results

The low-energy hadron parameters being calculated for three values of $\bar{\alpha}_s$ are presented in Table 1. The calculations needs numerical integrations of expressions containing Meijer functions with infinite upper limits. Therefore the presented data may contain uncertainties (not more than 3%). Note that integrals I_4, I_6, I_7 are evaluated analytically, and all the rest ones are calculated numerically. Let us recall, that only two parameters are in our disposal – the average strong coupling constant $\bar{\alpha}_s$ and dimensional constant of π-decay f_π. We use of course the Goldberger-Treiman relation (35), which is obtained in the framework of the present approach. No other outside information is used. In particular while calculating σ-meson width we substitute pion mass which was obtained here in the corresponding approximation and was quoted in the Table. We see from Table 1, that the approach gives reasonable correspondence to the existing facts and data. We present information from experiments and from phenomenological considerations in the last right-hand column of the table.

Value of current mass m_0 is essentially larger than customary values. This parameter is defined by value u_0 (12), which is sharply dependent on corrections of the next orders of $1/N$ expansion and on other details of the next approximations. For example, by a slight change in coefficient afore the second term in brackets in expression (11) one may obtain for m_0 value being two-three times smaller than that presented in Table 1. At this all other parameters practically do not change. Taking into account this fact we do not consider the deviation of m_0 as critical for the approach. In comparison to the traditional value the modulus of quark condensate is essentially smaller. While its calculation we have seen that in the leading approximation this parameter is zero and the values quoted correspond to loop corrections. It is possible, that there are other contributions to the quark condensate, which give essential changes. For example contribution of s-quark loop to the first term in (42) evidently enlarge it. This problem deserves a special study. However just the presented values give rise to an interesting observation. It comes out, that Gell-Mann – Renner relation [15] agrees sufficiently well with the parameters of Table 1. Indeed In the lowest line of the table for pion mass we write down its values calculated by the relation

$$m_\pi^2 \; = \; - \, \frac{2\, m_0}{f_\pi^2} \; < \bar{q}\, q > \; .$$

Values of m_π for m_0 and $< \bar{q}\, q >$ from the table are quite satisfactory in the range of a reasonable accuracy. Note also that values of the quark condensate being lower than the traditional one are discussed in the literature (see e.g. [16]). Lattice measurements [17] give rather wide interval of renormalization invariant value of the condensate $- < \bar{q}q >$ $= ((206 \pm 44 \pm 8 \pm 5)\,\mathrm{MeV})^3$, in which also the values presented in Table 1 also enter. Parameters of the σ-meson especially its width noticeably depend on a choice of value $\bar{\alpha}_s$. Unfortunately the spread of data [14] covers all presented results. Provided one choose results of one work then it is possible to do some conclusion on preferable value of $\bar{\alpha}_s$. For example in paper [18] the processing of the set of data leads to result $m_\sigma = (470 \pm 30)\,\mathrm{MeV}$, $\Gamma_\sigma = (590 \pm 40)\,\mathrm{MeV}$. Such values agree well with our results for $\bar{\alpha}_s \simeq 0.5$. At that the pion mass and the constituent quark mass are quite satisfactory.

So we may state a quantitative agreement for constituent quark mass and for parameters of mesons π and σ, whereas for current mass and for the quark condensate we may declare at least a qualitative agreement. One hardly could expect more from the first non-perturbative approximation with only two parameters, one of which f_π is strictly fixed. Therefore one have to admit the description of the low-energy data by the approach being satisfactory.

To conclude let us emphasize that the aim of the work is achieved. We have begun with the demonstration of the non-trivial solution of the compensation equation. The existence of scalar and pseudo-scalar excitations (mesons) in the same approximation is a consequence of its existence. The account of QCD interaction leads to the shift of their masses squared to the negative region, i.e. to the appearance of tachyons, which are necessary for scalar condensate to arise. As a result we obtain the standard scheme of chiral symmetry breaking with massive scalar and massless pion. Subsequent approximations of the approach are related to values of the quark condensate and the pion mass.

We have shown that the application of the method of work [1], which is based on Bogolubov quasi-averages approach, to the low-energy region of hadron physics leads to quite reasonable results. From this we would make two essential conclusions.

Firstly, a subsequent development of the present approach to the hadron physics quite deserves attention. In particular it is advisable to apply the approach to calculation of parameters of vector mesons ρ, ω, A_1, to consider hadrons containing s-quark, to take into account the $\pi - A_1$-mixing, to introduce diquarks etc.. In view of methods it would be desirable to improve a procedure of taking into account of particle masses. These problems comprise subjects for a forthcoming studies.

Secondly, the positive result of applicability test with Nambu – Jona-Lazinio model, being taken as an example, allows to hope for successful application of the approach to other problems. In particular we mean the problem of a dynamical breaking of the electroweak symmetry. A qualitative discussion of possible variants in this region is presented e.g. in works [19], [20].

The author is grateful to M.K. Volkov and S.B. Gerasimov for valuable discussions and to I.V. Zaitsev for some numerical calculations.

The work is partially supported by grant "Universities of Russia" UR.02.02.503.

A Appendix

We present below a set of formulas for angular integrals in four-dimensional Euclidean space, which are used in the work.

$$
\begin{aligned}
& x \,=\, p^2\,, \qquad y \,=\, q^2\,; \\
& \int \frac{d^4q\,F(q^2)}{(p-q)^2} \,=\, \pi^2 \int_0^\infty ydyF(y)\Big(\theta(x-y)\frac{1}{x}+\theta(y-x)\frac{1}{y}\Big)\,; \\
& \int \frac{d^4q\,F(q^2)\,(pq)}{(p-q)^2} \,=\, \frac{\pi^2}{2} \int_0^\infty ydyF(y)\Big(\theta(x-y)\frac{y}{x}+\theta(y-x)\frac{x}{y}\Big)\,; \\
& \int \frac{d^4q\,F(q^2)(pq)^2}{(p-q)^2} \,=\, \frac{\pi^2}{4} \int_0^\infty ydyF(y)\Big(\theta(x-y)\Big(\frac{y^2}{x}+y\Big)+\theta(y-x)\Big(x+\frac{x^2}{y}\Big)\Big)\,; \\
& \int \frac{d^4q\,F(q^2)(pq)^3}{(p-q)^2} \,=\, \frac{\pi^2}{8} \int_0^\infty ydyF(y)\Big(\theta(x-y)\Big(\frac{y^3}{x}+2y^2\Big)+\theta(y-x)\Big(2x^2+\frac{x^3}{y}\Big)\Big)\,; \\
& \int \frac{d^4q\,F(q^2)(p,p-q)}{((p-q)^2)^2} \,=\, \pi^2 \int_0^\infty ydyF(y)\theta(x-y)\frac{1}{x}\,; \\
& \int \frac{d^4q\,F(q^2)(q,q-p)}{((p-q)^2)^2} \,=\, \pi^2 \int_0^\infty ydyF(y)\theta(y-x)\frac{1}{y}\,; \\
& \int \frac{d^4q\,F(q^2)(p,p-q)(pq)}{((p-q)^2)^2} \,=\, \frac{\pi^2}{4} \int_0^\infty ydyF(y)\Big(\theta(x-y)\frac{3y}{x}-\theta(y-x)\frac{x}{y}\Big)\,; \\
& \int d^4qF(q^2)\log{(q-p)^2} \,=\, \pi^2 \int_0^\infty ydyF(y)\Big(\theta(x-y)\Big(\frac{y}{2x}+\log\,x\Big)+ \\
& +\theta(y-x)\Big(\log\,y+\frac{x}{2y}\Big)\Big)\,; \\
& \int d^4qF(q^2)(pq)\log{(q-p)^2} \,=\, \frac{\pi^2}{6} \int_0^\infty ydyF(y)\Big(\theta(x-y)\Big(\frac{y^2}{x}-3y\Big)+ \\
& +\theta(y-x)\Big(-3x+\frac{x^2}{y}\Big)\Big)\,; \qquad\qquad (44) \\
& \int d^4qF(q^2)(pq)^2\log{(q-p)^2} \,=\, \frac{\pi^2}{4} \int_0^\infty ydyF(y)\Big(\theta(x-y)\Big(\frac{y^3}{4x}+\,xy\log\,x\Big)+ \\
& +\theta(y-x)\Big(xy\log\,y+\frac{x^3}{4y}\Big)\Big)\,; \\
& \int d^4qF(q^2)(pq)^3\log{(q-p)^2} \,=\, \frac{\pi^2}{8} \int_0^\infty ydyF(y)\Big(\theta(x-y)\Big(\frac{y^4}{5x}+\frac{y^3}{3}-2y^2x\Big)+ \\
& +\theta(y-x)\Big(-\,2x^2y+\frac{x^3}{3}+\frac{x^4}{5y}\Big)\Big)\,.
\end{aligned}
$$

Integrals containing $\log{(q-p)^2}$ are evaluated using formulas from book [21].

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Table 1.
Parameters of the low-energy hadron physics for three values of effective coupling constant $\bar{\alpha}_s$.

$\bar{\alpha}_s$	0.46	0.61	0.73	exp/phen
f_π MeV	93	93	93	input
g	3.20	3.44	3.59	–
m_0 MeV	19.8	17.9	16.9	5 – 10
m MeV	298.5	321.3	335.5	300 – 350
$G_1^{-1/2}$ MeV	240.5	217.6	205.4	–
$- < \bar{q}q >^{1/3}$ MeV	0 167.0	0 155.4	0 148.7	220 – 240
m_π MeV	0 146.0	0 124.6	0 113.3	136 – 140
$g_{\sigma\pi\pi}/m$	8.04	8.27	8.40	–
m_σ MeV	440.1	469.1	486.3	400 – 1200
Γ_σ MeV	584.4	761.1	862.5	600 – 1000
u	0.001	0.002	0.003	

Figure captions

Fig. 1. Diagram representation of the compensation equation. Black spot corresponds to four-fermion vertex with a form-factor.

Fig. 2. Diagram representation of Bethe-Salpeter equation for scalar bound state, the later corresponding to a double line.

Fig. 3. Loop correction to the non-perturbative part of the quark condensate. The full line corresponds to the quark propagator with non-perturbative mass operator $(m - m_0)\Psi(p^2)$. The dotted line represents a gluon.

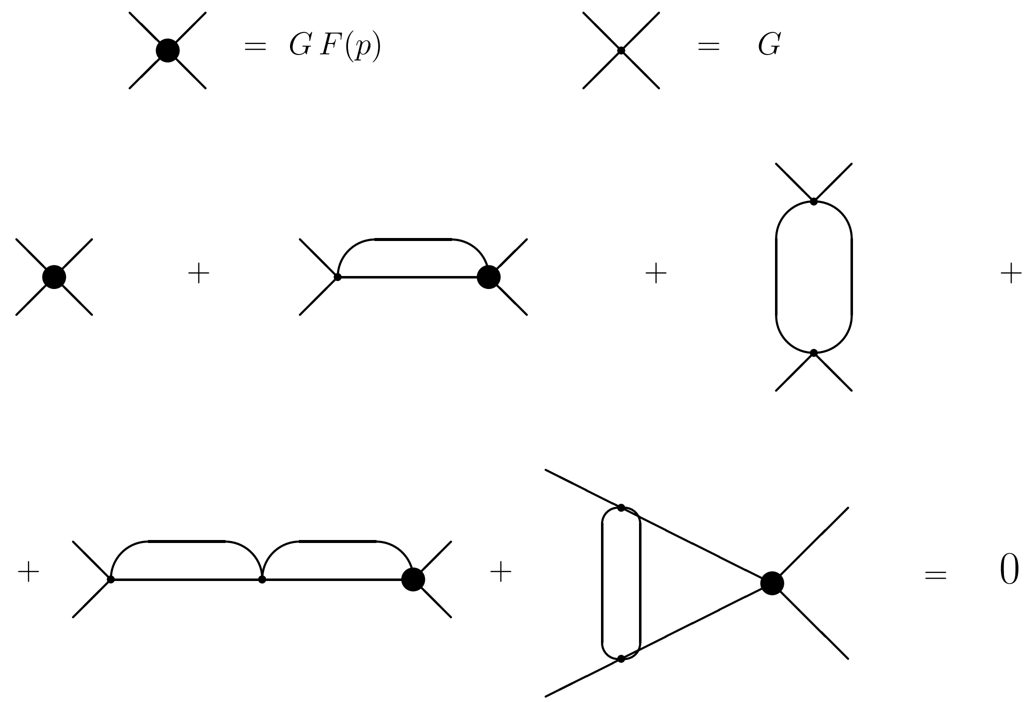

Fig. 1.

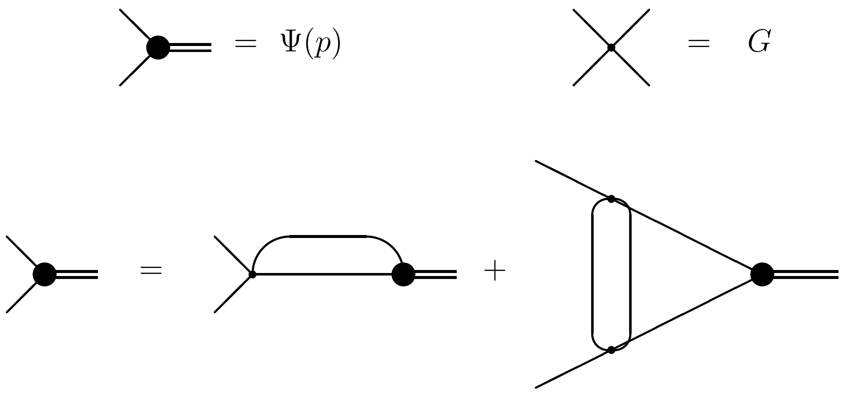

Fig. 2.

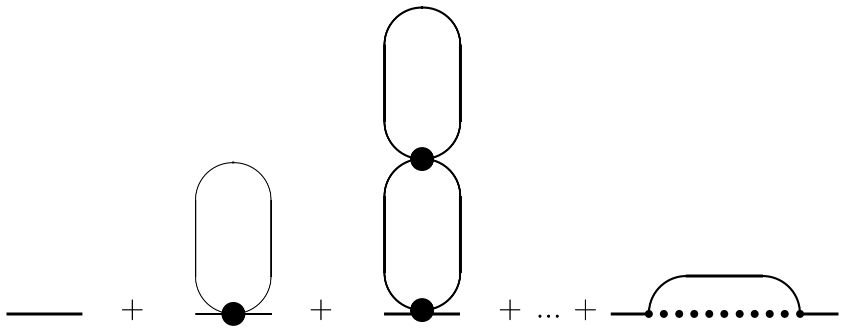

Fig. 3.