

# Productions of $X(1835)$ as baryonium with sizable gluon content

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## Abstract

The  $X(1835)$  has been treated as a baryonium with sizable gluon content, and to be almost flavor singlet. This picture allows us to rationally understand  $X(1835)$  production in  $J/\psi$  radiative decays, and its large couplings with  $p\bar{p}$ ,  $\eta'\pi\pi$ . The processes  $\Upsilon(1S) \rightarrow \gamma X(1835)$  and  $J/\psi \rightarrow \omega X(1835)$  have been examined. It has been found that  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))Br(X(1835) \rightarrow p\bar{p}) < 6.45 \times 10^{-7}$ , which is compatible with CLEO's recently experimental result (Phys.Rev.**D73** (2006) 032001;hep-ex/0510015). The branching fractions of  $Br(J/\psi \rightarrow \omega X(1835))$ ,  $Br(J/\psi \rightarrow \rho X(1835))$  with  $X(1835) \rightarrow p\bar{p}$  and  $X(1835) \rightarrow \eta'\pi^+\pi^-$  have been estimated by the quark-pair creation model. We show that they are heavily suppressed, so the signal of  $X(1835)$  is very difficult, if not impossible, to be observed in these processes. The experimental checks for these estimations are expected. The existence of the baryonium nonet is conjectured, and a model independent derivation of their production branching fractions is presented.

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## I. INTRODUCTION

Recently the BES collaboration has observed a new resonant state X(1835) in the  $\eta'\pi\pi$  invariant mass spectrum in the process  $J/\psi \rightarrow \gamma\pi^+\pi^-\eta'$  [1] with a statistical significance of  $7.7\sigma$ . The fit with the Breit-Wigner function yields mass  $M = 1833.7 \pm 6.1(\text{stat}) \pm 2.7(\text{syst})\text{MeV}/c^2$ , width  $\Gamma = 67.7 \pm 20.3(\text{stat}) \pm 7.7(\text{syst})\text{MeV}/c^2$  and the product branching fraction  $Br(J/\psi \rightarrow \gamma X(1835))Br(X(1835) \rightarrow \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})) \times 10^{-4}$ . A narrow near threshold enhancement in the proton-antiproton ( $p\bar{p}$ ) mass spectrum was observed from the radiative decay  $J/\psi \rightarrow \gamma p\bar{p}$  [2]. This enhancement can be fitted with either an S or P wave Breit-Wigner resonance function. In the case of S-wave fit, the peak mass is  $M = 1859_{-10}^{+3}(\text{stat})_{-25}^{+5}(\text{sys})\text{MeV}/c^2$  with the total width  $\Gamma < 30\text{MeV}/c^2$  at 90% confidential level and the product branching fraction  $Br(J/\psi \rightarrow \gamma X)Br(X \rightarrow p\bar{p}) = (7.0 \pm 0.4_{-0.8}^{+1.9}) \times 10^{-5}$ .

The masses of the two structures observed in both  $J/\psi \rightarrow \gamma p\bar{p}$  and  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  channels are overlap and  $0^{-+}$  quantum number for the resonance in  $\eta'\pi^+\pi^-$  channel is possible. A question arise if they are the same state, in Ref.[1] an argument is presented if the final state interaction is included in the fit of the  $p\bar{p}$  mass spectrum, the width of the resonance observed in  $\gamma p\bar{p}$  channel will become larger. Therefore, the X observed in both  $p\bar{p}$  and  $\eta'\pi^+\pi^-$  channels could be the same state and it is named as X(1835) in Ref.[1]. And this state couples strongly with  $p\bar{p}$  and  $\eta'\pi^+\pi^-$ , in the recent talk of BES [4], the estimation of  $Br(J/\psi \rightarrow \gamma X(1835)) \sim (0.5 - 2) \times 10^{-3}$ ,  $Br(X \rightarrow p\bar{p}) \sim (4 - 14)\%$  are presented.

However recently a negative experimental result has been reported by CLEO collaboration[5]. They claimed that in the radiative decay of  $\Upsilon(1S)$  the narrow enhancement observed by BES near  $p\bar{p}$  mass threshold is not seen. The upper limit of the product branching fraction for the decay  $\Upsilon(1s) \rightarrow \gamma X(1835)$ ,  $X(1835) \rightarrow \gamma p\bar{p}$  has been determined to be  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))Br(X(1835) \rightarrow p\bar{p}) < 5 \times 10^{-7}$ [5].

Moreover, another problem we would like to mention is that because  $Br(J/\psi \rightarrow \gamma X(1835)) \sim (0.5 - 2) \times 10^{-3}$  claimed by BES in [1] is rather larger among  $J/\psi$  decays and  $\omega$  is a photon like vector meson with negative  $G$  parity, an experimental measurement of  $Br(J/\psi \rightarrow \omega X(1835))$  seems to be practicable in BES, or at least the signal of  $J/\psi \rightarrow \omega X(1835)$  should be seen in BES. However, there are still not yet any results on this matter reported by BES, therefore it is urgent to discuss the problem that whether the

fact that the signal of  $J/\psi \rightarrow \omega X(1835)$  is not revealed at the present stage contradicts the existence of  $X(1835)$  or not.

In this case, the existence of  $X(1835)$  seems to become a puzzle. Therefore it is worth pursuing both the reasons why  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))$  is so small that  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))Br(X(1835) \rightarrow p\bar{p}) < 5 \times 10^{-7}$  and the reasons why there is still not yet any information on  $J/\psi \rightarrow \omega X(1835)$  reported by BES. In this work we try to answer the above questions, and try to illustrate that the absence of  $X(1835)$  signal from the two processes at present stage is due to the special structure of  $X(1835)$ .

The theoretical interpretation of this exotic state is a great challenge, and many proposals has been suggested [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Some of them interpret  $X(1835)$  as a  $p\bar{p}$  bound state [7, 10, 11, 12], and large enough binding energy to bind proton and antiproton together has been derived from the constitute quark models[12]. On the other hand, some authors identify  $X(1835)$  as a pseudoscalar glueball [13, 15], and in Refs.[11, 14] the authors claim that there is large gluon content in  $X(1835)$ . Also some authors suggest that the two structures observed in  $J/\psi \rightarrow \gamma p\bar{p}$  and  $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$  are not the same state, and identify  $X(1835)$  as the  $\eta$ 's second radial excitation [16]. Obviously, more theoretical and experimental efforts are needed to determine whether  $X(1835)$  exists or not, and to be sure that  $X(1835)$  is a  $p\bar{p}$  bound state or glueball or something else. Motivated by solving the puzzles mentioned above and getting the information about the structure of  $X(1835)$ , we investigate the productions of  $X(1835)$  in  $\Upsilon$  and  $J/\psi$  decays in this work. The production of  $X(1835)$  may provide significant information on the structure of  $X(1835)$ .

So far, the experiments strongly indicate that  $X(1835)$  is almost uniquely produced in  $J/\psi$  radiative decays and it has large coupling with  $p\bar{p}$  and  $\eta' \pi \pi$ . Whatever  $X(1835)$  is a glueball or  $p\bar{p}$  bound state or something else, it must meet these two significant experimental facts. In this work the possibility of  $X(1835)$  as a baryonium with sizable gluon content are investigated. In this picture, the puzzles mentioned in the above can be answered naturally.

The paper is organized as followings: In section II, we suggest  $X(1835)$  as a baryonium with sizable gluon content, whose gluon content is similar to that of  $\eta'$ . In this picture, we can easily understand the reasons why  $\Upsilon(1S) \rightarrow \gamma X(1835)$  and  $J/\psi \rightarrow \omega X(1835)$  are not be seen at the present stage. In section III we conjecture the existence of pseudoscalar baryonium nonet and study its production in  $J/\psi$  decay in a model independent way. Finally we briefly summary the results and give some discussions.

## II. THE POSSIBLE STRUCTURE OF X(1835) AND $\Upsilon(1S) \rightarrow \gamma X(1835)$ , $J/\psi \rightarrow \omega X(1835)$

The production of X(1835) in  $J/\psi$  radiative decay  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  may indicate that there is large gluon content in X(1835), as is shown in Ref.[11, 14]. Also  $J/\psi \rightarrow \gamma + gg$ ,  $gg \rightarrow hadrons$  provide an important search ground for the glueball[17], some people suggest that X(1835) is a  $0^{-+}$  glueball. However the lowest pseudoscalar glueball mass is  $2.1 \sim 2.5\text{GeV}$  from the quenched lattice approach [18], and  $2.05 \pm 0.19\text{GeV}$ ,  $2.2 \pm 0.2\text{GeV}$  in QCD sum rules [19] and it seems difficult to explain the large mass difference between 1835 MeV and the theoretical prediction mass. On the other hand, even if X(1835) is a pure glueball, it would mix with other mesonic states, such as  $\eta(1440)$ ,  $\eta(1295)$  and  $\eta_c(1S)$ .

Furthermore in Ref.[12] we shown that X(1835) can be possibly a baryonium and the relative large mass defect can be produced. In Ref.[11], we pointed out that there is sizeable gluon content in the Skyrmion-Baryonium  $X(1860)$  (*i.e.*,  $X(1835)$ ) by discussing the baryonium decay through baryon-antibaryon annihilation in the Skyrme model. Distinguishing from the naive (or old fashioned) baryonium in the Fermi-Yang type models[8, 20, 21, 22], the Skyrmion-Baryonium is constructed in the model inspired by QCD, and therefore the gluon inside the baryonium will play important role in the baryonium physics, e.g, the baryonium decays and productions. Therefore, the Skyrmion-Baryonium belongs to a sort of baryonium with sizable gluon content. We address that in the naive baryonium model framework, it is difficult to simultaneously explain the large branching fraction  $X(1835) \rightarrow p\bar{p}$ ,  $X(1835) \rightarrow \eta'\pi^+\pi^-$ . The gluon content in X(1835) should play essential role in the  $X(1835)$  decay[11]. So it is natural to treat X(1835) as a baryonium with sizable gluon content, which looks like  $\eta'$  in some sense, and mainly belongs to a SU(3) flavor singlet.

In the following two subsections, we will start with this view to examine the branching fractions of  $\Upsilon(1S) \rightarrow \gamma X(1835)$  and of  $J/\psi \rightarrow \omega X(1835)$  respectively. We will show that the branching fractions of both  $\Upsilon(1S) \rightarrow \gamma X(1835)$  and  $J/\psi \rightarrow \omega X(1835)$  are much smaller comparing to that of  $J/\psi \rightarrow \gamma X(1835)$ . We will also predict the branching fraction of  $J/\psi \rightarrow \rho X(1835)$  is very small, so we see that the process with visible  $X(1835)$  may only be the  $J/\psi$  radiative decay at present stage.

### A. $\Upsilon(1S) \rightarrow \gamma X(1835)$

According to Novikov et al.[23], for the  $J/\psi$  radiative decay, the photon is emitted by the  $c$  quark with a subsequent annihilation of the  $c\bar{c}$  into light quarks through the effect of the  $U(1)_A$  anomaly. The creation of the corresponding light quarks is controlled by the gluonic matrix element  $\langle \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} | P_i \rangle$  ( $P_i$  is a pseudoscalar, it can be  $\eta$ ,  $\eta'$ , and  $X(1835)$  and so on). Photon emission from the light quarks is negligible as can be seen from the smallness of the  $J/\psi \rightarrow \gamma\pi$  decay width, this mechanism leads to the following width for the  $J/\psi$  radiative decay into the pseudoscalar  $P_i$

$$\Gamma(J/\psi \rightarrow \gamma P_i) = \frac{2^5}{5^2 3^8} \pi e_c^2 \alpha_{em}^3 K[J/\psi \gamma P_i]^3 \left( \frac{M_{J/\psi}}{m_c^2} \right)^4 \frac{|\langle \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} | P_i \rangle|^2}{\Gamma(J/\psi \rightarrow e^+ e^-)} \quad (1)$$

where  $K[J/\psi \gamma P_i]$  is the momentum of the pseudoscalar  $P_i$  in the  $J/\psi$  rest frame, and  $K[J/\psi \gamma P_i] = \frac{M_{J/\psi}}{2} (1 - \frac{M_{P_i}^2}{M_{J/\psi}^2})$ . Then the ratio of the branching fractions between  $J/\psi \rightarrow \gamma X(1835)$  and  $J/\psi \rightarrow \gamma \eta'$  is

$$\frac{Br(J/\psi \rightarrow \gamma X(1835))}{Br(J/\psi \rightarrow \gamma \eta')} = \frac{K[J/\psi \gamma X(1835)]^3 |\langle \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} | X(1835) \rangle|^2}{K[J/\psi \gamma \eta']^3 |\langle \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} | \eta' \rangle|^2} \quad (2)$$

It is straightforward to extend the anomaly dominance to the case of the  $\Upsilon(1S)$  radiative decay[24, 25]. Then we have

$$\frac{Br(\Upsilon(1S) \rightarrow \gamma X(1835))}{Br(\Upsilon(1S) \rightarrow \gamma \eta')} = \frac{K[\Upsilon(1S) \gamma X(1835)]^3 |\langle \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} | X(1835) \rangle|^2}{K[\Upsilon(1S) \gamma \eta']^3 |\langle \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} | \eta' \rangle|^2}. \quad (3)$$

From Eq.(2) and Eq.(3), we get

$$Br(\Upsilon(1S) \rightarrow \gamma X(1835)) = \frac{K[\Upsilon(1S) \gamma X(1835)]^3}{K[\Upsilon(1S) \gamma \eta']^3} \frac{K[J/\psi \gamma \eta']^3}{K[J/\psi \gamma X(1835)]^3} \frac{Br(\Upsilon(1S) \rightarrow \gamma \eta')}{Br(J/\psi \rightarrow \gamma \eta')} \times Br(J/\psi \rightarrow \gamma X(1835)). \quad (4)$$

For the  $\Upsilon(1S)$  radiative decay  $\Upsilon(1S) \rightarrow \gamma \eta'$ , only upper limit has been obtained, which is  $Br(\Upsilon(1S) \rightarrow \gamma \eta') < 1.6 \times 10^{-5}$  at 90% confidential level[26]. Using again the Particle Data Group's value  $Br(J/\psi \rightarrow \gamma \eta') = (4.31 \pm 0.30) \times 10^{-3}$ [26], and substituting it into Eq.(4), we obtain

$$Br(\Upsilon(1S) \rightarrow \gamma X(1835)) < 9.22 \times 10^{-3} Br(J/\psi \rightarrow \gamma X(1835)). \quad (5)$$

Thus, since BES has already determined  $Br(J/\psi \rightarrow \gamma X(1835)) Br(X(1835) \rightarrow p\bar{p}) = (7.0 \pm 0.4_{-0.8}^{+1.9}) \times 10^{-5}$ [2], and by eq.(5), we finally get a reasonable estimation:

$$Br(\Upsilon(1S) \rightarrow \gamma X(1835)) Br(X(1835) \rightarrow p\bar{p}) < 6.45 \times 10^{-7}. \quad (6)$$

This estimation is compatible with CLEO collaboration's result of  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))Br(X(1835) \rightarrow p\bar{p}) < 5 \times 10^{-7}$ . So it is not surprising that the CLEO collaboration don't see the signal of X(1835) in the  $\Upsilon(1S)$  radiative decay, and it doesn't mean that X(1835) seen by BES is an experimental artifact.

### B. $J/\psi \rightarrow \omega X(1835)$

In this subsection, we examine the branching fraction of  $J/\psi \rightarrow \omega X(1835)$ . Since  $Br(J/\psi \rightarrow \gamma X(1835))$  is rather larger[1, 2], one could expect  $Br(J/\psi \rightarrow \omega X(1835))$  may also be reasonably large too, or at least be visible at present stage. In this way the existence of X(1835) may be rechecked in the non-radiative decay channel of  $J/\psi$ . However, this is only a naive conjecture, and there is not yet any data on this branching fraction experimentally. So a theoretical estimation on  $Br(J/\psi \rightarrow \omega X(1835))$  is necessary. Our estimations in this subsection are still based on the baryonium picture discussed in the above.

Unlike the radiative decays  $J/\psi \rightarrow \gamma X(1835)$  and  $\Upsilon(1S) \rightarrow \gamma X(1835)$ , where the gluon component plays important role due to the  $U_A(1)$  anomaly. For the decay  $J/\psi \rightarrow \omega X(1835)$ , the processes to which the  $U_A(1)$  anomaly contributes are suppressed, and the baryonic component dominates this process, the same is true for  $J/\psi \rightarrow \omega \eta'$ . We think the decay process  $J/\psi \rightarrow \omega X(1835)$  proceeds via two steps as illustrated in Fig.1. In first step, the  $c\bar{c}$  pair annihilate into three gluons, followed by the materialization of each gluon into a pair of quark-antiquark, this process can be calculated from perturbative QCD to the lowest order. Also a pair of quark-antiquark are created from the vacuum, and this process can be described by the quark pair creation model (the  $^3P_0$  model). Then in the second step the quarks and the antiquarks combine to form  $\omega$  and X(1835). Here, the nonperturbative dynamics is included by the hadron's wave functions in the naive quark model.

The quark pair creation model which describes the process that a pair of quark-antiquark with quantum number  $J^{PC} = 0^{++}$  is created from vacuum was first proposed by Micu[27] in 1969. In the 1970s, this model was developed by Yaouanc et al. [28, 29, 30, 31] and applied to study hadron decays extensively. The  $^3P_0$  quark pair creation model has proven to be a successful mechanism for describing strong decay of light mesons [32, 33, 34]. It also has been shown that the  $^3P_0$  quark pair creation mechanism may play important role for some exclusive decay in the charmonium sector[35, 36, 37]. In the  $^3P_0$  model, the created quark

pairs with any color and any flavor can be generated anywhere in space, but only those whose color-flavor wave functions and spatial wave functions overlap with those of outgoing hadrons can make a contribution to the final decay width. The hamiltonian for creating a quark pair can be defined in the  ${}^3P_0$  model in terms of quark and antiquark creation operators  $b^+$  and  $d^+$  [34],

$$H_I = \sum_{i,j,\alpha,\beta,s,s'} \int d^3k g_I [\bar{u}(\mathbf{k}, s) v(-\mathbf{k}, s')] b_{\alpha,i}^+(\mathbf{k}, s) d_{\beta,j}^+(-\mathbf{k}, s') \delta_{\alpha\beta} \hat{C}_I \quad (7)$$

where  $\alpha(\beta)$  and  $i(j)$  are the flavor and color indices of the created quarks (antiquarks), and  $u(\mathbf{k}, s)$  and  $v(\mathbf{k}', s')$  are free Dirac spinors for quarks and antiquarks respectively.  $\hat{C}_I = \delta_{ij}$  is the color operator for  $q\bar{q}$  and  $g_I$  is the strength of the decay interaction, which is assumed as a constant in these processes. In the nonrelativistic limit,  $g_I$  can be related to  $\gamma$ , the strength of the conventional  ${}^3P_0$  model, by  $g_I = 2m_q\gamma$ [34]. In order to cancel the large uncertainty in  $g_I$  and the overall constant dependence, we will calculate the ratio  $\frac{\Gamma(J/\psi \rightarrow \omega X(1835))}{\Gamma(J/\psi \rightarrow \omega\eta')}$  in the following. The process  $J/\psi \rightarrow \omega\eta'$  is schematically shown in Fig.2, where the electromagnetic decay process is not shown. For the  $J/\psi$  decaying into hadrons, the ratio between the hadronic decay width and the electromagnetic decay width is about 5 [38], i.e,  $\frac{\Gamma(J/\psi \rightarrow ggg \rightarrow \text{hadrons})}{\Gamma(J/\psi \rightarrow \gamma \rightarrow \text{hadrons})} \simeq 5$ . Thus we can include the contribution of the electromagnetic decay to  $J/\psi \rightarrow \omega\eta'$  though the above ratio.

For the  $J/\psi \rightarrow PV$  decays, the parity transformation is conserved, and the transitional amplitude square is  $\sum_{\Lambda} |M(\Lambda)|^2 = (1 + \cos^2\theta)|A_1|^2$ , where  $\Lambda$  is the  $J/\psi$  helicity, which is taken as  $\Lambda = \pm 1$  if it is produced from  $e^+e^-$  annihilations,  $A_1$  is the helicity amplitude with vector meson helicity equaling to 1, and  $\theta$  is the polar angle of the outgoing meson. The decay width  $\Gamma = \frac{|\mathbf{P}|}{6\pi M_{J/\psi}^2} |A_1|^2$ , here  $\mathbf{P}$  is the momentum of out-going mesons.

1.  $J/\psi \rightarrow \omega X(1835) \rightarrow \omega p\bar{p}$

The color factors for the fig.1 are:

- color factor for fig1.(a),  $c_a = \frac{5}{54}$
- color factor for fig1.(b),  $c_b = \frac{5}{432}$  (to be negligible)

The amplitude can be obtained according to the standard Feynman rules with the quark pair creation hamiltonian included, which is expressed as followings:

$$T_{\Lambda,s}(J/\psi \rightarrow \omega X(1835)) = C_0 c_a \alpha_s^{3/2} \langle \phi_\omega \phi_X | \bar{u}(\mathbf{p}_1, s_1) \gamma_\mu v(\mathbf{q}_1, \bar{s}_1) \bar{u}(\mathbf{p}_2, s_2) \gamma_\nu v(\mathbf{q}_2, \bar{s}_2) \bar{u}(\mathbf{p}_3, s_3) \gamma_\rho v(\mathbf{q}_3, \bar{s}_3) \epsilon_\psi^{(\Lambda)\lambda} \times \frac{g_{\mu\lambda} g_{\nu\rho} + g_{\nu\lambda} g_{\mu\rho} + g_{\rho\lambda} g_{\mu\nu}}{k_1^2 k_2^2 k_3^2} g_I \bar{u}(\mathbf{p}_4, s_4) v(\mathbf{q}_4, \bar{s}_4) | \phi_{J/\psi} \rangle \quad (8)$$

where  $C_0$  is coupling constant for  $J/\psi \rightarrow ggg$ , and  $\alpha_s$  is the the strong coupling constant.  $k_i$  is the gluonic momentum, and the normalization of Dirac spinor is taken as  $\bar{u}u = -\bar{v}v = m/E$ .  $\phi_\omega, \phi_X$  and  $\phi_{J/\psi}$  represent the wave functions of  $\omega, X(1835)$  and  $J/\psi$  respectively. And the helicity amplitude is:

$$A_{\Lambda,s}(J/\psi \rightarrow \omega X(1835)) = \int \prod_{i=1,4} \frac{d^3 \mathbf{q}_i}{(2\pi)^3} \frac{d^3 \mathbf{p}_i}{(2\pi)^3} \frac{d^3 \mathbf{t}_1}{(2\pi)^3} \frac{d^3 \mathbf{t}_2}{(2\pi)^3} T_{\Lambda,s}(J/\psi \rightarrow \omega X(1835)) \times (2\pi)^3 \delta^3(\mathbf{p}_\omega - \mathbf{p}_1 - \mathbf{q}_4) (2\pi)^3 \delta^3(\mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4 - \mathbf{t}_1) (2\pi)^3 \delta^3(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 - \mathbf{t}_2) \times (2\pi)^3 \delta^3(\mathbf{t}_1 + \mathbf{t}_2 - \mathbf{p}_X) (2\pi)^3 \delta^3(\mathbf{p}_4 + \mathbf{q}_4) \quad (9)$$

Here  $\Lambda$  and  $s$  denote the helicity of  $J/\psi$  and  $\omega$  respectively.

## 2. $J/\psi \rightarrow \omega \eta'$

The color factors for the fig.2 are:

- color factor for fig2 (a):  $c'_a = \frac{5\sqrt{3}}{54}$ .
- color factor for fig2 (b):  $c'_b = \frac{5\sqrt{3}}{216}$  (negligible).

The corresponding helicity amplitude is

$$A_{\Lambda,s}(J/\psi \rightarrow \omega \eta') = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \frac{d^3 \mathbf{p}_2}{(2\pi)^3} \frac{d^3 \mathbf{q}_1}{(2\pi)^3} T_{\Lambda,s}(J/\psi \rightarrow \omega \eta') \times (2\pi)^3 \delta^3(\mathbf{q}_\omega - \mathbf{p}_1 + \mathbf{p}_2) (2\pi)^3 \delta^3(\mathbf{q}_{\eta'} - \mathbf{q}_1 - \mathbf{p}_2) \quad (10)$$

where  $T_{\Lambda,s}(J/\psi \rightarrow \omega \eta')$  is the followings,

$$T_{\Lambda,s}(J/\psi \rightarrow \omega \eta') = C_0 c'_a \alpha_s^{3/2} \langle \phi_\omega(q_\omega, s) \phi_{\eta'}(q_{\eta'}) | \bar{u}(p_1, s_1) \gamma_\mu \frac{1}{\not{p}_1 - \not{k}_1 - m} \gamma_\nu \frac{1}{\not{q}_1 - \not{k}_2 - m} \gamma_\rho v(q_1, \bar{s}_1) \times \epsilon_\psi^{(\Lambda)\lambda} \frac{g_{\mu\lambda} g_{\nu\rho} + g_{\nu\lambda} g_{\mu\rho} + g_{\rho\lambda} g_{\mu\nu}}{k_1^2 k_2^2 k_3^2} g_I \bar{u}(p_2, s_2) v(-p_2, \bar{s}_2) | \phi_{J/\psi} \rangle \quad (11)$$



For simplicity, we make use of the on-shell approximation, *i.e.*,  $\frac{1}{k_1^2 k_2^2} \rightarrow -2\pi^2 \delta(k_1^2) \delta(k_2^2)$ , and with the replacement:

$$\int \frac{d^4 k_1 d^4 k_2}{k_1^2 k_2^2 k_3^2} \rightarrow -\frac{\pi^2}{2} \int_0^{M_\psi} dk_1^0 \int_0^{M_\psi - k_1^0} dk_2^0 \int d\Omega_1 d\Omega_2 \frac{k_1^0 k_2^0}{M_\psi^2 - 2k_1^0 M_\psi - 2k_2^0 M_\psi + 2k_1 \cdot k_2} \quad (12)$$

### 3. Numerical results

The spin-flavor wave functions of the mesons  $\omega$  and  $\eta'$  are well-known in quark model, and the spatial wave function is taken as the simple harmonic oscillator wave function, *i.e.*,  $\phi(\mathbf{k}) = \frac{(2\pi)^{3/2}}{(\pi\beta^2)^{3/4}} e^{-\mathbf{k}^2/2\beta^2}$ . Since X(1835) is assumed as a baryonium with  $J^{PC} = 0^{-+}$ , it's spatial wave function is the product of the proton spatial wave function, antiproton spatial wave function and the relative spatial wave function between them. It's expressed as followings,

$$\phi_X = \phi_p(\mathbf{p}_\rho, \mathbf{p}_\lambda) \phi_{\bar{p}}(\mathbf{q}_\rho, \mathbf{q}_\lambda) \phi_{p\bar{p}}(\mathbf{t}_1 - \mathbf{t}_2) \quad (13)$$

where  $\phi_p(\mathbf{p}_\rho, \mathbf{p}_\lambda) = \frac{(2\pi)^{3/2}}{(\pi\beta^2)^{3/4}} e^{-(\mathbf{p}_\rho^2 + \mathbf{p}_\lambda^2)/2\beta^2}$ , and  $\phi_{p\bar{p}}(\mathbf{t}_1 - \mathbf{t}_2)$  is of the same formalism as the simple harmonic oscillator wave function. with  $\mathbf{p}_\rho = \frac{1}{\sqrt{6}}(\mathbf{p}_2 + \mathbf{p}_3 - 2\mathbf{p}_4)$ ,  $\mathbf{q}_\rho = \frac{1}{\sqrt{6}}(\mathbf{q}_1 + \mathbf{q}_2 - 2\mathbf{q}_3)$ ,  $\mathbf{p}_\lambda = \frac{1}{\sqrt{2}}(\mathbf{p}_2 - \mathbf{p}_3)$ ,  $\mathbf{q}_\lambda = \frac{1}{\sqrt{2}}(\mathbf{q}_1 - \mathbf{q}_2)$ ,  $\mathbf{t}_1 = \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4$  and  $\mathbf{t}_2 = \mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3$ . The spin-flavor wave function is the followings,

$$\begin{aligned} & \frac{1}{2\sqrt{2}} \{ [\chi_p^\rho(\uparrow)\phi_p^\rho + \chi_p^\lambda(\uparrow)\phi_p^\lambda] [\chi_{\bar{p}}^\rho(\downarrow)\phi_{\bar{p}}^\rho + \chi_{\bar{p}}^\lambda(\downarrow)\phi_{\bar{p}}^\lambda] \\ & - [\chi_p^\rho(\downarrow)\phi_p^\rho + \chi_p^\lambda(\downarrow)\phi_p^\lambda] [\chi_{\bar{p}}^\rho(\uparrow)\phi_{\bar{p}}^\rho + \chi_{\bar{p}}^\lambda(\uparrow)\phi_{\bar{p}}^\lambda] \} \end{aligned} \quad (14)$$

where  $\phi_p^\rho$  and  $\phi_p^\lambda$  are  $\rho$ -type and  $\lambda$ -type nucleon flavor wave function respectively, and similarly for  $\chi_{p/\bar{p}}^\rho$  and  $\chi_{p/\bar{p}}^\lambda$ .

There are four parameters to be determined in our calculation, *i.e.*, the quark mass  $m_u, m_d$ , the harmonic oscillation parameter  $\beta$  for hadrons and X(1835). The quark mass are taken as  $m_u = m_d = 0.31\text{GeV}$ . In most calculations in quark model, the harmonic oscillation parameter is fitted to the hadron decay width, which gives  $\beta = 0.4\text{GeV}$  [33, 34]. The harmonic oscillation parameter of X(1835) is determined by assuming that the radius of  $p\bar{p}$  is about  $1 \sim 2\text{fm}$ , which corresponds to the the parameter  $\beta_X = 0.15 \sim 0.30\text{GeV}$ . The ratio of  $\Gamma(J/\psi \rightarrow \omega X(1835))/\Gamma(J/\psi \rightarrow \omega\eta')$  is calculated in terms of different set of harmonic oscillation parameters  $\beta$  and  $\beta_X$  as listed in Table 1. It is clear to see that the ratio is very sensitive the parameter  $\beta$  and not sensitive to  $\beta_X$ .

TABLE I: Numerical results of  $\frac{\Gamma(J/\psi \rightarrow \omega X(1835))}{\Gamma(J/\psi \rightarrow \omega \eta')}$  corresponding to the different set of harmonic oscillation parameters  $\beta$  and  $\beta_X$ , with the contribution of the electromagnetic process included, where the quark mass are taken as  $m_u = m_d = 0.31\text{GeV}$ .

$\beta(\text{GeV})$	$\beta_X(\text{GeV})$				Average of $\frac{\Gamma(J/\psi \rightarrow \omega X)}{\Gamma(J/\psi \rightarrow \omega \eta')}$
	0.15	0.20	0.25	0.30	
0.36	6.2	5.9	5.0	4.1	$5.2 \pm 1.0$
0.40	0.15	0.13	0.12	0.10	$0.12 \pm 0.02$
0.46	0.006	0.006	0.005	0.005	$0.004 \pm 0.0008$
0.52	$5.9 \times 10^{-4}$	$5.6 \times 10^{-4}$	$4.8 \times 10^{-4}$	$4.0 \times 10^{-4}$	$(5.0 \pm 0.9) \times 10^{-4}$

As most quark model studies on the meson decays, we use the parameters  $m_u = m_d = 0.31, \beta = 0.4\text{GeV}$  as our favorable parameters. In our calculation, the uncertainties are from the parameter  $\beta_X$ , the ignored decay modes depressed by color factor and the accuracy of the numerical calculation. From our estimation, the uncertainty of the  $\beta_X$  within our setting range is about 20%. The contribution from the fig.1(b) and fig.2(b) are of the same order as that from fig.1(a) and fig.2(a) respectively. The color depressed decay modes will bring in uncertainty of about 6%, the uncertainty of the numerical evaluation is about 8%, then the total uncertainty is about 22%. Including these uncertainties, we predict the ratio  $\Gamma(J/\psi \rightarrow \omega \eta')/\Gamma(J/\psi \rightarrow \omega X) = 0.12 \pm 0.02$ . Using the PDG value  $Br(J/\psi \rightarrow \omega \eta') = (1.67 \pm 0.25) \times 10^{-4}$ , we predict that  $Br(J/\psi \rightarrow \omega X(1835)) = (2.00 \pm 0.35) \times 10^{-5}$ .

Comparing this result with  $Br(J/\psi \rightarrow \gamma X(1835)) \sim (0.5 - 2) \times 10^{-3}[1]$ , we see that the production rate of  $X(1835)$  in the process  $J/\psi \rightarrow \omega X(1835)$  is less than that in  $J/\psi \rightarrow \gamma X(1835)$  about two orders. Therefore, it is not surprising that the signal of  $J/\psi \rightarrow \omega X(1835)$  has not be seen by BES or other laboratories so far. In other words, the absence of the signal in the decay  $J/\psi \rightarrow \omega X(1835)$  at present can not be thought as an evidence against the existence of  $X(1835)$ .

Using BES's estimations of  $Br(X(1835) \rightarrow p\bar{p}) \sim (4 - 14)\%$  and  $Br(X(1835) \rightarrow \eta'\pi^+\pi^-) \sim 3Br(X(1835) \rightarrow p\bar{p})[1, 4]$ , we get further two useful estimations about the product branching fractions:

$$8.00 \times 10^{-7} < Br(J/\psi \rightarrow \omega X(1835))Br(X(1835) \rightarrow p\bar{p}) < 2.80 \times 10^{-6}, \quad (15)$$

$$2.40 \times 10^{-6} < Br(J/\psi \rightarrow \omega X(1835))Br(X(1835) \rightarrow \eta'\pi^+\pi^-) < 8.40 \times 10^{-6}. \quad (16)$$

Comparing them with the data[1]  $Br(J/\psi \rightarrow \gamma X)Br(X \rightarrow p\bar{p}) = (7.0 \pm 0.4_{-0.8}^{+1.9}) \times 10^{-5}$  and  $Br(J/\psi \rightarrow \gamma X(1835))Br(X(1835) \rightarrow \pi^+\pi^-\eta') = (2.2 \pm 0.4(stat) \pm 0.4(syst)) \times 10^{-4}$  respectively, we see also both  $Br(J/\psi \rightarrow \omega X(1835))Br(X(1835) \rightarrow p\bar{p})$  and  $Br(J/\psi \rightarrow \omega X(1835))Br(X(1835) \rightarrow \eta'\pi^+\pi^-)$  are also very small. So the signal of X(1835) is very difficult, if not impossible, to be observed in the process  $J/\psi \rightarrow \omega X(1835)$  with  $X(1835) \rightarrow p\bar{p}$  or  $X(1835) \rightarrow \eta'\pi^+\pi^-$ .

Finally we discuss the production of X(1835) in the process  $J/\psi \rightarrow \rho X(1835)$ . In this process the G-parity is not conserved, and it proceeds through a virtual photon  $c\bar{c} \rightarrow \gamma^*$ . Contributions from the isospin-violating part of QCD are supposedly very small. Furthermore the masses of  $\rho$  and  $\omega$  are approximately equal, so  $Br(J/\psi \rightarrow \rho X(1835)) < Br(J/\psi \rightarrow \omega X(1835))$  (the same holds true for  $J/\psi \rightarrow \omega\eta'$  and  $J/\psi \rightarrow \rho\eta'$ , that is  $Br(J/\psi \rightarrow \omega\eta') > Br(J/\psi \rightarrow \rho\eta')$ ). This means that we also can not see X(1835) in the process  $J/\psi \rightarrow \rho X(1835)$ .

After the above analysis, we conclude that comparing to  $Br(J/\psi \rightarrow \gamma X(1835))$ , the branching fractions of  $J/\psi \rightarrow VX(1835)$  with  $V$  being  $\omega$  or  $\rho$  are heavily suppressed due to its special structure. The search for the X(1835) in these decays seems impossible at the present stage.

### III. BARYONIUM NONET AND ITS PRODUCTION IN A MODEL INDEPENDENT WAY

The BES collaboration has observed not only the  $p\bar{p}$  enhancement[1, 2], but also the  $p\bar{\Lambda}$  enhancement[3]. These two states can belong to flavor **1-plet**, **8-plet**, **10-plet**,  **$\overline{10}$ -plet**, or **27-plet**. It seems that at least a baryonium nonet exists. Theoretically, we have predicted existence of such a baryonium nonet in Ref.[12]. The baryonium nonet was also suggested from the Fermi-Yang-Sakata model in Ref.[9]. The nonet can be pseudoscalar or vector multiplet[9, 12], and the corresponding weight diagram is shown in Fig.3 and Fig.4. The pseudoscalar and vector enhancement octet are respectively denoted by  $E_{P_i}$  and  $E_{V_i}$  ( $i = 1 \cdots 8$ ) as follows

$$\begin{aligned} E_{\pi^\pm} &= \frac{1}{\sqrt{2}}(E_{P_1} \mp iE_{P_2}), \quad E_{\pi^0} = E_{P_3}, \quad E_{K^\pm} = \frac{1}{\sqrt{2}}(E_{P_4} \mp iE_{P_5}) \\ E_{K^0} &= \frac{1}{\sqrt{2}}(E_{P_6} - iE_{P_7}), \quad E_{\bar{K}^0} = \frac{1}{\sqrt{2}}(E_{P_6} + iE_{P_7}), \quad E_{\eta_8} = E_{P_8} \end{aligned} \quad (17)$$

It is useful to add the singlet to the octet  $E_P$  by defining  $E_{\eta_1} = E_{P_0}$ , thereby creating the nonet  $E_P = (E_{P_0}, E_{P_i})$ . If the pseudoscalar glueball and radially exciting states are ignored, the physics states  $E_{\eta'}$  and  $E_\eta$  are mixing of  $E_{\eta_8}$  and  $E_{\eta_0}$  with the mixing angle  $\varphi_P$

$$E_{\eta_8} = \cos \varphi_P E_\eta + \sin \varphi_P E_{\eta'}, \quad E_{\eta_1} = -\sin \varphi_P E_\eta + \cos \varphi_P E_{\eta'} \quad (18)$$

Similarly for the vector enhancement nonet  $E_V = (E_{\omega_1}, E_{V_i})$ , the physics states  $E_\omega$  and  $E_\phi$  are mixing of  $E_{\omega_8}$  and  $E_{\omega_1}$  with the mixing angle  $\varphi_V$

$$E_{\omega_8} = \cos \varphi_V E_\phi + \sin \varphi_V E_\omega, \quad E_{\omega_1} = -\sin \varphi_V E_\phi + \cos \varphi_V E_\omega \quad (19)$$

We identify the  $p\bar{p}$  enhancement X(1835) as the states  $E_{\eta'}$ , while the  $p\bar{\Lambda}$  enhancement should be  $E_{K^+}$  or  $E_{K^{*+}}$ , and  $E_{K^{*+}}$  is favored over  $E_{K^+}$  from the analysis of Ref.[12].

We can consider flavor SU(3) breaking by choosing a nonet pointing to a fixed direction in SU(3) space particularly for the desired breaking. We will consider two types of SU(3) breaking, first SU(3) is broken due to  $m_s \neq m_u, m_d$  ( $m_u = m_d$  is assumed), the quark mass term is  $m_d(d\bar{d} + u\bar{u}) + m_s s\bar{s} = m_0 q\bar{q} + \sqrt{\frac{1}{3}}(m_d - m_s)\bar{q}\lambda_8 q$ , where  $q = (u, d, s)$  and  $m_0 = \frac{1}{3}(2m_d + m_s)$ . We can see this SU(3) breaking corresponds to a nonet  $\mathbf{M}$ , pointing to the 8th direction, *i.e.*,  $M^a = \delta^{a8}$ . Second, the electromagnetic effects violate SU(3) invariance, since the photon coupling to quarks is  $\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s = \frac{1}{2}\bar{q}\gamma_\mu(\lambda_3 + \frac{\lambda_8}{\sqrt{3}})q$ . This symmetry breaking effect corresponds to a nonet  $\mathbf{E}$ , given by  $E^a = \delta^{a3} + \sqrt{\frac{1}{3}}\delta^{a8}$ . In the following we consider the process of  $J/\psi \rightarrow E_P P$ , that means  $J/\psi$  decays into a pseudoscalar baryonium ( $E_\pi, E_K, E_\eta, E_{\eta'}$ ) and a pseudoscalar ( $\pi, K, \eta, \eta'$ ), the process of  $J/\psi \rightarrow E_P V$ , *i.e.*,  $J/\psi$  decays to a pseudoscalar baryonium ( $E_\pi, E_K, E_\eta, E_{\eta'}$ ) and a vector ( $\rho, K^*, \omega, \phi$ ) and the process of  $J/\psi \rightarrow E_V P$ , *i.e.*,  $J/\psi$  decays to a vector baryonium ( $E_\rho, E_{K^*}, E_\omega, E_\phi$ ) and a pseudoscalar ( $\pi, K, \eta, \eta'$ ) in a model independent way via SU(3) symmetry with the effects of electromagnetic and mass breaking of the SU(3) symmetry being considered[39, 40]. Since the phase space factor is proportional to the cube of the final three-momentum, we define the reduced branching fraction  $\widetilde{Br}(J/\psi \rightarrow E_P P) = Br(J/\psi \rightarrow E_P P)/P_P^3$ , here  $P_P$  is the momentum of the pseudoscalar P in the  $J/\psi$  rest frame, and the reduced branching fractions  $\widetilde{Br}(J/\psi \rightarrow E_P V)$ ,  $\widetilde{Br}(J/\psi \rightarrow E_V P)$  are defined in the same way.

### A. $J/\psi \rightarrow E_P P$

These processes occur completely due to the SU(3) breaking effects, we can constructed charge conjugation invariant and SU(3) invariant effective lagrangian involving the symmetry breaking nonet  $\mathbf{E}$  or  $\mathbf{M}$ , which may be written as followings,

$$\mathcal{L}_{eff} = f^{abc} \Psi^\mu E_{P_a} \overset{\leftrightarrow}{\partial}_\mu P_b (g_M M^c + g_E E^c) \quad (20)$$

Here the new parameter  $g_M$  and  $g_E$  parametrize the SU(3) breaking effects. From Eq.(19) and Eq.(22) we can obtain the following reduced branching fractions

$$\begin{aligned} \widetilde{Br}(J/\psi \rightarrow E_{\pi^+} \pi^-) &= \widetilde{Br}(J/\psi \rightarrow E_{\pi^-} \pi^+) = |g_E|^2 \\ \widetilde{Br}(J/\psi \rightarrow E_{K^+} K^-) &= \widetilde{Br}(J/\psi \rightarrow E_{K^-} K^+) = \left| \frac{\sqrt{3}}{2} g_M + g_E \right|^2 \\ \widetilde{Br}(J/\psi \rightarrow E_{K^0} \overline{K}^0) &= \widetilde{Br}(J/\psi \rightarrow E_{\overline{K}^0} K^0) = \frac{3}{4} |g_M|^2 \\ \widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \pi^0) &= \widetilde{Br}(J/\psi \rightarrow E_\eta \pi^0) = \widetilde{Br}(J/\psi \rightarrow E_{\eta'} \pi^0) = 0 \end{aligned} \quad (21)$$

The last formula means that the process  $J/\psi \rightarrow \pi^0 X(1835)$ ,  $X(1835) \rightarrow p\bar{p}$  should be forbidden, which is indeed not observed [2], and it is forbidden because of C parity.

### B. $J/\psi \rightarrow E_P V$

Following the same way as in the discussion of  $J/\psi \rightarrow E_P P$ , we construct the charge conjugation invariant, SU(3) invariant effective lagrangian including the symmetry breaking nonet  $\mathbf{E}$  and  $\mathbf{M}$ , which may be written as follows:

$$\begin{aligned} \mathcal{L}_{eff} &= \varepsilon_{\mu\nu\alpha\beta} F_\Psi^{\mu\nu} \{ g_8 F_{V_a}^{\alpha\beta} E_{P_a} + g_1 F_{\omega_1}^{\alpha\beta} E_{\eta_1} + [g_{M,88} d^{abc} F_{V_a}^{\alpha\beta} E_{P_b} M^c + \sqrt{\frac{2}{3}} g_{M,18} F_{V_a}^{\alpha\beta} M^a E_{\eta_1} \\ &+ \sqrt{\frac{2}{3}} g_{M,81} F_{\omega_1}^{\alpha\beta} M^a E_{P_a}] + [g_{E,88} d^{abc} F_{V_a}^{\alpha\beta} E_{P_b} E^c + \sqrt{\frac{2}{3}} g_{E,18} F_{V_a}^{\alpha\beta} E^a E_{\eta_1} + \sqrt{\frac{2}{3}} g_{E,81} F_{\omega_1}^{\alpha\beta} E^a E_{P_a}] \} \end{aligned} \quad (22)$$

Where  $F_\Psi^{\mu\nu}$  is the strength of the  $J/\psi$  field with  $F_\Psi^{\mu\nu} = \partial^\mu \Psi^\nu - \partial^\nu \Psi^\mu$ , and  $F_{\omega_1}^{\alpha\beta}$ ,  $F_{V_a}^{\alpha\beta}$  are respectively the field strength of the vector field  $\omega_1$  and  $V_a$ . We assume nonet symmetry holds true within a reasonable approximation, which relate the octet to the singlet, then the relations  $g_8 = g_1 \equiv g$ , then we have the relations  $g_{M,88} = g_{M,81} = g_{M,18} \equiv g'_M$  and  $g_{E,88} = g_{E,81} = g_{E,18} \equiv g'_E$ . We take the parameters  $g$ ,  $g_M$  and  $g_E$  to be small and calculate

SU(3) breaking to first order in these parameters. From Eq.(17), Eq.(18), the  $\omega - \phi$  "ideal" mixing and the lagrangian Eq.(22) we get the following reduced branching fractions:

$$\begin{aligned}
\widetilde{Br}(J/\psi \rightarrow E_{\pi^+ \rho^-}) &= \widetilde{Br}(J/\psi \rightarrow E_{\pi^- \rho^+}) = \widetilde{Br}(J/\psi \rightarrow E_{\pi^0 \rho^0}) = |g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 \\
\widetilde{Br}(J/\psi \rightarrow E_{K^+ K^* -}) &= \widetilde{Br}(J/\psi \rightarrow E_{K^- K^* +}) = |g - \frac{1}{2\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 \\
\widetilde{Br}(J/\psi \rightarrow E_{K^0 \bar{K}^{*0}}) &= \widetilde{Br}(J/\psi \rightarrow E_{\bar{K}^0 K^*0}) = |g - \frac{1}{2\sqrt{3}}g'_M - \frac{2}{3}g'_E|^2 \\
\widetilde{Br}(J/\psi \rightarrow E_\eta \phi) &= |g - \frac{2}{\sqrt{3}}g'_M - \frac{2}{3}g'_E|^2 (\sqrt{\frac{2}{3}} \cos \varphi_P + \frac{1}{\sqrt{3}} \sin \varphi_P)^2 \\
\widetilde{Br}(J/\psi \rightarrow E_\eta \omega) &= |g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 (\sqrt{\frac{1}{3}} \cos \varphi_P - \sqrt{\frac{2}{3}} \sin \varphi_P)^2 \\
\widetilde{Br}(J/\psi \rightarrow E_\eta \rho^0) &= |g'_E|^2 (\sqrt{\frac{1}{3}} \cos \varphi_P - \sqrt{\frac{2}{3}} \sin \varphi_P)^2 \\
\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \phi) &= |g - \frac{2}{\sqrt{3}}g'_M - \frac{2}{3}g'_E|^2 (\sqrt{\frac{1}{3}} \cos \varphi_P - \sqrt{\frac{2}{3}} \sin \varphi_P)^2 \\
\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \omega) &= |g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 (\sqrt{\frac{1}{3}} \sin \varphi_P + \sqrt{\frac{2}{3}} \cos \varphi_P)^2 \\
\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \rho^0) &= |g'_E|^2 (\frac{1}{\sqrt{3}} \sin \varphi_P + \sqrt{\frac{2}{3}} \cos \varphi_P)^2 \\
\widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \phi) &= 0 \\
\widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \omega) &= |g'_E|^2
\end{aligned} \tag{23}$$

From the above reduced branching fractions, the following relations can be obtained

$$\begin{aligned}
\frac{\widetilde{Br}(J/\psi \rightarrow E_\eta \omega)}{\widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \rho^0)} &= \frac{\widetilde{Br}(J/\psi \rightarrow E_\eta \rho^0)}{\widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \omega)} = (\sqrt{\frac{1}{3}} \cos \varphi_P - \sqrt{\frac{2}{3}} \sin \varphi_P)^2 \\
\frac{\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \omega)}{\widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \rho^0)} &= \frac{\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \rho^0)}{\widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \omega)} = (\sqrt{\frac{1}{3}} \sin \varphi_P + \sqrt{\frac{2}{3}} \cos \varphi_P)^2
\end{aligned} \tag{24}$$

Both because  $Br(J/\psi \rightarrow \omega X(1835))$  is heavily suppressed (please see the previous section of this work) and because there are not yet any experimental reports on it, we take  $\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \omega) = |g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 (\sqrt{\frac{1}{3}} \sin \varphi_P + \sqrt{\frac{2}{3}} \cos \varphi_P)^2 \approx 0$ , this implies  $|g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 \approx 0$  or  $(\sqrt{\frac{1}{3}} \sin \varphi_P + \sqrt{\frac{2}{3}} \cos \varphi_P)^2 \approx 0$ . If  $|g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 \approx 0$ , we can see  $\widetilde{Br}(J/\psi \rightarrow E_{\pi^+ \rho^-}) = \widetilde{Br}(J/\psi \rightarrow E_{\pi^- \rho^+}) = \widetilde{Br}(J/\psi \rightarrow E_{\pi^0} \rho^0) \approx 0$ ; However if  $(\sqrt{\frac{1}{3}} \sin \varphi_P + \sqrt{\frac{2}{3}} \cos \varphi_P)^2 \approx 0$ , it indicates  $\widetilde{Br}(J/\psi \rightarrow E_{\eta'} \rho^0) \approx 0$ , and we can not observe X(1835) in the process  $J/\psi \rightarrow \rho^0 X(1835)$ , with  $X(1835) \rightarrow p\bar{p}$  or  $X(1835) \rightarrow \eta' \pi^+ \pi^-$ . This conclusion is consistent with

the result  $Br(J/\psi \rightarrow \rho X(1835)) < Br(J/\psi \rightarrow \omega X(1835))$  which has been obtained in the Sec.II.B.

### C. $J/\psi \rightarrow E_V P$

Similar to the above two cases, the effective lagrangian responsible for the decay is

$$\begin{aligned} \mathcal{L}_{eff} = & \varepsilon_{\mu\nu\alpha\beta} F_{\Psi}^{\mu\nu} \{g_8'' F_{E_{V_a}}^{\alpha\beta} P_a + g_1'' F_{E_{\omega_1}}^{\alpha\beta} P_{\eta_1} + [g_{M,88}'' d^{abc} F_{E_{V_a}}^{\alpha\beta} P_b M^c + \sqrt{\frac{2}{3}} g_{M,81}'' F_{E_{V_a}}^{\alpha\beta} M^a P_{\eta_1} \\ & + \sqrt{\frac{2}{3}} g_{M,18}'' F_{E_{\omega_1}}^{\alpha\beta} M^a P_a] + [g_{E,88}'' d^{abc} F_{E_{V_a}}^{\alpha\beta} P_b E^c + \sqrt{\frac{2}{3}} g_{E,81}'' F_{E_{V_a}}^{\alpha\beta} E^a P_{\eta_1} + \sqrt{\frac{2}{3}} g_{E,18}'' F_{E_{\omega_1}}^{\alpha\beta} E^a P_a]\} \end{aligned} \quad (25)$$

under the nonet symmetry, the relations  $g_8'' = g_1'' \equiv g''$ ,  $g_{M,88}'' = g_{M,81}'' = g_{M,18}'' \equiv g_M''$  and  $g_{E,88}'' = g_{E,81}'' = g_{E,18}'' \equiv g_E''$  hold. From Eq.(19) and the above lagrangian Eq.(25) we can obtain the following reduced branching fractions:

$$\begin{aligned} \widetilde{Br}(J/\psi \rightarrow \pi^+ E_{\rho^-}) &= \widetilde{Br}(J/\psi \rightarrow \pi^- E_{\rho^+}) = \widetilde{Br}(J/\psi \rightarrow \pi^0 E_{\rho^0}) = |g'' + \frac{1}{\sqrt{3}} g_M'' + \frac{1}{3} g_E''|^2 \\ \widetilde{Br}(J/\psi \rightarrow K^+ E_{K^{*-}}) &= \widetilde{Br}(J/\psi \rightarrow K^- E_{K^{*+}}) = |g'' - \frac{1}{2\sqrt{3}} g_M'' + \frac{1}{3} g_E''|^2 \\ \widetilde{Br}(J/\psi \rightarrow K^0 E_{\overline{K}^{*0}}) &= \widetilde{Br}(J/\psi \rightarrow \overline{K}^0 E_{K^{*0}}) = |g'' - \frac{1}{2\sqrt{3}} g_M'' - \frac{2}{3} g_E''|^2 \\ \widetilde{Br}(J/\psi \rightarrow \eta E_{\phi}) &= |g'' \cos(\theta_P - \varphi_V) - (g_M'' + \frac{1}{\sqrt{3}} g_E'') [\frac{1}{\sqrt{3}} \cos \theta_P \cos \varphi_V + \sqrt{\frac{2}{3}} \sin(\theta_P + \varphi_V)]|^2 \\ \widetilde{Br}(J/\psi \rightarrow \eta E_{\omega}) &= |g'' \sin(\varphi_V - \theta_P) + (g_M'' + \frac{1}{\sqrt{3}} g_E'') [-\frac{1}{\sqrt{3}} \cos \theta_P \sin \varphi_V + \sqrt{\frac{2}{3}} \cos(\theta_P + \varphi_V)]|^2 \\ \widetilde{Br}(J/\psi \rightarrow \eta E_{\rho^0}) &= |g_E''|^2 (\frac{1}{\sqrt{3}} \cos \theta_P - \sqrt{\frac{2}{3}} \sin \theta_P)^2 \\ \widetilde{Br}(J/\psi \rightarrow \eta' E_{\phi}) &= | -g'' \sin(\varphi_V - \theta_P) + (g_M'' + \frac{1}{\sqrt{3}} g_E'') [-\frac{1}{\sqrt{3}} \sin \theta_P \cos \varphi_V + \sqrt{\frac{2}{3}} \cos(\theta_P + \varphi_V)]|^2 \\ \widetilde{Br}(J/\psi \rightarrow \eta' E_{\omega}) &= |g'' \cos(\varphi_V - \theta_P) + (g_M'' + \frac{1}{\sqrt{3}} g_E'') [-\frac{1}{\sqrt{3}} \sin \theta_P \sin \varphi_V + \sqrt{\frac{2}{3}} \sin(\theta_P + \varphi_V)]|^2 \\ \widetilde{Br}(J/\psi \rightarrow \eta' E_{\rho^0}) &= |g_E''|^2 (\frac{1}{\sqrt{3}} \sin \theta_P + \sqrt{\frac{2}{3}} \cos \theta_P)^2 \\ \widetilde{Br}(J/\psi \rightarrow \pi^0 E_{\phi}) &= |g_E''|^2 (\frac{1}{\sqrt{3}} \cos \varphi_V - \sqrt{\frac{2}{3}} \sin \varphi_V)^2 \\ \widetilde{Br}(J/\psi \rightarrow \pi^0 E_{\omega}) &= |g_E''|^2 (\frac{1}{\sqrt{3}} \sin \varphi_V + \sqrt{\frac{2}{3}} \cos \varphi_V)^2 \end{aligned} \quad (26)$$

here  $\theta_P$  is the mixing angle of  $\eta$  and  $\eta'$  with  $\theta_P \approx -16.9^\circ \pm 1.7^\circ$ [41], and we can find the relation

$$\frac{\widetilde{Br}(J/\psi \rightarrow \pi^0 E_\phi)}{\widetilde{Br}(J/\psi \rightarrow \pi^0 E_\omega)} = \left( \frac{\frac{1}{\sqrt{3}} \cos \varphi_V - \sqrt{\frac{2}{3}} \sin \varphi_V}{\frac{1}{\sqrt{3}} \sin \varphi_V + \sqrt{\frac{2}{3}} \cos \varphi_V} \right)^2 = \left( \frac{1 - \sqrt{2} \tan \varphi_V}{\tan \varphi_V + \sqrt{2}} \right)^2 \quad (27)$$

In summary, the other exotic states in the weight diagram are expected to be observed in future, and these relations between the branching fractions can be served as a guide to the experimental search for these exotic states.

#### IV. CONCLUSION AND DISCUSSION

In conclusion, the processes  $\Upsilon \rightarrow \gamma X(1835)$  and  $J/\psi \rightarrow \omega X(1835)$  have been investigated. Considering the large coupling of  $X(1835)$  with  $p\bar{p}$  and  $\eta'\pi^+\pi^-$ , we propose that  $X(1835)$  is a baryonium with sizable gluon content, and mainly belongs to a  $SU(3)$  flavor singlet. In this scheme, we can finely understand the observation data both in the process  $J/\psi \rightarrow \gamma X(1835)$ ,  $X(1835) \rightarrow p\bar{p}$ [2] and in  $J/\psi \rightarrow \gamma X(1835)$ ,  $X(1835) \rightarrow \eta'\pi^+\pi^-$ [1]. We estimate that in the  $\Upsilon(1S)$  radiative decay the product branching fraction  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))Br(X(1835) \rightarrow p\bar{p}) < 6.45 \times 10^{-7}$ , which is compatible with the CLEO's experimental upper limit  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))Br(X(1835) \rightarrow p\bar{p}) < 5 \times 10^{-7}$ [5]. Noting that in these processes the gluon component plays important role due to the  $U_A(1)$  anomaly. Thus, we find out that the drastic smallness of  $Br(\Upsilon(1S) \rightarrow \gamma X(1835))$  is caused the special nature of  $X(1835)$ , and it does not contradict with the experimental evidence of  $X(1835)$  revealed in the process  $J/\psi \rightarrow \gamma X(1835)$  by BES.

In our baryonium scheme of  $X(1835)$ , we found out also that  $Br(J/\psi \rightarrow \omega X(1835)) = (2.00 \pm 0.35) \times 10^{-5}$ ,  $8.00 \times 10^{-7} < Br(J/\psi \rightarrow \omega X(1835))Br(X(1835) \rightarrow p\bar{p}) < 2.80 \times 10^{-6}$ , and  $2.40 \times 10^{-6} < Br(J/\psi \rightarrow \omega X(1835))Br(X(1835) \rightarrow \eta'\pi^+\pi^-) < 8.40 \times 10^{-6}$ . The production of  $X(1835)$  in the process  $J/\psi \rightarrow \omega X(1835)$  are heavily suppressed. We also point out that  $Br(J/\psi \rightarrow \rho X(1835)) < Br(J/\psi \rightarrow \omega X(1835))$ , so it is very difficult to observe  $X(1835)$  in the process  $J/\psi \rightarrow V X(1835)$ (here  $V$  is  $\omega$  or  $\rho$ ) with  $X(1835) \rightarrow p\bar{p}$  or  $X(1835) \rightarrow \eta'\pi^+\pi^-$ , and the  $J/\psi$  radiative decay is the most suitable place for searching  $X(1835)$ . We address that the baryonic component dominates the decay  $J/\psi \rightarrow V X(1835)$  with  $V$  is  $\omega$  or  $\rho$ , since the  $U_A(1)$  anomaly contributions are suppressed in these processes. The experimental check for the above results are expected.



Finally we conjecture the existence of baryonium nonet, which is supported in Ref.[12] and Ref.[9], and the nonet can be pseudoscalar( $E_{P_i}$ ) or vector( $E_{V_i}$ ). The  $p\bar{p}$  enhancement X(1835) is identified as  $E_{\eta'}$ , and the  $p\bar{\Lambda}$  enhancement[3] can be  $E_{K^{*+}}$  or  $E_{K^+}$ . We derive the reduced branching fractions of  $J/\psi \rightarrow E_P P$ ,  $J/\psi \rightarrow E_P V$  and  $J/\psi \rightarrow E_V P$  in a model independent way basing on SU(3) symmetry with the symmetry breaking included. The relations between the branching fractions can be served as a guide to the experimental search for these exotic states.

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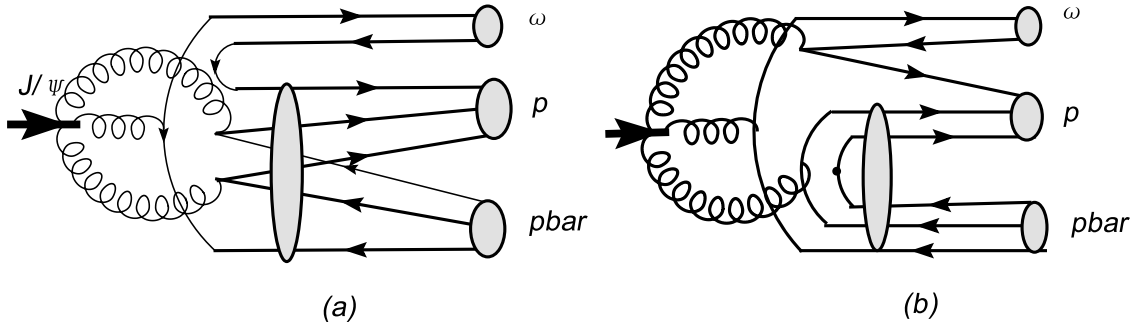


FIG. 1: A schematic diagram for  $J/\psi \rightarrow \omega X(1835) \rightarrow p\bar{p}$  in a mechanism with a  $^3P_0$  quark pair created in two configurations.

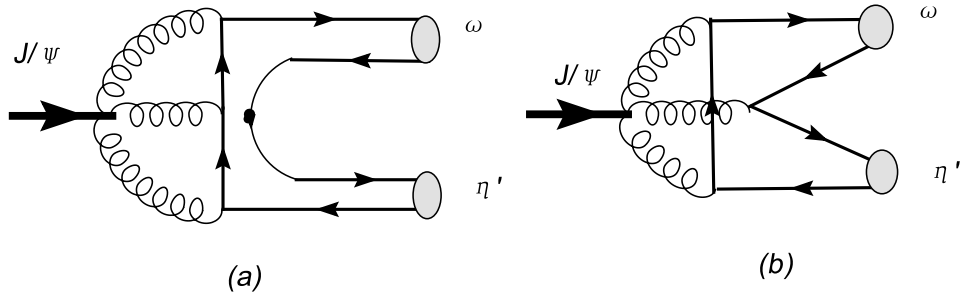


FIG. 2: A schematic diagram for  $J/\psi \rightarrow \omega\eta'$  decay mechanism (a) a  $^3P_0$  quark pair created;(b) no  $^3P_0$  quark pair created.

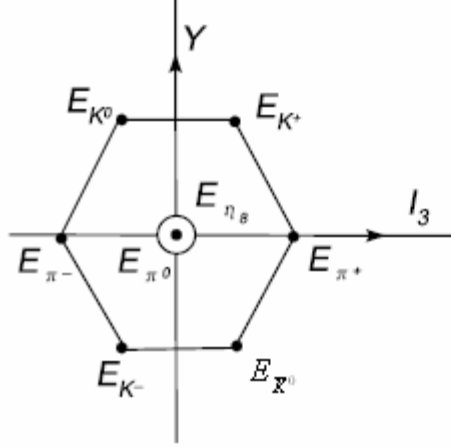


FIG. 3: The weight diagram for the pseudoscalar baryonium octet

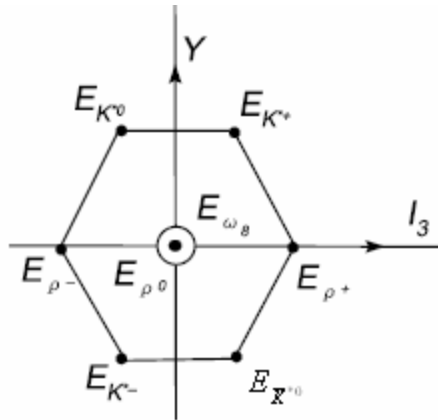


FIG. 4: The weight diagram for the vector baryonium octet