

## STRUCTURE OF THE $\sigma$ -MESON AND DIAMAGNETISM OF THE NUCLEON \*

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The structure of the  $\sigma$  meson and the diamagnetism of the nucleon are shown to be topics which are closely related to each other. Arguments are found that the  $\sigma$  meson couples to two photons via its non-strange  $q\bar{q}$  structure component. This ansatz leads to a quantitative explanation of the  $t$ -channel component of the difference of electromagnetic polarizabilities,  $(\alpha - \beta)^t$ , containing the diamagnetism of the nucleon. The prediction is  $(\alpha - \beta)_{p,n}^t = (5\alpha_E g_{\pi NN}) / (6\pi^2 m_\sigma^2 f_\pi) = 15.3$  in units of  $10^{-4} \text{fm}^3$  to be compared with the experimental values  $(\alpha - \beta)_p^t = 15.1 \pm 1.3$  for the proton and  $(\alpha - \beta)_n^t = 14.8 \pm 2.7$  for the neutron. The equivalent approach to exploit the  $\pi\pi$  structure component of the  $\sigma$  meson via the BEFT sum rule leads to  $(\alpha - \beta)_{p,n}^t = 14 \pm 2$ , what also is in agreement with the experimental results.

### 1. Introduction

Diamagnetism is one of the dominant properties of the nucleon, contrasting with the fact that this phenomenon still is barely understood. A clearcut information is provided by dispersion theory<sup>1,2,3,4</sup> where Compton scattering is described by six invariant amplitudes  $A_i(s, t)$ . The amplitudes  $A_i(s, t)$  are

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analytic functions in the two complex planes  $s$  and  $t$  and may be calculated from the singularities contained in these planes. The  $s$ -plane singularities are given by the meson-photoproduction cross sections contributing to the total photoabsorption cross section. The  $t$ -channel singularities were identified in the late 1950th and the beginning of the 1960th by Low, Jacob and Mathews (LJM<sup>5</sup>) and Hearn and Leader (HL<sup>6</sup>). These singularities reflect the degrees of freedom (d.o.f.) of the nucleon and we find it convenient to introduce the terms  $s$ -channel d.o.f and  $t$ -channel d.o.f. Translated into modern language, LJM<sup>5</sup> argued that instead of an excitation of the pion and constituent-quark structures of the nucleon, i.e. the  $s$ -channel d.o.f., the production of a  $\pi^0$  meson in the intermediate state may lead to Compton scattering. This  $\pi^0$ -pole contribution is now generally accepted as part of the backward spin-polarizability  $\gamma_\pi$ . On the other hand the scalar-isoscalar  $\pi\pi$   $t$ -channel introduced by HL<sup>6</sup> making a large contribution to  $(\alpha - \beta)$  is frequently ignored or insufficiently represented in theoretical approaches (see Ref.<sup>4</sup>).

## 2. Experimental status of electromagnetic polarizabilities

According to our recent analysis<sup>7,4</sup> the experimental polarizabilities may be summarized in the form given in Table 1. The quantities  $\alpha_p, \beta_p, \alpha_n, \beta_n$

Table 1. Summary on electromagnetic polarizabilities in units of  $10^{-4}\text{fm}^3$

1		proton	neutron
2	BL sum rule	$(\alpha + \beta)_p = 13.9 \pm 0.3$	$(\alpha + \beta)_n = 15.2 \pm 0.5$
3	Compton scattering	$(\alpha - \beta)_p = 10.1 \pm 0.9$	$(\alpha - \beta)_n = 9.8 \pm 2.5$
4	BEFT sum rule	$(\alpha - \beta)_p^s = -5.0 \pm 1.0$	$(\alpha - \beta)_n^s = -5.0 \pm 1.0$
5	line 2 – line 3	$(\alpha - \beta)_p^t = 15.1 \pm 1.3$	$(\alpha - \beta)_n^t = 14.8 \pm 2.7$
6	experimental	$\alpha_p = 12.0 \pm 0.6$	$\alpha_n = 12.5 \pm 1.7$
7	$s$ -channel only	$\alpha_p^s = 4.5 \pm 0.5$	$\alpha_n^s = 5.1 \pm 0.6$
8	$t$ -channel only	$\alpha_p^t = 7.5 \pm 0.8$	$\alpha_n^t = 7.4 \pm 1.8$
9	experimental	$\beta_p = 1.9 \mp 0.6$	$\beta_n = 2.7 \mp 1.8$
10	$s$ -channel only	$\beta_p^s = 9.5 \pm 0.5$	$\beta_n^s = 10.1 \pm 0.6$
11	$t$ -channel only	$\beta_p^t = -7.6 \pm 0.8$	$\beta_n^t = -7.4 \pm 1.9$

are the experimental electric and magnetic polarizabilities for the proton and neutron, respectively. The quantities with an upper label  $s$  are the corresponding electric and magnetic polarizabilities where only the  $s$ -channel d.o.f. are included. These latter quantities have been obtained by making use of the fact that  $(\alpha + \beta)$ , when calculated from forward-angle dispersion

theory as given by the Baldin or Baldin-Lapidus (BL) sum rule

$$(\alpha + \beta) = \frac{1}{2\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_{\text{tot}}(\omega)}{\omega^2} d\omega, \quad (1)$$

has no  $t$ -channel contribution, i.e.  $(\alpha + \beta) = (\alpha + \beta)^s$ , and by using the estimate  $(\alpha - \beta)_{p,n}^s = -5.0 \pm 1.0$  obtained from the  $s$ -channel part of the BEFT sum rule

$$(\alpha - \beta)^s = \frac{1}{2\pi^2} \int_{m_\pi + \frac{m_\pi^2}{2M}}^{\infty} \sqrt{1 + \frac{2\omega}{M}} [\sigma(\omega, E1, \dots) - \sigma(\omega, M1, \dots)] \frac{d\omega}{\omega^2} \quad (2)$$

both for the proton and the neutron (see Ref.<sup>4</sup>), where  $M$  is the nucleon mass. The numbers in line 5 of Table 1 are the  $t$ -channel contributions to  $(\alpha - \beta)$  obtained from the experimental values  $(\alpha - \beta)_p = 10.1 \pm 0.9$  and  $(\alpha - \beta)_n = 9.8 \pm 2.5$  and the estimate for  $(\alpha - \beta)_{p,n}^s$ . We see that the experimental values for  $\alpha$  are much larger than the  $s$ -channel contributions alone, whereas for the magnetic polarizabilities the opposite is true. For the magnetic polarizability it makes sense to identify the large difference between the experimental value and the  $s$ -channel contribution with the diamagnetic polarizability. This difference is filled up by  $\beta^t$  which, therefore, may be considered as the diamagnetic polarizability. It is important to notice that this definition of the diamagnetic polarizability has nothing in common with the ‘‘classical’’ diamagnetic term given e.g. in Eq. (12) of Ref.<sup>4</sup>. This latter term makes use of  $s$ -channel degrees of freedom only and, therefore, does not describe the physical origin of the diamagnetism.

### 3. The $\sigma$ -pole and the BEFT sum rule

The  $\sigma$  meson having quantum numbers  $I = 0$  and  $J = 0$  may be considered as a quasi-stable  $1/\sqrt{2}(|u\bar{u}\rangle + |d\bar{d}\rangle)$   $1^3P_0$  state in a confining potential which is coupled to a di-pion state in the continuum  $1/\sqrt{3}(|\pi^+\pi^-\rangle - |\pi^0\pi^0\rangle + |\pi^-\pi^+\rangle)$ , showing up as a relative  $S$ -wave of the two pions. By exploiting the non-strange  $q\bar{q}$  structure component we are led to the following relations for the  $\sigma$ -pole

$$F_{\sigma\gamma\gamma} = |M(\sigma \rightarrow 2\gamma)| = \frac{\alpha_e}{\pi f_\pi} N_c \left[ \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = \frac{5}{3} \frac{\alpha_e}{\pi f_\pi} \quad (3)$$

$$(\alpha - \beta)_{p,n}^t = \frac{g_{\sigma NN} F_{\sigma\gamma\gamma}}{2\pi m_\sigma^2} = \frac{5\alpha_e g_{\pi NN}}{6\pi^2 m_\sigma^2 f_\pi} = 15.3 \times 10^{-4} \text{fm}^3 \quad (4)$$

where  $\alpha_e = 1/137.04$ ,  $N_c = 3$ ,  $m_\sigma = 665$  MeV,  $f_\pi = (92.42 \pm 0.26)$  MeV and  $g_{\sigma NN} = g_{\pi NN} = 13.169 \pm 0.057$  Ref.<sup>8</sup>. The  $\pi\pi$  structure component

leads to the  $t$ -channel part of the BEFT sum rule in the following form

$$(\alpha - \beta)^t = \frac{1}{16\pi^2} \int_{4m_\pi^2}^{\infty} \frac{dt}{t^2} \frac{16}{4M^2 - t} \left( \frac{t - 4m_\pi^2}{t} \right)^{1/2} \times \left[ f_+^0(t) F_0^{0*}(t) - \left( M^2 - \frac{t}{4} \right) \left( \frac{t}{4} - m_\pi^2 \right) f_+^2(t) F_0^{2*}(t) \right] \quad (5)$$

where  $f_+^{J=0,2}(t)$  is the partial wave amplitude of the process  $N\bar{N} \rightarrow \pi\pi$  and  $F_{I=0}^{J=0,2}(t)$  the partial wave amplitude of the process  $\pi\pi \rightarrow \gamma\gamma$ . The predictions of the BEFT sum rule are  $(\alpha - \beta)_{p,n}^t = +(14 \pm 2)$  (Levchuk et al., see Ref.<sup>4</sup>),  $(\alpha - \beta)_p^t = +16.5$  (Drechsel et al.<sup>2</sup>) with the arithmetic average  $(\alpha - \beta)_{p,n}^t = +15.3 \pm 1.3$ .

Table 2 summarizes the results obtained for  $(\alpha - \beta)_{p,n}^t$ . It is remarkable

Table 2. Difference of electromagnetic polarizabilities  $(\alpha - \beta)_{p,n}^t$  in units of  $10^{-4}\text{fm}^3$

	$(\alpha - \beta)_p^t$	$(\alpha - \beta)_n^t$
experiment	$15.1 \pm 1.3$	$14.8 \pm 2.7$
$\sigma$ -pole	15.3	15.3
BEFT sum rule	$15.3 \pm 1.3$	$15.3 \pm 1.3$

that the experimental values are in agreement with the prediction of the  $\sigma$ -pole as well as the prediction of the BEFT sum rule. This leads to the tentative conclusion that both approaches are equivalent and to a confirmation of the expression given in (3) for  $F_{\sigma\gamma\gamma}$ , treating the two-photon coupling of the  $\sigma$ -meson analogous to the  $\pi^0$ -meson case.

## References

1. A.I. L'vov, V.A. Petrun'kin, M. Schumacher, *Phys. Rev.* **C55**, 359 (1997)
2. D. Drechsel, B. Pasquini, M. Vanderhaeghen, *Phys. Rep.* **378**, 99 (2003)
3. F. Wissmann, *Springer Tracts in Modern Physics*, **200**, 1 (2004)
4. M. Schumacher, *Progress in Particle and Nuclear Physics* **55**, 567 (2005) [hep-ph/0501167]
5. F.E. Low, *Phys. Rev.* **120** 582 (1960) (and reference therein); M. Jacob, J. Mathews, *Phys. Rev.* **117**, 854 (1960)
6. A.C. Hearn, E. Leader, *Phys. Rev.* **126**, 789 (1962) ; R. Köberle, *Phys. Rev.* **166**, 1558 (1968)
7. M. Schumacher, *Proceedings: P.A. Cherenkov and Modern Physics, Moscow, June 22-25*, (2004) [nucl-ex/0411048]
8. M. Nagy, M.D. Scadron, G.E. Hite, *Acta Physica Slovaca* **54**, 427 (2004) [hep-ph/0406009]; M.D. Scadron et al., *Phys. Rev.* **D 69**, 014010 (2004)