Background field method at finite temperature and density

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In this letter we make use of the Background Field Method (BFM) to compute the effective potential of an SU(2) gauge field theory, in the presence of chemical potential and temperature. The main idea is to consider the chemical potential as the background field. The gauge fixing condition required by the BFM turns out to be exactly the one we found in a previous article in a different context.

INTRODUCTION

The background field method (BFM) is an easy and common tool for quantizing gauge fields without breaking explicitly the gauge invariance. This method, well described in [\[1,](#page-5-0) [2,](#page-5-1) [3\]](#page-5-2), can be used to find, perturbatively, in a simple way, the effective action [\[4](#page-5-3)]. The BRST, the Slavnov-Taylor and the Ward identities are preserved [\[5\]](#page-5-4). It is also possible to show, in this frame, the renormalization of the standard model [\[6](#page-5-5)].

The extension of the BFM to theories at finite temperature, and/or densities, has not been properly formulated. For example, there are ambiguities in the formulation of the renormalization group, so that, finally there is not a unique answer for the thermal/density behavior of the running coupling constants [\[7](#page-5-6)]. The first attempt to extend the BFM to finite temperature was proposed in [\[8](#page-5-7)] employing the thermal renormalization group [\[9](#page-5-8)].

Recently we have discussed how to compute the thermodynamical potential (Ω) of the standard model, in the presence of finite chemical potentials and temperature, using a new gauge fixing condition that allows to separate the contribution of the different fields [\[10\]](#page-5-9). This gauge fixing condition can be interpreted as an extension of the well known R_{ξ} gauge introduced by 't Hooft. In this letter, we show that this gauge fixing condition emerges in a natural way from a description based on the BFM, by interpreting the chemical potentials as background fields. Notice that this is a new way of handling chemical potentials, since in the BFM they are not introduced as Lagrange multipliers associated to conserved charges. An advantage of this way of handling chemical potentials is the fact it is not necessary to compute the conserved charges, and to integrate over the canonical field momenta, as it is usually done, see for example [\[11\]](#page-5-10).

We will concentrate our discussion on a pure $SU(2)$ Yang-Mills theory, considering, afterwards, the inclusion of scalar and fermion fields. We show how to compute the effective potential according to the BFM prescription, in the presence of chemical potential and temperature.

PURE $SU(2)$ GAUGE THEORY

As it is well known, the generating functional for a non Abelian gauge theory is given by

$$
Z[J] = \int \mathcal{D}A \det \left[\frac{\delta G^a}{\delta w^b} \right] \exp i [S[A] - \frac{1}{2\xi} G \cdot G + J \cdot A \right], \qquad (1)
$$

where A^a_μ is the $SU(2)$ gauge field, G^a is the gauge fixing condition, w^b are the infinitesimal gauge parameters and the classical action $S[A]$ is given by

$$
S = -\frac{1}{4} \int d^4x F^a_{\mu\nu} F_a^{\mu\nu}, \qquad (2)
$$

where

$$
F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c. \tag{3}
$$

According to the BFM, we must shift our field $A_\mu^a \rightarrow$ $A_{\mu}^{a} + B_{\mu}^{a}$, where B_{μ}^{a} is the background field. Then, the new generating functional will be

$$
\tilde{Z}[J, B] = \int \mathcal{D}A \det \left[\frac{\delta \tilde{G}^a}{\delta w^b} \right] \exp i[S[A + B] - \frac{1}{2\xi} \tilde{G} \cdot \tilde{G} + J \cdot A], \tag{4}
$$

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where \tilde{G}^a is the modified gauge fixing condition. In order to maintain gauge symmetry, we need a \tilde{G}^a of the following shape

$$
\tilde{G}^a = \partial^{\mu} A^a_{\mu} + g \epsilon^{abc} B^b_{\mu} A^{\mu}_c \equiv \bar{D}_{\mu} A^{\mu}_a. \tag{5}
$$

The new infinitesimal transformations, will be defined according to

$$
\delta A_{\mu}^{a} \equiv -\epsilon^{abc} w^{b} A_{\mu}^{c}
$$

\n
$$
\delta B_{\mu}^{a} \equiv -\epsilon^{abc} w^{b} B_{\mu}^{c} + \frac{1}{g} \partial_{\mu} w^{a}
$$

\n
$$
\delta J_{\mu}^{a} \equiv -\epsilon^{abc} w^{b} J_{\mu}^{c},
$$
\n(6)

so that the gauge symmetry remains unbroken, including the $\tilde{G} \cdot \tilde{G}$ term. Notice that the gauge field transforms as a matter field and the gauge parameters appear associated to the background field [\[3](#page-5-2)]. The sum of both fields transforms in the usual way

$$
\delta(A^a_\mu + B^a_\mu) = -\epsilon^{abc} w^b (A^c_\mu + B^c_\mu) + \frac{1}{g} \partial_\mu w^a, \qquad (7)
$$

i.e. as gauge fields, so that the classical action in [\(4\)](#page-0-3) remains invariant. In the loop calculation of the effective action, the background field (B_μ^a) appears as external amputated legs, whereas the quantum gauge fields (A_μ^a) and the ghost fields (η^a) live only in internal lines.

The modified Lagrangian, including the ghost fields will read

$$
\mathcal{L}_{mod} = \mathcal{L}_{A_{\mu}} + \mathcal{L}_{ghost} + \mathcal{L}_{GF},\tag{8}
$$

such that

$$
\mathcal{L}_{A_{\mu}} = -\frac{1}{4} \left(B_{\mu\nu}^{a} + \bar{D}_{\mu} A_{\nu}^{a} - \bar{D}_{\nu} A_{\mu}^{a} + g \epsilon^{abc} A_{\mu}^{b} A_{\nu}^{c} \right)^{2},
$$
\n
$$
\mathcal{L}_{ghost} = -(\bar{D}_{\mu} \eta_{a}^{*})(\bar{D}^{\mu} \eta_{a} - g \epsilon^{abc} \eta_{b} A_{c}^{\mu}),
$$
\n
$$
\mathcal{L}_{GF} = -\frac{1}{2\xi} (\bar{D}_{\mu} A_{a}^{\mu})^{2},
$$
\n(9)

where

$$
\bar{D}_{\mu}A_{\nu}^{a} \equiv \partial_{\mu}A_{\nu}^{a} + g\epsilon^{abc}B_{\mu}^{b}A_{\nu}^{c}, \qquad (10)
$$

$$
\bar{D}_{\mu}\eta^{a} \equiv \partial_{\mu}\eta^{a} + g\epsilon^{abc}B_{\mu}^{b}\eta^{c}, \qquad (11)
$$

and

$$
B^{a}_{\mu\nu} \equiv \partial_{\mu}B^{a}_{\nu} - \partial_{\nu}B^{a}_{\mu} + g\epsilon^{abc}B^{b}_{\mu}B^{c}_{\nu}.
$$
 (12)

CHEMICAL POTENTIALS AS BACKGROUND FIELDS

The basic idea of this section is to introduce the chemical potential (μ) as the background field. This is different from the usual approach, where chemical potentials appear in covariant derivatives as constant external time component gauge fields. We will use the following prescription

$$
B^a_\mu = \frac{\mu}{g} v_\mu \delta^{a3},\tag{13}
$$

where v_{μ} is a 4-velocity with respect to the thermal bath, that allows us to keep a formal covariant language, although, finally we have to choose the frame of reference where the heat bath is at rest, i.e. $v_{\mu} = (1, 0, 0, 0)$. We have chosen the third component of the internal $SU(2)$ group. This restriction corresponds to a simple orientation in the group manifold. The appearance of the quotient μ/g is a consequence of keeping the usual BFM relation between Z_B and Z_g , the background field and coupling constant renormalization factors, respectively, given by

$$
Z_g = Z_B^{-1/2} \tag{14}
$$

The gauge fixing conditions acquires the form

$$
G^a = \partial_\mu A^a_\mu + \mu \epsilon^{abc} A^c_\mu v^\mu \delta^{3b}.
$$
 (15)

We would like to emphasize that the same gauge fixing condition was found in our previous paper [\[10](#page-5-9)], using a complete different approach. In [\[10\]](#page-5-9), we found an exact expression for the effective potential of the Weinberg-Salam model in the presence of chemical potentials and thermal effects. The idea was to diagonalize the effective potential to get separate contributions from each field. The splitting of the effective potential is not possible without our gauge fixing condition. For example, in [\[13](#page-5-11)] the author gives an expression for the effective potential, but only in the high temperature expansion.

To show the efficiency of this method, we will proceed with the calculation in the one loop approximation of the effective potential for a pure gauge theory. As it is well known the one-loop thermal effective action is given by

$$
\exp\Gamma_1^{\beta}[\phi_c] = \int D[Fields] \exp \int_0^{\beta} d\tau \int d^3x \mathcal{L}_q(\bar{x}), (16)
$$

where we shifted to Euclidean metric $(\tau = it)$, with

$$
\bar{x} = (-i\tau, \mathbf{x}), \qquad \bar{p} = (i\omega_n, \mathbf{p}), \tag{17}
$$

with $\omega_n = 2\pi n/\beta$ for bosons and $\omega_n = 2\pi (n+1)/\beta$ for fermions. In equation [\(16\)](#page-1-0) the \mathcal{L}_q denotes the quadratic Lagrangian for the A^a_μ and η^a fields. Now, as it is well known, the effective potential corresponds to the effective action, by taking the classical field as a constant.

$$
\Gamma_1^{\beta}[\phi_c = constant] = -\beta \int d^3x \ \Omega_{eff}^{\beta}.
$$
 (18)

Since the internal lines in the Feynman diagrams are associated to the gauge and the ghosts fields, we will need to find the quadratic Lagrangian in both fields, in order to form

$$
\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 x \mathcal{L}_q = -\frac{1}{2} \int \mathrm{d}\bar{x}' \int \mathrm{d}\bar{x} A_\mu^a(\bar{x}') B_{ab}^{\mu\nu}(\bar{x}', \bar{x}) A_\nu^b(\bar{x})
$$

$$
- \int \mathrm{d}\bar{x}' \int \mathrm{d}\bar{x} \omega^*(\bar{x}') C(\bar{x}', \bar{x}) \omega(\bar{x}), \quad (19)
$$

Now, the effective potential will be given by

$$
-\beta \int d^3x \ \Omega_{eff} = -\frac{1}{2} \text{Tr} \ln \mathbf{B} + \text{Tr} \ln \mathbf{C}.
$$
 (20)

the quadratic Lagrangian for a pure gauge theory is

$$
\mathcal{L}_{q} = -\frac{1}{4} (\bar{D}_{\mu} A_{\nu}^{a} - \bar{D}_{\nu} A_{\mu}^{a})^{2} - \frac{1}{2} B_{a}^{\mu \nu} \epsilon^{abc} A_{\mu}^{b} A_{\nu}^{c} - \frac{1}{2\xi} (\bar{D}_{\mu} A_{a}^{\mu})^{2} - (\bar{D}_{\mu} \omega_{a}^{*})(\bar{D}^{\mu} \omega_{a}).
$$
\n(21)

Taking into account the choice [\(13\)](#page-1-1), we notice that $B^a_{\mu\nu} = 0$. Since $Z_B^{1/2}$ multiplies $B^a_{\mu\nu}$, we need three different chemical potentials μ^a associated to a particular flavor. Otherwise $B_{\mu\nu}^a$ vanishes and we are not able to carry on the renormalization procedure [\[3\]](#page-5-2). The idea is that renormalizability has already been proved, and then we are free to select one direction in the isospin space in order to compute the effective potential. This remind us the problem that appears when quantizing gauge field theories, between the R_{ξ} and the unitary gauge.

A very simple way to calculate the effective potential is to write each field explicitly. Let us choose, to simplify the calculations, $\xi = 1$. The sum of all contributions of the gauge fields will give

$$
\mathcal{L}_{q} = -\frac{1}{2} [\partial_{\mu} A_{\nu}^{1} \partial^{\mu} A_{1}^{\nu} + \mu^{2} A_{\nu}^{1} A_{1}^{\nu}] \n- \frac{1}{2} [\partial_{\mu} A_{\nu}^{2} \partial^{\mu} A_{2}^{\nu} + \mu^{2} A_{\nu}^{2} A_{2}^{\nu}] \n- \frac{1}{2} [\partial_{\mu} A_{\nu}^{3} \partial^{\mu} A_{3}^{\nu}] \n- \mu v_{\mu} (A_{1}^{\nu} \partial^{\mu} A_{\nu}^{2} - A_{2}^{\nu} \partial^{\mu} A_{\nu}^{1}).
$$
\n(22)

As usual, we will write this lagrangian in Euclidean metric, and the calculation of the effective potential will be given by taking the traces of [\(22\)](#page-2-0). After evaluating the sums over the Matsubara frequencies, we find the gauge field contribution to the thermal effective potential

$$
\Omega_{A_{\mu}}^{\beta} = \frac{1}{2\beta} \int d^{3}k \left(8\ln\left[(1 - e^{-\beta(|\mathbf{k}| + \mu)}) (1 - e^{-\beta(|\mathbf{k}| - \mu)}) \right] + 8\ln(1 - e^{-\beta|\mathbf{k}|}) \right). \tag{23}
$$

Although it seems that we have extra degrees of freedom, we must not forget that we still need to calculate the contribution of the ghost fields, which reads

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$$
\Omega_{ghosts}^{\beta} = -\frac{1}{\beta} \int d^3k \left(2\ln\left[(1 - e^{-\beta(|\mathbf{k}| + \mu)}) (1 - e^{-\beta(|\mathbf{k}| - \mu)}) \right] + 2\ln(1 - e^{-\beta|\mathbf{k}|}) \right). \tag{24}
$$

So, the final result for the effective potential is

$$
\Omega_{eff}^{\beta} = \frac{1}{\beta} \int d^3k \left(2 \ln \left[(1 - e^{-\beta(|\mathbf{k}| + \mu)}) (1 - e^{-\beta(|\mathbf{k}| - \mu)}) \right] + 2 \ln (1 - e^{-\beta |\mathbf{k}|}) \right). \tag{25}
$$

This is exactly the result we wanted to obtain. We can see that we a have massless and chargeless gauge boson with two degrees of freedom, and two massless and charged gauge bosons with two degrees of freedom each.

SCALAR AND FERMION $SU(2)$ GAUGE SYMMETRY

First we will concentrate our attention on a scalar $SU(2)$ gauge invariant theory, given by the following Lagrangian

$$
\mathcal{L} = \frac{1}{2} (D_{\mu} \phi^{a})^{T} (D^{\mu} \phi^{a}) - \frac{1}{4} F_{\mu \nu}^{a} F_{a}^{\mu \nu} - V(\phi), \qquad (26)
$$

where ϕ belongs to the adjoint representation, i.e. it is given by a real scalar triplet

$$
\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{27}
$$

and

$$
D_{\mu}\phi^{a} = (\partial_{\mu} - igA_{\mu})\phi^{a}, \qquad (28)
$$

$$
F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c, \tag{29}
$$

with

$$
A_{\mu} = A_{\mu}^{b} T^{b} \tag{30}
$$

The classical potential is given by

$$
V(\phi) = \frac{m^2}{2}\phi^T\phi + \frac{\lambda}{4}(\phi^T\phi)^2,
$$
\n(31)

In the $SU(2)$ case, the group generators will be given by $T^a = \omega^a/2$, where

$$
\omega_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \omega_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad (32)
$$

$$
\omega_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (33)
$$

Now, following the BFM prescription, we will expand our fields in the following way

$$
\phi^a \to \phi^a + \bar{\phi}^a,
$$

\n
$$
A^a_\mu \to A^a_\mu + B^a_\mu,
$$
\n(34)

where

$$
\bar{\phi}^a = \begin{pmatrix} 0 \\ 0 \\ \nu \end{pmatrix}, \tag{35}
$$

and

$$
B^a_\mu = \frac{\mu}{g} v_\mu \delta^{a3}.
$$
 (36)

Here $\bar{\phi}^a$ is the vacuum expectation value of the scalar fields and B^a_μ is the background field associated to the chemical potential. These two background fields are constant. The Lagrangian will now read

$$
\mathcal{L} = \frac{1}{2} [\tilde{D}_{\mu} (\phi^a + \bar{\phi}^a)]^T [\tilde{D}^{\mu} (\phi^a + \bar{\phi}^a)] - V(\phi) \n- \frac{1}{4} (\bar{D}_{\mu} A_{\nu}^a - \bar{D}_{\nu} A_{\mu}^a + g \epsilon^{abc} A_{\mu}^b A_{\nu}^c)^2, \tag{37}
$$

with

$$
\tilde{D}_{\mu}\phi^{a} = (\partial_{\mu} - igT^{b}(A^{b}_{\mu} + B^{b}_{\mu}))\phi^{a}, \n\bar{D}_{\mu}A^{a}_{a} = \partial_{\mu}A^{a}_{a} + ge^{abc}B^{b}_{\mu}A^{a}_{c}.
$$
\n(38)

The gauge fixing condition should be treated in the same way as before, but this time incorporating the scalar fields

$$
\mathcal{L}_{GF} = -\frac{1}{2\xi} [\bar{D}_{\mu} A^{\mu}_{a} - ig\xi \phi^{T} T^{a} \bar{\phi}]^{2}.
$$
 (39)

Since

$$
[T^a, T^b] = i\epsilon^{abc} T^c,\t\t(40)
$$

and the Goldstone theorem says

$$
i T^a \bar{\phi} = 0, \qquad \text{for every unbroken symmetry,}
$$

$$
i T^a \bar{\phi} \neq 0, \qquad \text{for every broken symmetry,}
$$
 (41)

we can see that this gauge fixing condition will remove every quadratic mixing between the scalar and the gauge fields.

For the calculation of the effective potential we only need the quadratic terms in the Lagrangian, $\mathcal{L}^q = \mathcal{L}^q_{\phi}$ + $\mathcal{L}_{A_{\mu}}^{q} + \mathcal{L}_{GF}^{q} + \mathcal{L}_{ghost}^{q}$, with

$$
\mathcal{L}_{\phi}^{q} = \frac{1}{2} \left[\partial_{\mu} \phi^{T} \partial^{\mu} \phi + B_{\mu} B^{\mu} \phi^{T} \phi \right] + i (\phi^{T} B_{\mu} \partial^{\mu} \phi - \partial_{\mu} \phi^{T} B^{\mu} \phi) \right], \qquad (42)
$$

$$
\mathcal{L}_{A_{\mu}}^{q} = -\frac{1}{4} (\bar{D}_{\mu} A_{\nu}^{a} - \bar{D}_{\nu} A_{\mu}^{a})^{2} + \bar{\phi}^{\dagger} A_{\mu} A^{\mu} \bar{\phi}, \quad (43)
$$

$$
\mathcal{L}_{GF}^{q} = -\frac{1}{2\xi} [(\partial_{\mu}A_{a}^{\mu})^{2} + 2C^{abc}\partial_{\mu}A_{a}^{\mu}\bar{A}_{\nu}^{b}A_{c}^{\nu} + (C^{abc}A_{\mu}^{b}A_{c}^{\mu})^{2} + \xi^{2}(\phi^{\dagger}T^{a}\bar{\phi})^{2}], \qquad (44)
$$

$$
\mathcal{L}_{ghost}^q = -\bar{D}_{\mu} \eta_a^* \bar{D}^{\mu} \eta_a + \xi g^2 \eta_a^* \bar{\phi}^T T^a T^b \bar{\phi} \eta_b. \quad (45)
$$

Because of the Goldstone theorem, the number of scalar bosons that acquire a gauge dependent mass and the number of massive gauge fields should be the same as the number of spontaneously broken symmetries. If we define

$$
(M_A^{ab})^2 = \bar{\phi}^T T^a T^b \bar{\phi},\tag{46}
$$

so that

$$
M_A^2 = \frac{g^2 \nu^2}{4},\tag{47}
$$

the masses of the fields involved are given by

$$
m_{\phi_{1,2}}^2 = m^2 + \lambda \nu^2 + \xi M_A^2 \equiv m_1^2,
$$

\n
$$
m_{\phi_3}^2 = m^2 + 3\lambda \nu^2 \equiv m_3^2,
$$

\n
$$
m_{A_{1,2}}^2 = M_A^2,
$$

\n
$$
m_{A_3}^2 = 0,
$$

\n
$$
m_{\eta_{1,2}}^2 = \xi M_A^2,
$$

\n
$$
m_{\eta_3}^2 = 0.
$$
\n(48)

Choosing $\xi = 1$, and writing the Lagrangian in the form

$$
\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 x \mathcal{L}_q = -\frac{1}{2} \int \! \mathrm{d}\bar{x}' \! \int \! \mathrm{d}\bar{x} \phi^a(\bar{x}') A_{ab}(\bar{x}', \bar{x}) \phi^b(\bar{x})
$$

$$
- \frac{1}{2} \int \! \mathrm{d}\bar{x}' \! \int \! \mathrm{d}\bar{x} A_\mu^a(\bar{x}') B_{ab}^{\mu\nu}(\bar{x}', \bar{x}) A_\nu^b(\bar{x})
$$

$$
- \int \! \mathrm{d}\bar{x}' \! \int \! \mathrm{d}\bar{x} \omega^*(\bar{x}') C(\bar{x}', \bar{x}) \omega(\bar{x}), \quad (49)
$$

we have that the thermodynamical effective potential is given by

$$
-\beta \int d^3x \ \Omega_{eff} = -\frac{1}{2} \text{Tr} \ln \mathbf{A} - \frac{1}{2} \text{Tr} \ln \mathbf{B} + \text{Tr} \ln \mathbf{C}. \tag{50}
$$

A straightforward calculation leads us to the several thermal contributions from the different fields. For the ϕ_3 boson we have

$$
\Omega_{\phi_3}^{\beta} = \frac{1}{\beta} \int d^3k \ln(1 - e^{-\beta(\sqrt{\mathbf{k}^2 + m_3^2})}), \quad (51)
$$

and for ϕ_1 and ϕ_2

$$
\Omega^{\beta}_{\phi_{1,2}} = \frac{1}{\beta} \int d^3k \left[\ln(1 - e^{-\beta(\sqrt{\mathbf{k}^2 + m_1^2} + \frac{\mu}{2}})) + \ln(1 - e^{-\beta(\sqrt{\mathbf{k}^2 + m_1^2} - \frac{\mu}{2}})) \right].
$$
 (52)

Notice that $\mathcal{L}_{A_\mu}^q$ is very similar to that calculated in [\(22\)](#page-2-0), but now two of the gauge fields are massive, because of the $\bar{\phi}A_{\mu}A^{\mu}\phi$ term in [\(43\)](#page-3-0). The contribution of these fields is

$$
\Omega_{A_{\mu}}^{\beta} = \frac{1}{\beta} \int d^{3}k \left(4\ln\left[(1 - e^{-\beta(\sqrt{\mathbf{k}^{2} + M_{A}^{2}} + \mu)}) (1 - e^{-\beta(\sqrt{\mathbf{k}^{2} + M_{A}^{2}} - \mu)}) \right] + 4\ln(1 - e^{-\beta|\mathbf{k}|}) \right). \tag{53}
$$

When we add the contribution of the Faddeev-Popov Lagrangian we obtain

$$
\Omega_{A_{\mu},\eta}^{\beta} = \frac{1}{\beta} \int d^{3}k \left(2 \ln \left[(1 - e^{-\beta(\sqrt{\mathbf{k}^{2} + M_{A}^{2}} + \mu)}) (1 - e^{-\beta(\sqrt{\mathbf{k}^{2} + M_{A}^{2}} - \mu)}) \right] + 2 \ln(1 - e^{-\beta|\mathbf{k}|}) \right). \tag{54}
$$

Notice that the number of degrees of freedom is the expected one. If we choose $\nu^2 = -m^2/\lambda$ we recover the usual Higgs-Kibble mechanism. In the presence of chemical potential, however, we have a lesser number of Goldstone bosons, as was shown in [\[14\]](#page-5-12). Our calculation confirms this picture.

The treatment of the fermion fields is equivalent to

the usual procedure, where the chemical potentials appear as external zero component gauge fields forming a new covariant derivative. This a consequence of the fact that the conserved fermionic charge does not depend on the derivatives of the fields, i.e. there is no need to integrate over the conjugate momenta to pass from the Hamiltonian picture to the Lagrangian formalism. For

the fermion fields we have

$$
\mathcal{L}_{\psi} = i\bar{\psi}(\partial - igA - igB)\psi + m_{\psi}^{2}\bar{\psi}\psi, \tag{55}
$$

where m_{ψ}^2 is the mass of the fermions due to the Higgs-Kibble mechanism. The calculation of their contribution to the thermodynamical potential is straightforward. We found the well known result

$$
\Omega_{\psi}^{\beta} = -\frac{1}{\beta} \int d^3k \left[2\ln(1 + e^{-\beta(\sqrt{\mathbf{k}^2 + m_{\psi}^2} + \frac{\mu}{2}})) + 2\ln(1 + e^{-\beta(\sqrt{\mathbf{k}^2 + m_{\psi}^2} - \frac{\mu}{2}})) \right].
$$
 (56)

The final one loop effective potential in a $SU(2)$ gauge theory with scalars and fermions will be given by the sum of equations [\(51\)](#page-4-0), [\(52\)](#page-4-1), [\(54\)](#page-4-2) and [\(56\)](#page-5-13).

In this letter we have shown that the gauge fixing condition that enables to diagonalize the effective potential for a system including gauge, scalar and/or fermion fields emerges naturally from the description based on the BFM method.

This gauge fixing condition has been only explored for small gauge field configurations. The analysis of the existence of Gribov copies will be carried on in a future work.

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