

Neutrino Mixing and Oscillations in Quantum Field Theory

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Abstract

We show that the generator of field mixing transformations in Quantum Field Theory induces a non trivial structure in the vacuum which turns out to be a coherent state, both for bosons and for fermions, although with a different condensate structure. The Fock space for mixed fields is unitarily inequivalent to the Fock space of the massive (free) fields in the infinite volume limit. As a practical application we study neutrino mixing and oscillations. A new oscillation formula is found where the oscillation amplitude is depressed, with respect to the usual one, by a factor which is momentum and mass dependent. In the relativistic limit, the usual formula is recovered. We finally discuss in some detail phenomenological features of the modified oscillation formula.

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Novel features of field mixing transformations in Quantum Field Theory (QFT) have been recently [1,2,3] discovered. In particular it has been shown [1,3] that the generator of such transformations induces a non trivial structure in the vacuum which turns out to be a coherent state, both for bosons and for fermions, although with a different condensate structure. The Fock space for mixed fields has been explicitly constructed and it has been shown that, in the infinite volume limit, it is unitarily inequivalent (orthogonal) to the Fock space of the corresponding massive (free) fields.

As explained below, such new and almost unexpected features find their origin in the existence, in QFT, of infinitely many inequivalent representations of the canonical (anti-)commutation relations [4,5].

The question arises, however, if such a new and rich structure leads to any new and possibly testable effect. For such a purpose, neutrino mixing and oscillations [6,7] have been investigated in ref.[1,2] as a practical example and a new oscillation formula (different from the usual one) has been found. In particular, we have found a correction on the oscillation amplitude which turns out to be momentum and mass dependent. However, in the relativistic limit, the usual formula is recovered; this is in general agreement with other studies of neutrino oscillations in the non relativistic region [8].

The aim of the present paper is to report on such results and to discuss in some detail phenomenological features of the modified neutrino oscillation formula.

In the simple case of two flavor mixing [7] (for the case of three flavors see ref. 1) the mixing relations are:

$$\begin{aligned}\nu_e(x, t) &= \nu_1(x, t) \cos \theta + \nu_2(x, t) \sin \theta \\ \nu_\mu(x, t) &= -\nu_1(x, t) \sin \theta + \nu_2(x, t) \cos \theta ,\end{aligned}\tag{1}$$

where x denotes the (three) spatial coordinates; $\nu_e(x, t)$ and $\nu_\mu(x, t)$ are the (Dirac) neutrino fields with definite flavors. $\nu_1(x, t)$ and $\nu_2(x, t)$ are the (free) neutrino fields with definite masses m_1 and m_2 , respectively. Here we do not need to distinguish between left-handed and right-handed components. The fields $\nu_1(x, t)$ and $\nu_2(x, t)$ are written as

$$\nu_i(x, t) = \frac{1}{\sqrt{V}} \sum_{k,r} [u_{k,i}^r(t) \alpha_{k,i}^r e^{ikx} + v_{k,i}^r(t) \beta_{k,i}^{r\dagger} e^{-ikx}], \quad i = 1, 2 .\tag{2}$$

$\alpha_{k,i}^r$ and $\beta_{k,i}^r$, $i = 1, 2$, $r = 1, 2$ are the annihilator operators for the vacuum state $|0\rangle_{1,2}$: $\alpha_{k,i}^r|0\rangle_{12} = \beta_{k,i}^r|0\rangle_{12} = 0$. For simplicity, we use the same symbol for the vector k and for its modulus. The anticommutation relations are:

$$\{\nu_i^\alpha(x, t), \nu_j^{\beta\dagger}(y, t')\}_{t=t'} = \delta^3(x - y)\delta_{\alpha\beta}\delta_{ij}, \quad \alpha, \beta = 1, \dots, 4, \quad (3)$$

and

$$\{\alpha_{k,i}^r, \alpha_{q,j}^{s\dagger}\} = \delta_{kq}\delta_{rs}\delta_{ij}; \quad \{\beta_{k,i}^r, \beta_{q,j}^{s\dagger}\} = \delta_{kq}\delta_{rs}\delta_{ij}, \quad i, j = 1, 2. \quad (4)$$

All other anticommutators are zero. The orthonormality and completeness relations are the usual ones.

Eqs.(1) relate the hamiltonians $H_{1,2}$ (we consider only the mass terms) and $H_{e,\mu}$ [7]:

$$H_{1,2} = m_1 \bar{\nu}_1\nu_1 + m_2 \bar{\nu}_2\nu_2 \quad (5)$$

$$H_{e,\mu} = m_{ee} \bar{\nu}_e\nu_e + m_{\mu\mu} \bar{\nu}_\mu\nu_\mu + m_{e\mu} (\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_e) \quad (6)$$

where $m_{ee} = m_1 \cos^2 \theta + m_2 \sin^2 \theta$, $m_{\mu\mu} = m_1 \sin^2 \theta + m_2 \cos^2 \theta$ and $m_{e\mu} = (m_2 - m_1) \sin \theta \cos \theta$.

In the LSZ formalism of QFT [4] observables are expressed in terms of asymptotic in- (or out-) fields. These fields, also called free or physical fields, are obtained by the weak limit of the Heisenberg or interacting fields for $t \rightarrow -(or+)\infty$. The system Lagrangian and the resulting field equations are given in terms of the Heisenberg fields and therefore the meaning of the weak limit is to provide a realization of the basic dynamics in terms of the asymptotic fields. The weak limit is however not unique since infinitely many representations of the canonical (anti-)commutation relations exist in QFT [4,5] and as a consequence the realization of the basic dynamics in terms of the asymptotic fields is not unique. Therefore, in order to avoid ambiguities, it is of crucial importance to investigate with much care the mapping among Heisenberg fields and free fields (generally known as dynamical mapping or Haag expansion) [4,5].

For example, since unitarily inequivalent representations describe physically different phases, in theories with spontaneous symmetry breaking the same set of Heisenberg field equations describes the normal (symmetric) phase as well as the symmetry broken phase, according to the representation one chooses for the asymptotic fields.

It should be observed that no problem arises with uniqueness of the asymptotic limit in quantum mechanics, namely for finite volume systems. In such a case indeed the von Neumann theorem ensures that the representations of the canonical commutation relations are each other unitary equivalent. However, the von Neumann theorem does not hold in QFT since infinite number of degrees of freedom is there considered and much attention is then required when considering any mapping among interacting and free fields [4,5].

For these reasons, intrinsic to the QFT structure, the mixing relations (1), which can be seen as a mapping among Heisenberg fields and free fields, deserve a careful analysis.

To this aim, we can rewrite the mixing relations (1) in the form:

$$\begin{aligned}\nu_e^\alpha(x, t) &= G^{-1}(\theta, t) \nu_1^\alpha(x, t) G(\theta, t) \\ \nu_\mu^\alpha(x, t) &= G^{-1}(\theta, t) \nu_2^\alpha(x, t) G(\theta, t) ,\end{aligned}\tag{7}$$

and the generator $G(\theta, t)$ can be written as:

$$G(\theta, t) = \exp[\theta(S_+(t) - S_-(t))] ,\tag{8}$$

with

$$S_+(t) \equiv \int d^3x \nu_1^\dagger(x, t) \nu_2(x, t) , \quad S_-(t) \equiv \int d^3x \nu_2^\dagger(x, t) \nu_1(x, t) = (S_+(t))^\dagger .\tag{9}$$

In the following we will omit for simplicity the time dependence. It is easy to see, by introducing $S_3 \equiv \frac{1}{2} \int d^3x (\nu_1^\dagger(x) \nu_1(x) - \nu_2^\dagger(x) \nu_2(x))$, that the $su(2)$ algebra is closed:

$$[S_+, S_-] = 2S_3 \quad , \quad [S_3, S_\pm] = \pm S_\pm .\tag{10}$$

The main point (see ref.[1] for details) is that the above generator of mixing transformations does not leave invariant the vacuum of the free fields $\nu_{1,2}$, say $|0\rangle_{1,2}$, since it induces an $SU(2)$ coherent state structure of neutrino-antineutrino pairs in this state [9,1]. This coherent state is the vacuum for the fields $\nu_{e,\mu}$, which we denote by $|0\rangle_{e,\mu}$:

$$|0\rangle_{e,\mu} = G^{-1}(\theta) |0\rangle_{1,2} .\tag{11}$$

It is then possible [1] to construct explicitly the Fock space for the mixed operators which can be rewritten in the form:

$$\nu_e(x, t) = \frac{1}{\sqrt{V}} \sum_{k,r} e^{ikx} [u_{k,1}^r(t) \alpha_{k,e}^r(t) + v_{-k,1}^r(t) \beta_{-k,e}^{r\dagger}(t)] \quad (12a)$$

$$\nu_\mu(x, t) = \frac{1}{\sqrt{V}} \sum_{k,r} e^{ikx} [u_{k,2}^r(t) \alpha_{k,\mu}^r(t) + v_{-k,2}^r(t) \beta_{-k,\mu}^{r\dagger}(t)] \quad (12b)$$

where the wave functions for the massive fields have been used [1,3] and (in the reference frame $k = (0, 0, |k|)$) the creation and annihilation operators for the mixed fields are given by:

$$\alpha_{k,e}^r(t) = \cos \theta \alpha_{k,1}^r + \sin \theta \left(U_k^*(t) \alpha_{k,2}^r + \epsilon^r V_k(t) \beta_{-k,2}^{r\dagger} \right) \quad (13a)$$

$$\alpha_{k,\mu}^r(t) = \cos \theta \alpha_{k,2}^r - \sin \theta \left(U_k(t) \alpha_{k,1}^r - \epsilon^r V_k(t) \beta_{-k,1}^{r\dagger} \right) \quad (13b)$$

$$\beta_{-k,e}^r(t) = \cos \theta \beta_{-k,1}^r + \sin \theta \left(U_k^*(t) \beta_{-k,2}^r - \epsilon^r V_k(t) \alpha_{k,2}^{r\dagger} \right) \quad (13c)$$

$$\beta_{-k,\mu}^r(t) = \cos \theta \beta_{-k,2}^r - \sin \theta \left(U_k(t) \beta_{-k,1}^r + \epsilon^r V_k(t) \alpha_{k,1}^{r\dagger} \right) \quad (13d)$$

with $\epsilon^r = (-1)^r$ and

$$V_k(t) = |V_k| e^{i(\omega_{k,2} + \omega_{k,1})t} \quad , \quad U_k(t) = |U_k| e^{i(\omega_{k,2} - \omega_{k,1})t} \quad (14)$$

$$|U_k| = \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left(1 + \frac{k^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right) \quad (15a)$$

$$|V_k| = \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left(\frac{k}{(\omega_{k,2} + m_2)} - \frac{k}{(\omega_{k,1} + m_1)} \right) \quad (15b)$$

$$|U_k|^2 + |V_k|^2 = 1 \quad (16)$$

$$|V_k|^2 = |V(k, m_1, m_2)|^2 = \frac{k^2 [(\omega_{k,2} + m_2) - (\omega_{k,1} + m_1)]^2}{4 \omega_{k,1} \omega_{k,2} (\omega_{k,1} + m_1) (\omega_{k,2} + m_2)} \quad (17)$$

where $\omega_{k,i} = \sqrt{k^2 + m_i^2}$.

By using eqs.(13) the expectation value of the number operator $N_{\sigma_l}^{k,r}$ is obtained as:

$${}_{1,2} \langle 0 | N_{\sigma_l}^{k,r} | 0 \rangle_{1,2} = \sin^2 \theta |V_k|^2 \quad , \quad \sigma = \alpha, \beta \quad , \quad l = e, \mu \quad , \quad (18)$$

Eq.(18) gives the condensation density of the vacuum state $|0\rangle_{1,2}$ as a function of the mixing angle θ , of the masses m_1 and m_2 and of the momentum modulus k , and it is in contrast with the usual approximation case where one puts $|0\rangle_{e,\mu} = |0\rangle_{1,2} \equiv |0\rangle$ and it is $\langle 0| N_{\alpha_e}^{k,r} |0\rangle = \langle 0| N_{\alpha_\mu}^{k,r} |0\rangle = 0$. Also note that ${}_{1,2}\langle 0| N_{\sigma_l}^{k,r} |0\rangle_{1,2}$ plays the role of zero point contribution when considering the energy contribution of $\sigma_l^{k,r}$ particles [1].

The oscillation formula is obtained by using the mixing mappings (13) [1]:

$$\begin{aligned} & \langle \alpha_{k,e}^r(t) | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r(t) \rangle = \\ & = 1 - \sin^2 \theta |V_k|^2 - |U_k|^2 \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega_k}{2} t \right). \end{aligned} \quad (19)$$

The fraction of $\alpha_\mu^{k,r}$ particles in the same state is

$$\begin{aligned} & \langle \alpha_{k,e}^r(t) | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r(t) \rangle = \\ & = |U_k|^2 \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega_k}{2} t \right) + \sin^2 \theta |V_k|^2 \left(1 - \sin^2 \theta |V_k|^2 \right). \end{aligned} \quad (20)$$

The terms with $|V_k|^2$ and $|U_k|^2$ in (19) and (20) denote the contribution from the vacuum condensate. We have

$$\begin{aligned} & \langle \alpha_{k,e}^r(t) | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r(t) \rangle + \langle \alpha_{k,e}^r(t) | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r(t) \rangle = \\ & \langle \alpha_{k,e}^r | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r \rangle + \langle \alpha_{k,e}^r | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r \rangle. \end{aligned} \quad (21)$$

where $|\alpha_{k,e}^r\rangle = |\alpha_{k,e}^r(t=0)\rangle$, which shows the conservation of the number $(N_{\alpha_e}^{k,r} + N_{\alpha_\mu}^{k,r})$. The expectation value of this number in the state $|0\rangle_{1,2}$ is not zero due to the condensate contribution.

Eqs.(19) and (20) are to be compared with the approximated ones in the conventional treatment:

$$\langle \alpha_{k,e}^r(t) | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r(t) \rangle = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega_k}{2} t \right) \quad (22)$$

and

$$\langle \alpha_{k,e}^r(t) | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r(t) \rangle = \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega_k}{2} t \right), \quad (23)$$

respectively.

Eqs.(19) and (20) reproduce the conventional ones (22) and (23) when $|U_k| \rightarrow 1$ (and $|V_k| \rightarrow 0$).

In conclusion, in the proper QFT treatment we obtain corrections to the flavor oscillations which come from the condensate contributions. The conventional (approximate) results (22) and (23) are recovered when the condensate contributions are missing (in the $|V_k| \rightarrow 0$ limit).

The phenomenological implications of the results (19) and (20) have been discussed in ref.[2] where we have studied the function $|V_k|^2$.

Here we note that $|V_k|^2$ depends on k only through its modulus and it is always in the interval $[0, \frac{1}{2}[$. It has a maximum for $k = \sqrt{m_1 m_2}$. Also, $|V_k|^2 \rightarrow 0$ when $k \rightarrow \infty$. Moreover, $|V_k|^2 = 0$ when $m_1 = m_2$ (no mixing occurs in Pontecorvo theory in this case).

This last feature is remarkable since the corrections to the oscillations depend on the modulus k through $|V_k|^2$ (and $|U_k|^2 = 1 - |V_k|^2$). So, these corrections disappear in the infinite momentum or relativistic limit $k \gg \sqrt{m_1 m_2}$ (note that $\sqrt{m_1 m_2}$ is the scale of the condensation density).

However, for finite k , the oscillation amplitude is depressed, or "squeezed", by a factor $|U_k|^2$: the squeezing factor ranges from 1 to $\frac{1}{2}$ depending on k and on the masses values. The values of the squeezing factor may therefore have not negligible effects in experimental findings and the dependence of the flavor oscillation amplitude on the momentum could thus be tested.

To better estimate the effects of the momentum dependence we rewrite the $|V_k|^2$ function as

$$|V_k|^2 \equiv |V(p, a)|^2 = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1 + a \left(\frac{p}{p^2+1} \right)^2}} \right) \quad (24)$$

with

$$p = \frac{k}{\sqrt{m_1 m_2}} \quad , \quad a = \frac{(\Delta m)^2}{m_1 m_2} \quad , \quad 0 \leq a < +\infty \quad , \quad (25)$$

where $\Delta m \equiv m_2 - m_1$ (we take $m_1 \leq m_2$).

At $p = 1$, $|V(p, a)|^2$ reaches its maximum value $|V(1, a)|^2$, which goes asymptotically to $1/2$ when $a \rightarrow \infty$.

It is useful to calculate the value of p , say p_ϵ , at which the function

$|V(p, a)|^2$ becomes a fraction ϵ of its maximum value $V(1, a)$:

$$p_\epsilon = \sqrt{-c + \sqrt{c^2 - 1}} \quad , \quad c \equiv \frac{b^2(a+2) - 2}{2(b^2 - 1)} \quad , \quad b \equiv 1 - \epsilon \left(1 - \frac{2}{\sqrt{a+4}} \right) . \quad (26)$$

The values of $\sqrt{m_1 m_2}$ and of a corresponding to some given values of m_1 and m_2 chosen below the current experimental bounds are reported in Tab. I.

Three sets of values of $|U(p_\epsilon, a)|^2$ and of k_ϵ , for $\epsilon = 1, \frac{1}{2}, \frac{1}{10}$, corresponding to the values of m_1 and m_2 given in Tab. I, are reported in Tab. II (see also Fig. 1). We used $|U(p_\epsilon, a)|^2 = 1 - \epsilon + \epsilon |U(1, a)|^2$ and $k_\epsilon = p_\epsilon \sqrt{m_1 m_2}$.

We note that neutrinos of not very large momentum may have sensible squeezing factors for the oscillation amplitudes. Larger deviations from the usual oscillation formula may thus be expected in these low momentum ranges. We note that observations of neutrino oscillations by large passive detectors include neutrino momentum as low as few hundreds of KeV [6].

We observe that an indication on the neutrino masses may be given by the dependence, if experimentally tested, of the oscillating amplitude on the momentum since the function $|U_k|^2$ (cf. eqs.(16) and (19)) has a minimum at $k = \sqrt{m_1 m_2}$.

Another interesting case not considered in ref.[2] occurs when one of the two masses, say m_1 , goes to zero. In this case, the maximum of the condensation density (the function $|V_k|^2$) occurs at $k = 0$; however, since $a \rightarrow \infty$ when $m_1 \rightarrow 0$, it is still possible to have non negligible effects at rather "large" momenta; m_2 should be large in order to provide appreciable corrections. The situation is illustrated in Tab. III, where for the calculation we used $m_1 = 10^{-10} eV$.

Let us also observe that since the vacuum condensate induces the correction factor, the vacuum acts as a "momentum (or spectrum) analyzer" for the oscillating neutrinos: neutrinos with $k \gg \sqrt{m_1 m_2}$ have oscillation amplitude larger than neutrinos with $k \simeq \sqrt{m_1 m_2}$, due to the vacuum structure. Such a vacuum spectral analysis effect may sum up to other effects (such as MSW effect [10] in the matter) in depressing or enhancing neutrino oscillations; in this connection see ref.[1], where the above scheme is also generalized to the oscillations in the matter.

On the basis of the above discussion and results we can conclude that probing the non relativistic momentum domain seems promising in order to

obtain new insights in neutrino physics.

Further studies on neutrino oscillations in the framework here discussed are in progress [11].

Finally, let us mention that the study of the mixing of boson fields shows [3] that relations analogous to eqs. (13) and (18) hold and the vacuum also acquires a non trivial condensate structure. In the boson case we find $|U_k| = \cosh\sigma_k$ and $|V_k| = \sinh\sigma_k$ with $\sigma_k = \frac{1}{2}\log\left(\frac{\omega_{k,1}}{\omega_{k,2}}\right)$ where $\omega_{k,i}, i = 1, 2$ is the boson energy.

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Table I: The values of $\sqrt{m_1 m_2}$ and of a for given values of m_1 and m_2 .

	$m_1 (eV)$	$m_2 (KeV)$	$\sqrt{m_1 m_2} (KeV)$	a
<i>A</i>	5	250	1.12	$\sim 5 \cdot 10^4$
<i>B</i>	2.5	250	0.79	$\sim 1 \cdot 10^5$
<i>C</i>	5	200	1	$\sim 4 \cdot 10^4$
<i>D</i>	1	100	0.32	$\sim 1 \cdot 10^5$
<i>E</i>	0.5	50	0.15	$\sim 1 \cdot 10^5$
<i>F</i>	0.5	1	0.02	$\sim 2 \cdot 10^3$

Table II: $|U(p_\epsilon, a)|^2$ vs. k_ϵ .

	$ U(1, a) ^2$	$k_1 (KeV)$	$ U(p_{1/2}, a) ^2$	$k_{1/2} (KeV)$	$ U(p_{1/10}, a) ^2$	$k_{1/10} (KeV)$
<i>A</i>	$\simeq 0.5$	1.12	$\simeq 0.75$	$\simeq 146$	$\simeq 0.95$	$\simeq 519$
<i>B</i>	$\simeq 0.5$	0.79	$\simeq 0.75$	$\simeq 145$	$\simeq 0.95$	$\simeq 518$
<i>C</i>	$\simeq 0.5$	1	$\simeq 0.75$	$\simeq 117$	$\simeq 0.95$	$\simeq 415$
<i>D</i>	$\simeq 0.5$	0.32	$\simeq 0.75$	$\simeq 58$	$\simeq 0.95$	$\simeq 206$
<i>E</i>	$\simeq 0.5$	0.16	$\simeq 0.75$	$\simeq 29$	$\simeq 0.95$	$\simeq 104$
<i>F</i>	$\simeq 0.5$	0.02	$\simeq 0.75$	$\simeq 0.6$	$\simeq 0.95$	$\simeq 2$

Table III: $|U(p_\epsilon, a)|^2$ vs. k_ϵ for $m_1 \simeq 0$ and different values of m_2 .

$m_1 (eV)$	$m_2 (KeV)$	$ U(p_{1/2}, a) ^2$	$k_{1/2} (KeV)$	$ U(p_{1/10}, a) ^2$	$k_{1/10} (KeV)$
$\simeq 0$	250	$\simeq 0.75$	$\simeq 144$	$\simeq 0.95$	$\simeq 516$
$\simeq 0$	200	$\simeq 0.75$	$\simeq 115$	$\simeq 0.95$	$\simeq 413$
$\simeq 0$	100	$\simeq 0.75$	$\simeq 57$	$\simeq 0.95$	$\simeq 206$
$\simeq 0$	50	$\simeq 0.75$	$\simeq 29$	$\simeq 0.95$	$\simeq 103$

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Figure caption

Fig. 1: The function $|U(p, a)|^2$ for the values of parameters of Tabs.1,2: A (continuous line), B,D,E (dashed line), C (small-dashed line), F (dotted line).

