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What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?

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Inflation generates gravitational waves, which may be observable in the low multipoles of the cosmic microwave background (cmb) anisotropy but only if the inflaton field variation is at least of order the Planck scale. Such a large variation would imply that the model of inflation cannot be part of an ordinary extension of the standard model, and combined with the detection of the waves it would also suggest that the inflaton field cannot be one of the superstring moduli. Another implication of observable gravitational waves would be a potential $V^{1/4} = 2$ to 4×10^{16} GeV, which is orders of magnitude bigger than is expected on the basis of particle theory. It might emerge in a hybrid inflation model where most of the energy density comes from the Higgs sector of a GUT, but only if both the vacuum expectation values *and the masses* of the Higgs fields are of this order.

I. INTRODUCTION

Inflation generates a density perturbation and gravitational waves. The density perturbation is thought to be responsible for large scale structure and, together with a possible gravitational wave contribution, for the cosmic microwave background (cmb) anisotropy. It is well known that the detection of a gravitational wave contribution to the cmb anisotropy would immediately determine the value V and slope V' of the potential while relevant scales are leaving the horizon during inflation [1], with an eventual measurement of the spectral index of the density perturbation fixing V'' [2,1] and additional data providing limited additional information about the shape of V [3]. Here I point out that a detection would also tell one that the inflaton field variation during inflation is at least of order the Planck scale, and go on to discuss the theoretical implications of both this result and the value of V.

If δ_H^2 is the spectrum of the curvature perturbation associated with the density perturbation, and $\mathcal{P}_g$ is the spectrum of the gravitational waves (as defined for example in [1]), it is convenient to consider the ratio $r(k) = 0.139\mathcal{P}_g/\delta_H^2$. The spectra are in general scale dependent, and $r(k)$ has been normalized so that it in an analytic approximation [4] it gives the ratio of the two contributions to the mean-square quadrupole of the cmb anisotropy seen by a randomly placed observer. For higher multipoles the corresponding ratio is roughly constant in the range $1 < l \lesssim 100$, but then it falls off sharply so that it will be detected if at all in the above range.

The standard slow-roll paradigm of inflation [5] predicts [7–9,4]

$$\delta_H^2(k) = \frac{1}{75\pi^2 m_{Pl}^6}\frac{V^3}{V'^2} \tag{1}$$

$$r(k) = 6.9 m_{Pl}^2 (V'/V)^2 \tag{2}$$

where $m_{Pl} = (8\pi G)^{-1/2} = 2.4\times10^{18}$ GeV is the Planck scale. The right hand sides are evaluated when $k = aH$ where k/a is the wavenumber, a is the scale factor and $H = \dot{a}/a$.

In an interval $\Delta\ln k \sim 1$, the fractional changes in δ_H^2 and $\mathcal{P}_g$ are predicted to be $\ll 1$. Since the lth multipole of the cmb anisotropy corresponds to a scale $k^{-1} \simeq 2/(H_0 l)$ the relevant range $1 < l \lesssim 100$ corresponds to only $\Delta\ln k \simeq 4.6$ so $r(k)$ will have a roughly constant value which from now on will be denoted simply by r. Ignoring any variation one can show [10] that because of cosmic variance a value $r > .07$ is necessary in in order to have a better than even chance of eventually detecting the gravitational wave contribution, and approximately the same result should hold for the average even if there is some variation.

At present observation provides only a weak upper bound on r, which has not been quantified properly but is something like $r \lesssim 1$ [12]. The COBE observations give a good normalization, [13] $\delta_H \simeq 1.9(1+r)^{1/2}\times10^{-5}$, and using it one finds [11]

$$V^{1/4} \simeq (r/.07)^{1/4}\times1.8\times10^{16}\,\mathrm{GeV} \tag{3}$$

Thus a detection of r would give a value $V^{1/4} = 2$ to 4×10^{16} GeV.

The slow-roll paradigm also gives

$$\frac{1}{m_{Pl}}\left|\frac{d\phi}{dN}\right| = m_{Pl}\left|\frac{V'}{V}\right| = \left(\frac{r}{6.9}\right)^{\frac{1}{2}} \tag{4}$$

where $d\phi$ is the change in the inflaton field in $dN = Hdt \simeq d\ln a$ Hubble times. While the scales corresponding to $1 < l \lesssim 100$ are leaving the horizon $\Delta N \simeq 4.6$, so the corresponding field variation is

$$\Delta\phi/m_{Pl} \simeq 4.6(r/6.9)^{1/2} = 0.46(r/.07)^{1/2} \tag{5}$$

We see that a detectable r requires $\Delta\phi \gtrsim 0.5 m_{Pl}$. This is a minimum estimate for the total field variation, because inflation continues afterwards for some number N of e-folds. The standard estimate [1] is $N \simeq 50$, but with late reheating and a single epoch of thermal inflation [15,16] $N \simeq 25$. In either case it is clear that $r(k)$ can increase significantly on smaller scales, making the total field variation much bigger than the estimate (5). In fact, there is a whole class of models where the increase is typically so strong that a detectable r requires $\Delta\phi \gg m_{Pl}$. These are the models where the inflaton field is near a maximum of the potential.*

Now let us consider inflation model-building in the light of all this. In the earliest models [17] the inflaton field is rolling towards a vacuum expectation value (vev) $\langle\phi\rangle \ll m_{Pl}$, making $\Delta\phi \ll m_{Pl}$ and r negligible. These models were at best unattractive because the inflaton field had to be very weakly coupled, so 'primordial' models were suggested, where the vev and $\Delta\phi$ are of order m_{Pl} [18] or bigger [19]; these still give negligible r because inflation takes place near a maximum of the potential. Then power-law potentials $V \propto \phi^p$ were considered [20], where the field during inflation is rolling towards the origin with a value and a variation of order $10m_{Pl}$, giving a detectable r. Finally (confining ourselves to the case of Einstein gravity) 'hybrid' inflation has been proposed [21], where the inflaton field is accompanied by another field responsible for most of the potential energy, inflation ending when it is destabilized. Like the earliest models, typical hybrid inflation models have [22] $\Delta\phi \ll m_{Pl}$ (and therefore negligible r) but unlike them they need not involve very small couplings [23].

Should we care whether the field variation is big or small, when building a model of inflation? In the context of global supersymmetry (or no supersymmetry) the answer would be no, because m_{Pl} makes no appearance in the field theory. However, according to present ideas, the extension of the standard model chosen by nature is likely to involve supergravity. In that context, one expects the potential to have an infinite power-series expansion in each field,

$$V = V_0 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \lambda' m_{Pl}^{-2}\phi^6 + \lambda'' m_{Pl}^{-4}\phi^8 + \cdots \tag{6}$$

(For simplicity I am supposing that odd powers are excluded by a symmetry.) Ordinary field theory corresponds to a truncation at low order, which is justified if all fields are small. This is indeed the case for the usual applications of field theory, involving the standard model, its minimal supersymmetric extension and more ambitious extensions invoking such things as neutrino masses, Peccei-Quinn symmetry or a GUT.

So the answer to the question is that we should care very much. Small-field models, which in practice seems to mean hybrid inflation models, are under relatively good control; it will be enough to keep one or two dominant, low-order terms in expansion (6) of V (with perhaps quantum corrections [17,26]) and one can hope to further restrict V by requiring that the fields relevant for inflation already appear in an extension of the standard model designed for some other purpose [27].

If a gravitational wave effect is detected in the cmb anisotropy, we shall need a model of inflation in which the inflaton field is of order m_{Pl} or bigger. For a generic field one has no idea what to expect in this regime. The only exception is for the superstring moduli, where superstring theory provides some guidance. The moduli potential looks [28,29] as if it might be marginally capable of supporting inflation, in that the expected values of $m_{Pl}^2(V'/V)^2$ and $m_{Pl}^2 V''/V$ at a generic point are of order 1 so that there could be an exceptional region in the moduli space where these quantities are both small. Investigations using specific models [28,30,31] have actually concluded that viable inflation does not occur, but even if it does it will probably not give a detectable r. The reason is that one expects the size of the region in field space where inflation can occur to be only of order m_{Pl}, and in order to motivate the initial condition by invoking eternal inflation [32] one will probably want to start inflation near a maximum of the potential. As we saw earlier, the combination of these two requirements will probably not give a detectable r.

The conclusion is that a model of inflation giving a detectable r will probably live in uncharted territory, where there is as yet no theoretical guidance as to the form of the potential. There is no particular reason to invoke the usually-considered forms $V \simeq A \pm B\phi^p$, though of course one should still test such forms against observation by measuring both r and the spectral index of the density perturbation [1].

Finally let us return to the result that $V^{1/4}$ will have to be a few times $10^{16}\,\mathrm{GeV}$ if there is a detection. It has pointed out by several authors [33,34,29,35,27] that such a big inflationary potential is difficult to understand on the basis of particle theory, which might generically suggest a scale of order $(mm_{Pl})^{1/2}$ or $(mm_{Pl}^2)^{1/3}$ with $m \sim 10^2\,\mathrm{GeV}$. More particularly, one does not *expect* such a potential to be generated by the Higgs sector of a GUT, because this would give (at the maximum) $V \sim m_h^2\langle\phi_h\rangle^2$ and although coupling constant unifica-

*If the potential is $V \simeq V_0 - \frac{1}{2}m^2\phi^2$ one has $(\Delta\phi/m_{Pl})^2 \simeq V_0/(m_{Pl}^2 m^2) = 2/(1-n) \gg 1$ (where $1-n$ is the spectral index), but $r \simeq (6.9/2)(1-n)e^{-(1-n)N} < .051(25/N)$ which is undetectable. If $V \simeq V_0[1-(\phi/M)^p]$, with $p > 2$ and $M \gtrsim m_{Pl}$, one has $\Delta\phi \simeq M \gtrsim m_{Pl}$ but $r = 6.9p^2(M/m_{Pl})^{\frac{2p}{p-2}}[Np(p-2)]^{-\frac{2p-2}{p-2}}$ which is detectable only if M is very big ($M > 6.3m_{Pl}$ if $p=3$ and $M > 8.4m_{Pl}$ if $p=4$). Similar results hold if V is a mixture of terms, say quadratic at small ϕ and quartic at larger ϕ, provided that all terms have the same sign.

tion suggests vevs $\langle\phi_h\rangle \sim 10^{16}$ GeV there is no reason for the masses m_h to be so big [36]. But in the face of a measured $V^{1/4}$ of this order one might set aside all prejudice, and look at the viablity of a hybrid inflation model with a GUT higgs as the non-inflaton field and a *large* inflaton field variation.

To summarize, the observation of a gravitational wave signal in the cmb anisotropy would require a revision of current thinking about the likely form of the inflationary potential, in respect of both the field variation and the height of the potential. Turning the viewpoint around, it is fair to say that there is at present a considerable theoretical prejudice against the likelyhood of such an observation.

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