

Evading the Cosmological Domain Wall Problem

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Abstract

Discrete symmetries are commonplace in field theoretical models but pose a severe problem for cosmology since they lead to the formation of domain walls during spontaneous symmetry breaking in the early universe. However if one of the vacua is favoured over the others, either energetically, or because of initial conditions, it will eventually come to dominate the universe. Using numerical methods, we study the evolution of the domain wall network for a variety of field configurations in two and three dimensions and quantify the rate at which the walls disappear. Good agreement is found with a recent analytic estimate of the termination of the scaling regime of the wall network. We conclude that there is no domain wall problem in the post-inflationary universe for a weakly coupled field which is not in thermal equilibrium.

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I. INTRODUCTION

It was first noted by Zel'dovich, Kobzarev and Okun [1] that the restoration of spontaneously broken *discrete* symmetries at high temperatures in the early universe poses severe problems for its subsequent evolution.¹ If the manifold \mathcal{M} of degenerate vacua of the theory is disconnected such that the homotopy group $\pi_0(\mathcal{M})$ is non-trivial, then sheet-like topological defects — domain walls — form at the boundaries of the different degenerate vacua during the symmetry breaking phase transition, due to the existence of a causal particle horizon in a decelerating universe [5]. The subsequent evolution of the wall network can be studied using the techniques of percolation theory [6] and is such that the energy density in the walls eventually comes to dominate the total energy density. The consequence is an inflationary phase which suffers from a ‘graceful exit’ problem in that it creates an universe devoid of any matter [7,8]. This can be avoided if the energy scale associated with the discrete symmetry breaking is low enough; however in order to avoid generating excessive anisotropy in the cosmic microwave background, the energy scale is further restricted to be smaller than ~ 1 MeV [8]. Thus walls formed at any higher energy scale must be *unstable* and have decayed away long before the present epoch.

Attempts to introduce instability in domain walls which are expected in field theoretical models have usually focussed on the role of non-renormalizable operators in the Lagrangian which may arise due to violation of global symmetries by Planck-scale gravitational effects [9]. These can ‘tilt’ the potential so as to favour one particular minimum which comes to dominate the universe because the *pressure* of the favoured vacuum eventually wins out over the restraining tension of the domain walls [1,10]. An example from particle physics is the addition of a dimension-5 operator to the Lagrangian of the next-to-minimal supersymmetric standard model, in which the usual supersymmetric Higgs sector is supplemented by a gauge singlet superfield. Here Z_3 domain walls are expected to be created in the Higgs fields during the electroweak symmetry breaking phase transition at $T \sim m_W$ [11]. However the dissipation of the wall network in this case releases the contained energy as high energy interacting particles which can interfere adversely with cosmological processes such as primordial nucleosynthesis [12], hence the walls must disappear well *before* this epoch [13]. In order to quantify this constraint, it is necessary to study the rate at which the wall network dissipates for a given pressure difference between the different vacua.

Another way in which the domain wall network may be destabilized is through a suitable choice of initial conditions, viz. if the probability distribution has a *bias* for one vacuum over the others. This can be due to a prior non-equilibrium phase transition, for example cosmological inflation, which can displace weakly coupled light fields from the minima of their potential [14], leading to a biased initial state [15]. It has been suggested that the resulting unstable domain walls in a light scalar field may be relevant for the formation of large-scale structure in the universe [16]. Recently, both an analytic [17] and a numerical [18] study of such biased domain walls have been carried out.

¹It has been argued recently [2], following earlier work [3], that symmetry restoration may not necessarily occur, particularly if the scalar field has no gauge interactions. Whether this is still true when non-perturbative effects are taken into account is currently under investigation [4].

In this paper we make a systematic investigation of domain wall network evolution for both cases, viz. ‘pressure’ (§III (A)) and ‘bias’ (§III (B)) and compare our results with simple analytic estimates of the rate at which the walls disappear, as well as with previous numerical work [17,18]. We also study (§III (C)) the effects of biased initial field configurations resulting from a period of primordial inflation. We conclude (§IV) with a discussion of the physical implications of our results.

II. NUMERICAL TECHNIQUES

The generic potential which exhibits the problem under discussion is

$$V(\phi) = V_0 \left(\frac{\phi^2}{\phi_0^2} - 1 \right)^2. \quad (1)$$

This has two degenerate vacua, $\phi = \pm\phi_0$ separated by a potential barrier V_0 , so that the spontaneous breaking of the Z_2 symmetry when the universe cools below the temperature $T \sim V_0$ will result in the formation of domain walls. We wish to simulate the evolution of the resulting wall network by solving the field equations on a specified lattice. The basic problem in such numerical studies is that there are two very different length scales involved, viz. the wall width,

$$w_0 \sim \frac{\phi_0}{\sqrt{V_0}}, \quad (2)$$

and the size of the simulation box. The first is constant in physical coordinates whilst the second must be large relative to a typical domain size, which scales with the expansion of the Universe. However the wall thickness is in general much smaller than the size of the wall network so one can sensibly assume that the walls behave like two-dimensional relativistic membranes, given that their internal structure is expected to be of negligible importance for the evolution. Two routes have so far been pursued in applying this idea to computer simulations of domain wall dynamics.

Kawano [19] considered an effective action obtained by expanding in w_0/R , where R is the radius of curvature of the wall, and retaining only the zeroth order term. The resultant Nambu-type action yields [20] the evolution equation

$$\ddot{R} + 3\frac{\dot{a}}{a}\dot{R}(1 - \dot{R}^2) = -\frac{2(1 - \dot{R}^2)}{R}, \quad (3)$$

where a is the cosmological scale-factor of the Robertson-Walker metric for an Einstein-DeSitter universe, $g_{\mu\nu} = \text{diag}[-1, a(t)^2, a(t)^2, a(t)^2]$. Unfortunately this otherwise attractive approach leads to severe numerical instabilities (of the type discussed in a slightly different context [21]), hence cannot be fruitfully pursued.

The second approach, which we use in the present work is due to Press, Ryden and Spergel [22,23]. Consider the classical equation of motion for the Higgs field in an expanding background:

$$\frac{\partial^2 \phi}{\partial \eta^2} + 2\frac{d \ln a}{d \ln \eta} \frac{1}{\eta} \frac{\partial \phi}{\partial \eta} - \nabla^2 \phi = -a^2 \frac{\partial V}{\partial \phi}, \quad (4)$$

where η is the conformal time ($d\eta \equiv dt/a(t)$) which measures the *comoving* distance traversed by light since the big bang. Here the numerical problem is manifest in the a^2 term on the rhs which makes the potential barrier appear higher as time goes on, resulting in a wall solution which grows increasingly narrow (in comoving coordinates) with time. In attempting to evolve an equation of this type directly one would find that the wall solutions in the ϕ field quickly become so narrow as to be unresolvable. The solution is to generalize the equation of motion to:

$$\frac{\partial^2 \phi}{\partial \eta^2} + \alpha \frac{d \ln a}{d \ln \eta} \frac{1}{\eta} \frac{\partial \phi}{\partial \eta} - \nabla^2 \phi = -a^\beta \frac{\partial V}{\partial \phi} , \quad (5)$$

and then set $\beta = 0$ in order to freeze the wall size in comoving coordinates. We also set $\alpha = 3$ to ensure momentum conservation since this requires that we have [22]

$$\alpha + \frac{\beta}{2} = 3 . \quad (6)$$

Press *et al.* have discussed in some detail the justification for this approximation and demonstrated that it has a negligible effect on the evolution by performing simulations comparing $\alpha = \beta = 2$ and $\alpha = 3, \beta = 0$ results over a limited range of η . We perform a test of a complementary nature as described below which supports their contention that the numerical method is robust and unlikely to give spurious results.

Following ref. [22] we implement this procedure using the standard finite differencing scheme embodied in the expressions

$$\delta \equiv \frac{1}{2} \alpha \frac{\Delta \eta}{\eta} \frac{d \ln a}{d \ln \eta} , \quad (7)$$

$$(\nabla^2 \phi)_{i,j,k} \equiv \phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,j+1,k} + \phi_{i,j-1,k} + \phi_{i,j,k+1} + \phi_{i,j,k-1} - 6\phi_{i,j,k} , \quad (8)$$

$$\dot{\phi}_{ijk}^{n+1/2} = \frac{(1-\delta)\dot{\phi}_{ijk}^{n-0.5} + \Delta\eta(\nabla^2\phi_{ijk}^n - \partial V/\partial\phi_{ijk}^n)}{1+\delta} , \quad (9)$$

$$\phi_{ijk}^{n+1} = \phi_{ijk}^n + \Delta\eta\dot{\phi}_{ijk}^{n+0.5} . \quad (10)$$

We also use the same algorithm [22] for measuring the total wall area in the ϕ field. Our simulations were run in both two and three dimensions, on a $128 \times 128 \times 128$ grid ($D = 3$) and a 1024×1024 grid ($D = 2$).

III. WALL NETWORK EVOLUTION

The general evolution of the domain wall network is well known [8]; the most interesting phase is that of ‘Kibble scaling’, in which the walls have a correlation length ξ in comoving units given by

$$\xi \simeq v\eta , \quad (11)$$

where v is the velocity of wall propagation. The simplest variable which tracks the evolution is then the comoving area of the wall network per unit volume, (A/V) , which scales as

$$(A/V) \simeq (A/V)_0 \frac{\xi_0}{\xi} \simeq \frac{\xi_0}{v(\eta - \eta_0)} , \quad (12)$$

while the physical energy density contained in walls behaves as a^{-1} times this. We find that there is indeed a scaling region with

$$(A/V) \propto \eta^{-\nu}, \quad \nu = \begin{cases} 0.95 \pm 0.08 & \text{for } D = 2, \\ 0.92 \pm 0.08 & \text{for } D = 3, \end{cases} \quad (13)$$

in agreement both with Eq.(12) and other simulations [22,18]. (The errors quoted here are purely statistical). Hence we can calculate v , as defined in Eq.(11), to be

$$v = \begin{cases} 0.7 \pm 0.1 & \text{for } D = 2, \\ 0.5 \pm 0.1 & \text{for } D = 3, \end{cases} \quad (14)$$

(Note that the velocity which was found previously to be ~ 0.4 in three dimensions [22], was somewhat differently defined.) We shall henceforth simplify our algebra by normalising the values of the comoving energy density and scale factor to be unity at the initial time of our simulation, here denoted with the subscript 0. The evolution of the network thus proceeds as follows. For the initial few timesteps, with $\eta \lesssim 10$, the correlation length is less than the wall thickness, before the network enters the scaling regime of equation (12) which ends at $\eta \sim 100$ when it becomes comparable to the size of the lattice.

To verify the suitability of the numerical method used we ran a suite of simulations for $D = 3$ where β is increased in steps of 0.1 from zero and α is adjusted according to Eq.(6). In order for the domain wall to be resolvable, it should occupy at least one (preferably several) lattice spacing(s); this corresponds to a limiting value of $\beta \simeq 0.66$ in our simulations. In figure 1 we show that the exponent ν in Eq.(13) does not change significantly as β is increased up to this value.² Overall the gross features of the evolution appear to be insensitive to the numerical approximation used; in particular the onset and termination of the scaling regime are *not* dependent on β . This adds further weight to the conclusion of Press *et al.* that there is no difference between the dynamics of ‘real’ walls which have constant physical thickness, and the ‘artificial’ walls in these simulations which have constant comoving thickness.

A. Pressure

A well known way [10,24] to evade the causal limit on the disappearance of the domain walls is to include a pressure term in the potential, viz.

$$V(\phi) = V_0 \left[\left(\frac{\phi^2}{\phi_0^2} - 1 \right)^2 + \mu \frac{\phi}{\phi_0} \right], \quad (15)$$

The dynamics is now expected to be dominated by two competing forces, the surface tension σ/R where $R \simeq a\xi$ is the radius of curvature of the wall structure, and the pressure which

²There appears to be a small systematic trend of decreasing ν with increasing β . Although not statistically significant, this may just reflect the increasing efficiency of the area measuring algorithm at picking up small bubbles at late times, as β is increased.

is equal to the differences in energy density of the two minima of the potential, δV . Then we expect simple scaling behaviour (12) until some conformal time η_c at which the pressure becomes comparable to the surface tension, and the wall network disappears exponentially fast. Now η_c is given by

$$\sigma \frac{a(\eta_0)}{(\eta_c - \eta_0)a(\eta_c)} = \delta V , \quad (16)$$

where the subscript 0 indicates the value at the beginning of the evolution. In the present case we are scaling the fields with time so as to remove factors of a and set $\eta_0 = 1$ so this equation will reduce simply to

$$\frac{\sigma}{\eta_c} = 2\mu . \quad (17)$$

We display this behaviour in figure 2, where the comoving area density is plotted against conformal time for several choices of ε . As expected, there is an exponential fall-off from the scaling regime at some value of η_c which decreases with increasing μ . The ‘bounces’ in the curves occur as the bubbles of the disfavoured vacuum collapse and radiate away the energy contained in the walls in the form of Goldstone bosons; as noted earlier [25] a large fraction of the energy is lost in a few bounces. The relation between μ and η_c is shown in figures 3 and 4, which show excellent agreement with the expected behaviour (17) to within a factor of 2. (Here η_c is defined as the value of η at which the product $\eta(A/V)$ has fallen off by some factor taken here to be 10 and 100 for illustration.) The actual rate of the decay of the wall network after the exponential decay has set in is hard to measure. We find here that assuming a behaviour of form $(A/V)_c \sim \eta^{-1} \exp[-\kappa(\mu\eta)^n]$ with κ some constant, n is $\approx 2 \pm 1$ ($D = 2$) and $\approx 3 \pm 1$ ($D = 3$).

The implication of this is that we can regard the walls as simply disappearing as a consequence of the difference in energy between the two minima of the potential δV at a correlation length $R_c = \sigma/\delta V$ (in physical, not comoving, units), where again the constant of proportionality is around unity. Hence, if we have walls forming at the electroweak scale and we require that they disappear before nucleosynthesis [12], we find

$$\frac{\delta V}{\sigma} > \frac{1}{10^{10} \text{ cm}} \sim 10^{-24} \text{ GeV} , \quad (18)$$

in good agreement with previous estimates made from physical arguments as to the time of pressure domination [10] and 2-dimensional thin wall simulations [26].

B. Bias

Next we consider the possibility of a deviation from scaling behaviour due to a *biased* initial probability distribution. For concreteness we consider the Z_2 case where there are only two distinct vacua and we generate the initial configuration with a probability

$$p_+ = 0.5 + \varepsilon \quad (19)$$

that each initial domain is in the $+$ phase, where $\varepsilon > 0$. A similar exercise has been performed recently by Coulson, Lalak and Ovrut [18] who find that the domain wall network

then evolves much more rapidly than in the usual case of Eq.(12), and the favoured domain rapidly dominates the universe. Such a scenario is interesting because a bias in a light scalar field which is not in thermal equilibrium can be generated naturally by a previous epoch of inflation. Other possible ways in which a scalar field is more likely to find itself in one minimum than the other are if the symmetry is approximate, so that the two minima are not exactly degenerate, or if the symmetry breaking occurs through some intermediate phase which allows one minimum to be preferentially populated.

The evolution is shown in figure 5, where we plot comoving area density against conformal time for several choices of ε , ranging upto 0.03. Typically, the behaviour is similar to the usual case of no bias, with an exponential decay of the comoving energy density starting at some critical timescale η_c . (For $\eta \lesssim 10$ the behaviour is unphysical since ξ is less than the domain wall thickness (here scaled to be 5), while for $\eta \gtrsim 100$, ξ is approaching the physical size of the box, and hence the behaviour is again unphysical.) Notice that the wall network dissipates rapidly as ε is increased above zero, as was also found in ref. [18]. (Again we see the ‘bounces’ associated with the radiating away of the energy contained in the wall network.)

Hindmarsh [17] has recently made an analytic study of biased domain walls. He considers the evolution of the domain walls in terms of a scalar field constructed so that its zeros correspond to the locations of the domain walls, and whose evolution can be calculated in terms of Gaussian average field configurations; the expectation for the wall surface area density in D dimensions is

$$(A/V) \sim \frac{1}{\eta} \exp\left(-\kappa\varepsilon^2\eta^D\right), \quad (20)$$

where κ is some constant which should, in our units, be of order unity. The above result was obtained using rather sophisticated techniques but we can derive it from a much simpler (but perhaps less trustworthy) counting argument. We consider the wall evolution to proceed in such a way that the universe naturally divides itself up into domains of size $\xi \simeq v\eta$ (in comoving coordinates). Such domains have been causally connected, and so have had time enough to organize themselves to be either all + or all -. Hence we can see that the comoving area density in the absence of bias will behave as $(A/V) \simeq \xi^{-1} \simeq (v\eta)^{-1}$ as expected. We can now examine the behaviour of the comoving area density in the more complicated situation when there is a bias. Now we expect a domain of conformal size ξ to become a + domain if most of its ξ^3 subdomains are +, and a - domain otherwise. (We have normalized so that the size of the initial domains is unity.) If the probability of each subdomain entering the positive minimum is p_+ , the probability for the whole domain to be - is given by a sum over values of a Poisson distribution which can, in any interesting case, be taken to be a Gaussian integral, viz.

$$\mathcal{P}(\text{domain of } N \text{ sites, mostly } -) = \text{erf}\left(\frac{\bar{N} - N/2}{\sigma_N}\right) = \text{erf}(\sqrt{2\varepsilon}\sqrt{N}), \quad (21)$$

where $\bar{N} = (0.5+\varepsilon)N$ is the mean number of + sites and $\sigma_N = \sqrt{N}$ is the standard deviation. Here the Gaussian integral $\text{erf}(x) \equiv \int_x^\infty dt e^{-t^2/2}/\sqrt{2\pi}$ can be adequately approximated for large x by

$$\text{erf}(x) = \frac{1}{x\sqrt{2\pi}}e^{-x^2/2}. \quad (22)$$

We see that for a box with ξ sites, the probability given in Eq.(21) is just $\text{erf}(\sqrt{2}\varepsilon\xi^{D/2})$ and hence we expect exactly the exponential behaviour shown earlier in Eq.(20). The extra factor of $(\xi^{D/2})^{-1}$ is cancelled by a combinatoric factor from the many different ways in which we can lay down our domains upon the overall network. This uses the fact that a cluster containing N sites will typically scale as $N^{1/2}$, using the results of percolation theory as recently applied to similar problems in cosmology [27].

To verify this behaviour, we define η_c to be the conformal time at which the product $\eta(A/V)$ has decreased by some substantial factor, which we choose to be 10 and 100 for illustration. The values of $\eta_c^{-D/2}$ are plotted against ε for both 2 and 3 dimensions in figures 6 and 7 and we find good agreement with the theoretical prediction of a straight line. Note that for both cases, we have cut off the figures at $\eta \simeq 10$ since before this conformal time the domain wall network is not yet fully formed. We also expect unreliable results when η reaches 100 or 1000 in three and two dimensions respectively, since then the correlation length is approaching the size of our whole simulation. While we can measure the relationship between η_c and the bias ε , it is again harder to measure the actual rate of the exponential fall off, i.e. the exponent of η in the exponential. We find this exponent to be $\approx 1 - 2$ ($D = 2$) and $\approx 2 - 3$ ($D = 3$), in acceptable agreement with theory.

Therefore we can, for cosmological purposes, regard the wall evolution as being in the Kibble scaling regime for early times, and modelled subsequently by an exponential fall off at some conformal time η_c , which will lead to an almost immediate collapse of the wall network. The required value of ε is readily calculable in three dimensions from

$$\varepsilon = \left(\frac{\eta_c}{\eta_0}\right)^{-3/2} = \left(\frac{T_0}{T_c}\right)^{-3/2}, \quad (23)$$

where η_c (T_c) is the scale factor (temperature) at the time of disappearance and η_0 (T_0) at the time of domain formation. (Note that here we put $\kappa \simeq 1$ from our simulation result.) For example, in the event of wall formation at the weak scale of order 100 GeV, with the requirement that the walls disappear before the nucleosynthesis era which starts at ~ 1 MeV [12], we find that $\eta_c/\eta_0 \sim 10^5$, and hence that the bias in the initial probability distribution must be of order 10^{-8} or greater. However if the walls form subsequent to reheating following inflation, at an energy scale of say 10^6 GeV, the bias need only be about 10^{-14} .

C. Realistic Field Configurations

It is usually assumed [5] that following cosmological discrete symmetry breaking in a scalar field, the field values will be uncorrelated on scales larger than the causal (particle) horizon, so the degenerate vacua will be populated with equal probability. However, a prior period of cosmological inflation *can* result in correlations on (apparently) super-horizon scales if the field is sufficiently weakly coupled so as not to be in thermal equilibrium. This is because quantum fluctuations during inflation (with Hubble parameter H) will induce long-wavelength fluctuations in all scalar fields with mass $m \ll H$ on spatial scales $k^{-1} \gg H^{-1}$

[14]. For a minimally coupled field with vanishing potential (e.g. a (pseudo-) goldstone boson), this leads to the formation of a classical inhomogeneous field, with gaussian probability distribution [28]

$$\mathcal{P}(\phi) \propto \exp \left[-\frac{(\phi - \bar{\phi}_k)^2}{2\sigma_k^2} \right], \quad \sigma_k^2 \simeq \frac{H^2}{4\pi^2} \int_k^{k_c} d \ln k. \quad (24)$$

where $\bar{\phi}_k$ is the mean value of the field averaged over a domain of size k^{-1} and k_c is an effective ultraviolet cutoff imposed by physical considerations (dependent on the nature of the field). Since inflation blows up the scale factor exponentially fast, this provides a natural mechanism for bias since the global ensemble average of ϕ may not be realized even over regions as big as our present universe. During the post-inflationary phase, the above distribution (24) averaged over a chosen scale thus becomes skewed [15]. We parametrize this for the Z_2 case by drawing probabilities for populating the two vacua from the distribution

$$\mathcal{P}(\phi) = \frac{1}{\sqrt{2\pi\sigma}} \left[(0.5 - \varepsilon) e^{-(\phi+1)^2/2\sigma^2} + (0.5 + \varepsilon) e^{-(\phi-1)^2/2\sigma^2} \right], \quad (25)$$

where ε is the effective bias over the chosen scale. (Presumably its value can be calculated in principle if the field and inflationary parameters are specified.) In figures 8 and 9 we show the comoving area density plotted against conformal time for several choices of σ with $\varepsilon = 0, 0.005$. Surprisingly, the exponential fall-off from the scaling regime appears to be quite insensitive to the width of the probability distribution. This supports the suggestion [15] that this is an efficient way to eliminate the domain wall network.

IV. CONCLUSIONS

We have considered two possible ways in which a cosmological domain wall network can be made unstable. The first of these is that of pressure, i.e. a small breaking of the degeneracy between the minima. The physical implications of this are well understood and we have simply confirmed numerically the usual argument [10] that when the typical domain size R has grown to a critical value R_c , the wall energy density decays exponentially fast, so the network disappears essentially instantaneously. The value of R_c is

$$R_c = \frac{\sigma}{\delta\rho}, \quad (26)$$

where σ is the wall surface energy density and $\delta\rho$ is the difference in energy density between the two minima. (There may be an additional effect due to the bias induced at the phase transition by the energy difference between the two minima.)

The second mechanism is that of bias, where the initial field configuration is not symmetric between the various minima of the potential. Here we confirm previous analytical [17] and numerical [18] studies which predict an exponential decay in the energy density with a characteristic conformal time η_c . This is an attractive solution to the domain wall problem if we can find a natural mechanism to bias the initial distribution, particularly since we find the evolution to be insensitive to the width of the distribution. Since such a bias must appear on (apparently) causally disconnected scales, it can only occur if either the

minima are genuinely inequivalent (and so have non-degenerate energy densities) or else if we are in a post-inflationary phase in which the field has correlations on super-horizon scales. This will allow the universe to more efficiently organise itself into one preferred minimum everywhere, thus enabling the wall network to decay away. Now an initial field configuration with correlations on all wavelengths can be characterized by an effective bias $\varepsilon(R)$ averaged over a box of size R , as high frequency modes will average out to zero. We have seen that a region of size R will decay away exponentially after a conformal time $\eta_c(R) \sim \varepsilon(R)^{-D/2}$. For a radiation dominated universe, the time $t(R)$ at which the domain will decay is then

$$t(R) \sim \varepsilon(R)^{-3} , \tag{27}$$

which must grow more slowly than R to ensure that the ordinary causal scaling bound is exceeded. Hence for bias to provide a realistic means of eliminating wall networks through an initially non-equilibrium field distribution, we must have that $\varepsilon(R)$ falls off with R more slowly than $R^{-1/3}$. Since the spectrum of fluctuations from inflation is (nearly) scale-invariant, we may expect that the bias $\varepsilon(R)$ does stay approximately constant with R , and hence that this mechanism provides a way of eliminating domain walls in fields which are sufficiently weakly coupled so as not to be in thermal equilibrium.

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FIGURES

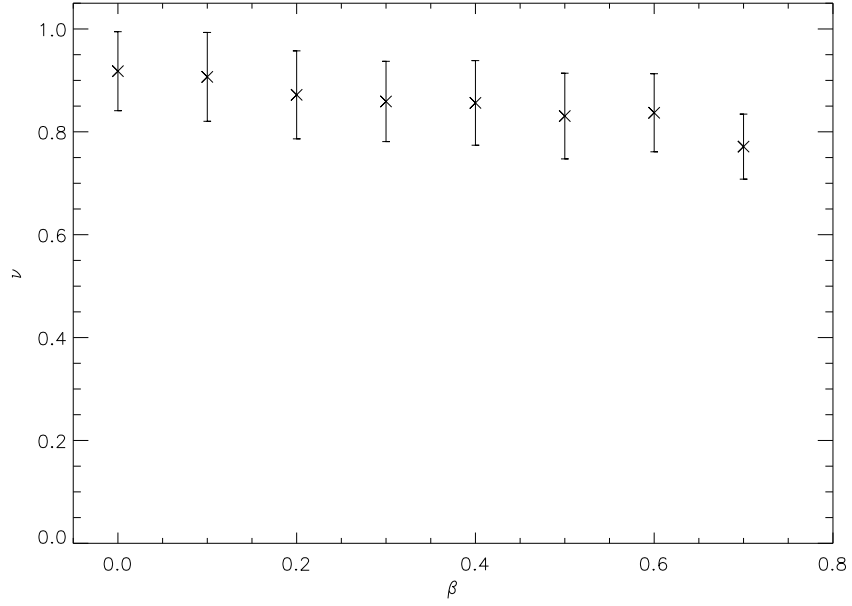


FIG. 1. A test of the numerical approximation used. The scaling index ν of the wall network is seen not to vary significantly as the parameter β in the equation of motion is varied.

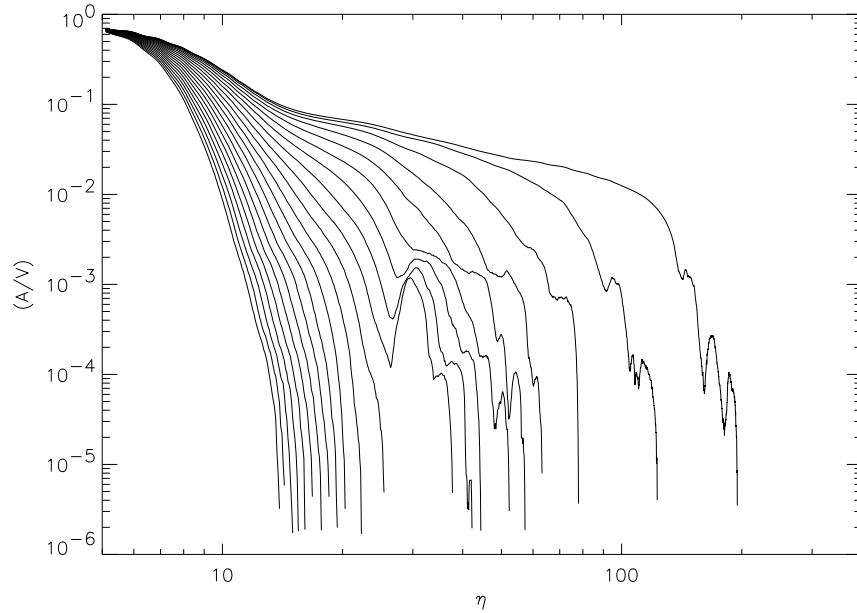


FIG. 2. Comoving area against conformal time in 3 dimensions, with the pressure μ in the range 0 – 0.2.

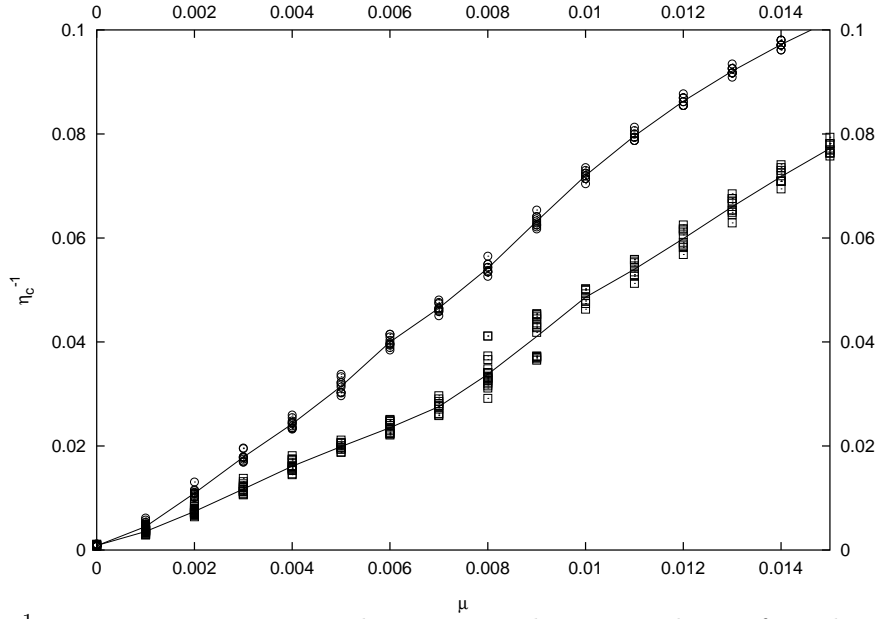


FIG. 3. η_c^{-1} against pressure μ in 2 dimensions, where η_c is the conformal time at which the product of η and the comoving area density has fallen by a factor 10 (circles) or 100 (squares). The line connects the mean value of η_c over all the runs.

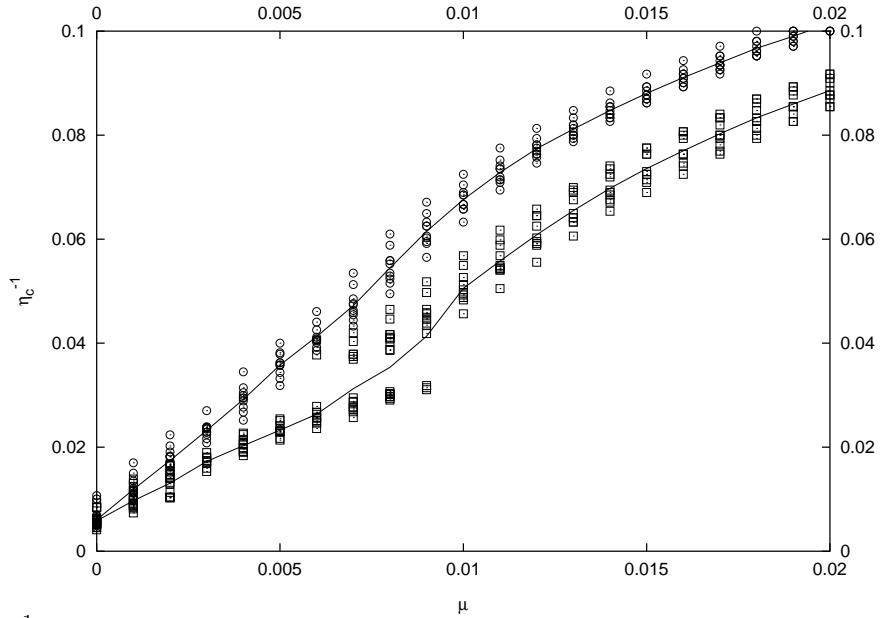


FIG. 4. η_c^{-1} against pressure μ in 3 dimensions, where η_c is the conformal time at which the product of η and the comoving area density has fallen by a factor 10 (circles) or 100 (squares). The line connects the mean value of η_c over all the runs.

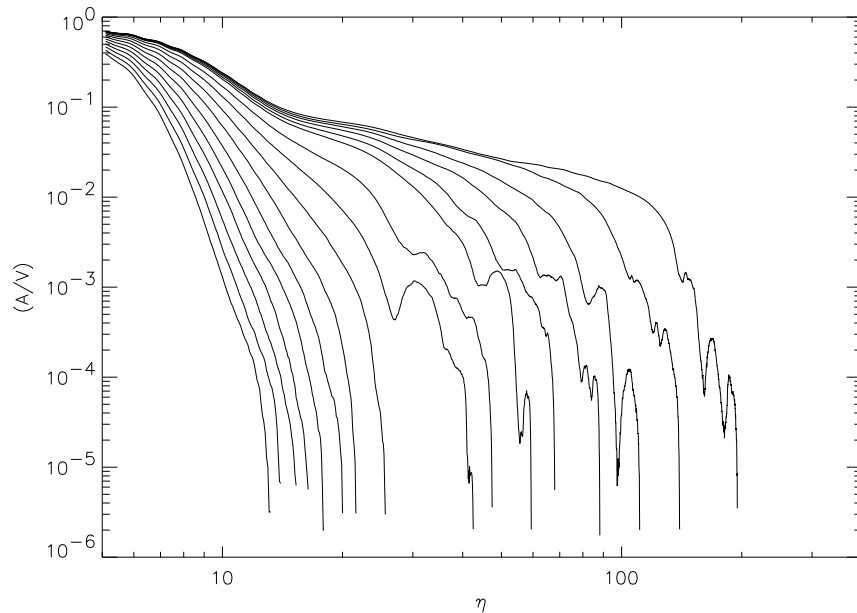


FIG. 5. Comoving area against conformal time in 3 dimensions with the bias ε in the range $0 - 0.03$.

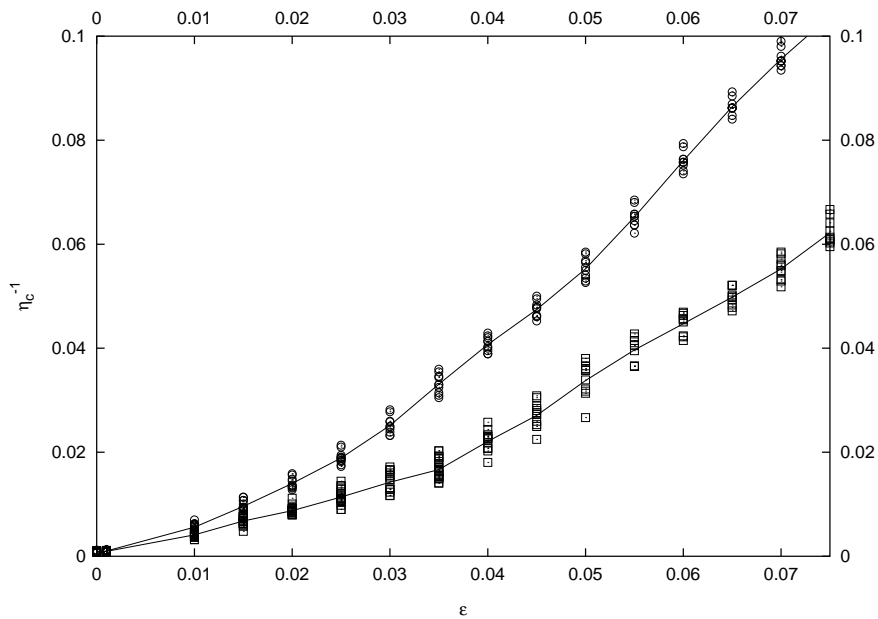


FIG. 6. η_c^{-1} against bias ε in 2 dimensions, where η_c is the conformal time at which the product of η and the comoving area density has fallen by a factor 10 (circles) or 100 (squares). The line connects the mean value of η_c over all the runs.

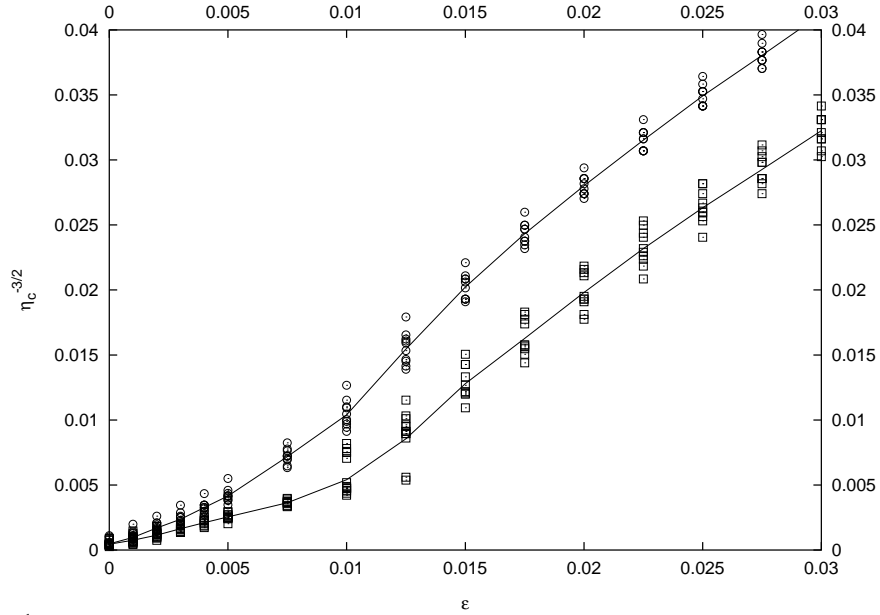


FIG. 7. η_c^{-1} against bias ε in 3 dimensions, where η_c is the conformal time at which the product of η and the comoving area density has fallen by a factor 10 (circles) or 100 (squares). The line connects the mean value of η_c over all the runs.

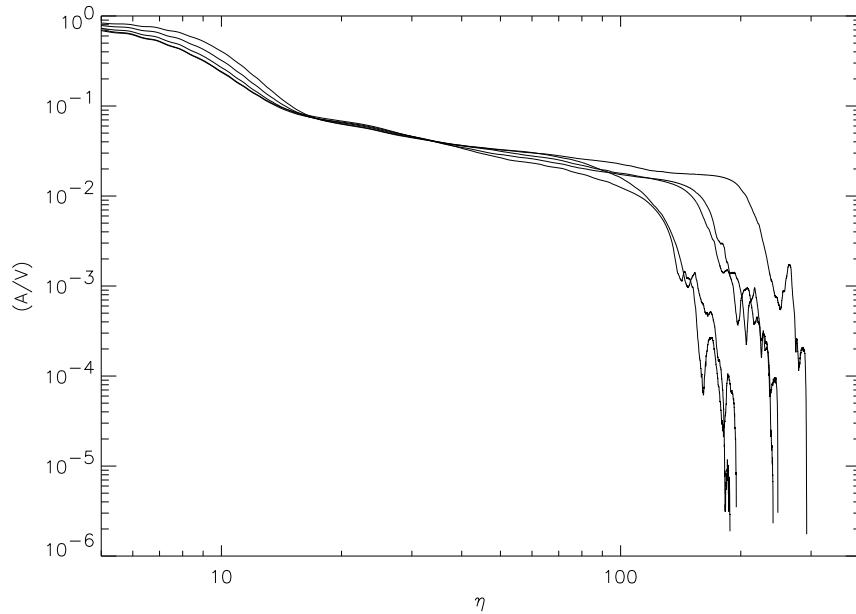


FIG. 8. Comoving area against conformal time for post-inflationary field configurations, parametrized by the width $\sigma = 0, 0.5, 1, 1.5, 2$ with no bias.

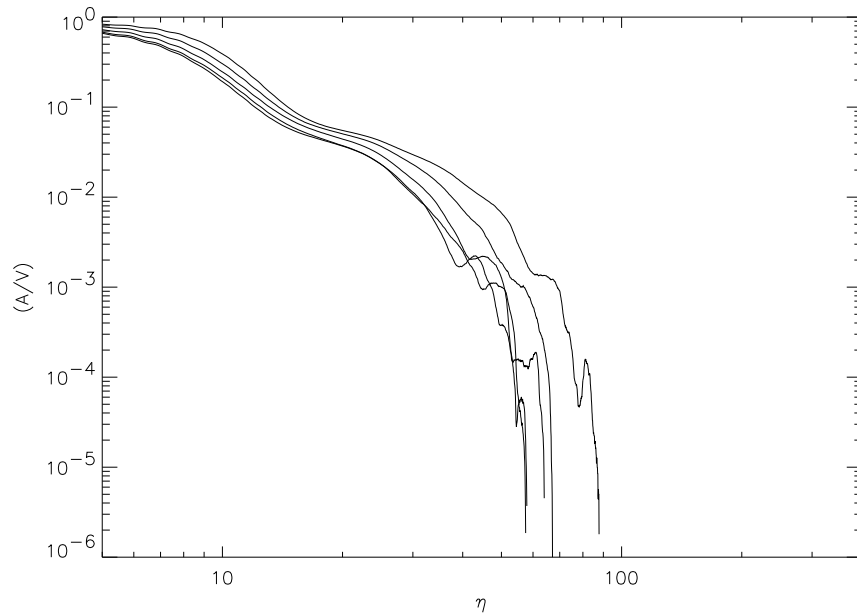


FIG. 9. Comoving area against conformal time for post-inflationary field configurations, parametrized by the width $\sigma = 0, 0.5, 1, 1.5, 2$ with bias $\varepsilon = 0.005$.