A MODEL OF COMPOSITE ELECTRONS AND PHOTONS

T. PRADHAN

Institute of Physics, Bhubaneswar-751005, INDIA

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ABSTRACT

It is shown that electrons and photons can be considered as composities of particles belonging to the fundamental representations of the extended Lorentz group $SU(3) \otimes$ SU(3) in (8+1) dimensional space-time which are held together by force mediated by gauge particles belonging to the regular representation of this group. In this theory, spin states of electron, positron and photon form a 8 \otimes 8 representation of the group. One can also accommodate the electron neutrinos, W and Z bosons as well super partners of all these particles in this model. The theory allows electron decay into photon and neutrino. There are only two parameters in this theory. These are the mass of the fundamental particles and the fine structure constant of their coupling with gauge particles. The former, estimated from the experimental limit on the electron life-time works out to be greater than 10^{-22} GeV and the latter works out to be unity by making use of Dirac ground state energy formula for the composite neutrino which is taken to be massless. These indicate that the compositeness of electron, photon and other particles will be revealed at Planckian energies.

Introduction of the concept of isospin space by Heisenberg [1] and symmetry of the Hamiltonian with respect to rotation in this space provided a high degree of order in the description of interaction among elementary particles. Extension of this space and the symmetry by Gell-Mann and Ne'eman [2] revealed that all hadrons are composed of quarks [3] held together by force provided by coloured gluone [4]. Experimental evidence of this composite nature of hadrons from inelastic electron scattering and jet events [5] is by now well established. However, leptons are still regarded as elementary particles. The highly successful Standard Model [6] of particle interactions treats electrons as fundamental stable particles. However experiments put a limit to its stability $(\tau > 2.7 \times 10^{23})$ years) [7]. This is an indication of a substructure of the electron with composition provided by more fundamental objects. Quest for such objects have been made through invocation of new internal symmetries [8]. Can it be that this may be better achieved through extension of space-time symmetry similar to Gell-Mann-Ne'eman extension of Heisenberg's internal symmetry which revealed quark structure of hadrons? It is the purpose of this communication to make an attempt towards this end. To accomplish this we extend the Dirac equation of the (3+1) dimensional space-time with invariance under the Lorentz group $SU(2) \otimes SU(2)$ to an (8+1) dimensional space-time with invariance under the extended Lorentz group $SU(3) \otimes SU(3)$.

The Dirac equation in spinor representation [9]

$$(p_0 - \vec{\sigma} \cdot \vec{p})\xi_p = m\eta_p$$

$$(p_0 + \vec{\sigma} \cdot \vec{p})\eta_p = m\xi_p$$
(1)

where

$$\xi_p = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}, \ \eta_p = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$
(2)

are form-invariant under the Lorentz group of transformations

$$\delta p_i = \varepsilon_{ijk} p_j \theta_k - p_0 \phi_i$$

$$\delta p_0 = -p_i \phi_i$$

$$i, j = 1, 2, 3$$
(3)

under which

$$\delta\xi_p = i\frac{\sigma_i}{2}(\theta_i - i\phi_i)\xi_p = i[K_i, \xi_p](\theta_i - i\phi_i)$$

$$\delta\eta_p = i\frac{\sigma_i}{2}(\theta_i + i\phi_i)\eta_p = i[L_i, \eta_p](\theta_i + i\phi_i)$$
(4)

.

where the generators

$$K_{i} = \frac{1}{2} \xi_{p}^{+} \sigma_{i} \xi_{p}$$

$$L_{i} = \frac{1}{2} \eta_{p}^{+} \sigma_{i} \eta_{p}$$
(5)

of the Lorentz group obey $SU_2(\xi)\otimes SU_2(\eta)$ algebra

$$[K_i, K_j] = i\varepsilon_{ijk}K_k$$

$$[L_i, L_j] = i\varepsilon_{ijk}L_k$$

$$[K_i, L_j] = 0$$

$$i, j, k = 1, 2, 3$$
(6)

We shall now extend this algebra (6) to $SU_3(\xi) \otimes SU_3(\eta)$:

$$[K_{a}, K_{b}] = if_{abc}K_{c}$$

$$[L_{a}, L_{b}] = if_{abc}L_{c}$$

$$[K_{a}, L_{b}] = 0$$

$$a, b, c = 1, 2, 3.....8$$
(7)

and extend equations (1) to

$$(p_0 - \frac{1}{2}\lambda_a p_a)\xi = m\eta$$

$$(p_0 + \frac{1}{2}\lambda_a p_a)\eta = m\xi$$
(8)

where

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_2 \end{pmatrix} \eta = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$
(9)

$$K_a = \frac{1}{2}\xi^+ \lambda_a \xi, \quad L_a = \frac{1}{2}\eta^+ \lambda_a \eta \tag{10}$$

 λ_a being 3×3 SU(3) matrices. It can be verified that the pair equations (8) are equivalent to three pairs of equations one of which is equation (1) and the rest two are

$$(p_0 - \vec{\sigma}.\vec{\pi})\xi_{\pi} = m\eta_{\pi}, \qquad (p_0 - \vec{\sigma}.\vec{\kappa})\xi_{\kappa} = m\eta_{\kappa}$$

$$(p_0 + \vec{\sigma}.\vec{\pi})\eta_{\pi} = m\xi_{\pi}, \qquad (p_0 + \vec{\sigma}.\vec{\kappa})\eta_{\kappa} = m\xi_{\kappa}$$
(11)

where

$$\xi_{\pi} = \begin{pmatrix} \xi_1 \\ \xi_3 \end{pmatrix}, \ \eta_{\pi} = \begin{pmatrix} \eta_1 \\ \eta_3 \end{pmatrix}, \ \xi_{\kappa} = \begin{pmatrix} \xi_2 \\ \xi_3 \end{pmatrix}, \ \eta_{\kappa} = \begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}$$
(12)

and

$$\vec{\pi} = (p_4, p_5, \frac{p_3}{\sqrt{3}}) \quad \vec{\kappa} = (p_6, p_7, \frac{p_8}{\sqrt{3}})$$
 (13)

The three pairs equations appearing in (1) and (11) represent the dynamics of fundamental building blocks of our model. Noting that $K_8 = \frac{1}{2}\xi^+\lambda_8\xi = \frac{1}{\sqrt{3}}(\xi_p^+\xi_p - \frac{1}{2}\xi_\pi^+\xi_\pi - \frac{1}{2}\xi_\kappa^+\xi_\kappa)$, their weight diagram will be as given in Fig.1.

Fig.1

Our next task is to constructed electrons positrons and photons as well as neutrino, Z and W bosons using the fundamental particles discussed above. In this connection we note that since under parity transformation $\xi \leftrightarrow \eta$ it is adequate to construct composites in ξ -space only. In this space the electron and the positron can be constructed from the $SU_p(2)$ doublets $\xi_p^{(\mu)}, \xi_p^{(d)}, \xi_{\pi}$ and ξ_{κ} of this group. The superscripts u and d stand for spin up and down.

$$e^{(u)} = \frac{1}{\sqrt{2}} \xi_p^{(u)} (\xi_\pi^+ + \xi_\kappa^+)$$

$$e^{(d)} = \frac{1}{\sqrt{2}} \xi_p^{(d)} (\xi_\pi^+ + \xi_\kappa^+)$$

$$\overline{e}^{(u)} = \frac{1}{\sqrt{2}} \xi_p^{+(u)} (\xi_\pi + \xi_\kappa)$$

$$\overline{e}^{(u)} = \frac{1}{\sqrt{2}} \xi_p^{+(d)} (\xi_\pi + \xi_\kappa)$$
(14)

The right and left circularly polarized photon states can be constructed from ξ_p and ξ_p^+ :

$$A^{(R)} = \xi_p^{(u)} \xi_p^{+(d)}$$

$$A^{(L)} = \xi_p^{(d)} \xi_p^{+(u)}$$
(15)

The states

$$A^{(3)} = \frac{1}{\sqrt{2}} (\xi_p^{(u)} \xi_p^{+(u)} - \xi_p^{(d)} \xi_p^{+(d)})$$

$$A^{(8)} = \frac{1}{\sqrt{6}} (\xi_p^{(u)} \xi_p^{+(u)} + \xi_p^{(d)} \xi_p^{+(d)} - 2\xi_\pi \xi_\pi^+)$$
(16)

do not bind as the gauge coupling is repulsive for antiparallel spins [10]. The composites given in (14), (15) and (16) from an octet and are represented in the weight diagram given in Fig.2.

Fig.2

The neutrinos states are obtained by replacing $(\xi_{\pi} + \xi_{\kappa}) \frac{1}{\sqrt{2}}$ in (14) by $\frac{1}{\sqrt{2}} (\xi_{\pi} - \xi_{\kappa})$. For accomodating Z and W bosons it would be necessary modify the representation (15) and (16) for photon states to:

$$A = (\xi_p \xi_p^+)(\xi_\pi^+ \xi_\pi) = \Psi_p \Phi_p \tag{17}$$

where $\Psi_p = \xi_p \xi_p^+$ is a second rank spinor and $\phi_{\pi} = \xi_{\pi}^+ \xi_{\pi}$ is a scalar in the (p_0, p_1, p_2, p_3) space. In that case

$$Z = \Psi_p \Phi_\kappa$$

$$W = \Psi_p \Phi_{\kappa\pi}$$
(18)

where $\Phi_{\kappa} = \xi_{\kappa}^{+} \xi_{\kappa}$ and $\Phi_{\kappa, \subset} = \xi_{\kappa}^{+} \xi_{\pi}$ are scalars. It is to be noted that there exist composite scalar particles.

$$\tilde{e} = \frac{\phi_p}{\sqrt{2}} (\xi_{\pi}^+ + \xi_{\kappa}^+)
\tilde{\nu} = \frac{1}{\sqrt{2}} \phi_p (\xi_{\pi}^+ - \xi_{\kappa}^+)$$
(19)

(where $\phi_p = \xi_p^+ \xi_p$ is a scalar) which can be taken as super-partners of electron and neutrino. Similarly there exist spin $\frac{1}{2}$ composites

$$\tilde{A} = \xi_p \phi_{\pi}^+$$

$$\tilde{Z} = \xi_p \phi_{\kappa}^+$$

$$\tilde{W} = \xi_p \phi_{\kappa\pi}^+$$
(20)

which can be taken as super partners of A, Z and W.

The gauge coupling of the fundamental particles is obtained from local version of eqns.(1), (8) and (11) which are obtained by the replacements

$$p_0 \longrightarrow (p_0 - gG_0) \qquad \vec{p} \longrightarrow (\vec{p} - g\vec{G}_p)$$
$$\vec{\pi} \longrightarrow (\vec{\pi} - g\vec{G}_\pi) \qquad \vec{\kappa} \longrightarrow (\vec{\kappa} - g\vec{G}_k)$$
(21)

in the respective equations. The interaction Lagrangian obtained from the local equations work out to be

$$\mathcal{L}_{int} = g \overline{\psi}_{p} \gamma_{\mu} (v_{\mu}^{(p)} + \gamma_{5} a_{\mu}^{(p)}) \psi_{p}$$

$$+ g \overline{\psi}_{\pi} \gamma_{\mu} (v_{\mu}^{(\pi)} + \gamma_{5} a_{\mu}^{(\pi)}) \psi_{\pi}$$

$$+ g \overline{\psi}_{\kappa} \gamma_{\mu} (v_{\mu}^{(\kappa)} + \gamma_{5} a_{\mu}^{(\kappa)}) \psi_{\kappa}$$
(22)

where

$$\psi_p = \begin{pmatrix} \xi_p \\ \eta_p \end{pmatrix} \psi_\pi = \begin{pmatrix} \xi_\pi \\ \eta_\pi \end{pmatrix} \psi_\kappa = \begin{pmatrix} \xi_\kappa \\ \eta_\kappa \end{pmatrix}$$
(23*a*)

$$v_{\mu}^{(p)} = \frac{1}{2} [G_{\mu}^{(p)}(\xi) + G_{\mu}^{(p)}(\eta)]$$

$$a_{\mu}^{(p)} = \frac{1}{2} [G_{\mu}^{(p)}(\eta) - G_{\mu}^{(p)}(\xi)]$$
(23b)

and similar definitions for $v_{\mu}^{(\pi)}, a_{\mu}^{(\pi)}, v_{\mu}^{(\kappa)}$ and $a_{\mu}^{(\kappa)}$ with $\mu = 0, 4, 5, 8$ for π fields and $\mu = 0, 6, 7, 8$ for κ fields.

Equation of motion for the gauge fields

$$f_p = \frac{1}{2}\sigma_i(e_i + ib_i)$$
(24)
$$a = 1, 2, 3, \dots, 8$$

where e_i and b_i are components of the field tensor

$$F_{\mu\nu} = \partial_{\mu}v_{\nu} - \partial_{\nu}v_{\mu} + \varepsilon_{\mu\nu\lambda\xi}\partial_{\lambda}a_{\xi}$$
(25)

can be written as [11]

$$(p_0 - \vec{\sigma}.\vec{p})f_p = 0 \tag{26a}$$

In the extended space this can be written as

$$(p_0 - \frac{1}{2}\lambda_a p_a)f = 0 \tag{26b}$$

with $f = \frac{1}{2}\lambda_a(e_a + ib_a) = \lambda_a f_a$. This equation is equivalent to the following six equations (27) and (29) if we set $f_3 = f_8 = 0$.

$$(p_0 - \vec{\sigma} \cdot \vec{p}) f_p = 0$$

$$(p_0 - \vec{\sigma} \cdot \vec{\pi}) f_\pi = 0$$

$$(p_0 - \vec{\sigma} \cdot \vec{\kappa}) f_\kappa = 0$$
(27)

where

$$f_p = \frac{1}{2}\vec{\sigma} \cdot \vec{f_p} \qquad f_\pi = \frac{1}{2}\vec{\sigma} \cdot \vec{f_\pi} \qquad f_\kappa = \frac{1}{2}\vec{\sigma} \cdot \vec{f_\kappa}$$
(28*a*)

with

$$\vec{f_p} = (f_1, f_2, f_3) \qquad f_\pi = (f_4, f_5, \frac{f_8}{\sqrt{3}}) \qquad \vec{f_\kappa} = (f_6, f_7, \frac{f_8}{\sqrt{3}})$$
(28b)

and

$$(p_0 - \vec{\sigma} \cdot \vec{p})h_p = 0$$

$$(p_0 - \vec{\sigma} \cdot \vec{\pi})h_\pi = 0$$

$$(p_0 - \vec{\sigma} \cdot \vec{\kappa})h_\kappa = 0$$
(29)

where

$$h_p = \begin{pmatrix} f_{\pi}^{(d)} \\ f_{\kappa}^{(d)} \end{pmatrix} \quad h_{\pi} = \begin{pmatrix} f_p^{(d)} \\ f_{\kappa}^{(u)} \end{pmatrix} \quad h_{\kappa} = \begin{pmatrix} f_p^{(u)} \\ f_{\pi}^{(u)} \end{pmatrix}$$
(30a)

with

$$f_p^{(u)} = (f_1 + if_2) \qquad f_\pi^{(u)} = (f_4 + if_5) \qquad f_\kappa^{(u)} = (f_6 + if_7)$$

$$f_p^{(d)} = (f_1 - if_2) \qquad f_\pi^{(d)} = (f_4 - if_5) \qquad f_\kappa^{(d)} = (f_6 - if_7)$$
(30b)

It will be noted that while equations (27) describe dynamics of p, π and κ space triplet (bosonic) gauge fields equations (29) describe the dynamics of the corresponding doublet (fermionic) gauge fields. One of these three octats is represented in the weight diagram given in Fig.3. Fig.3

Local versions of eqns(27) and (29) can be obtained by adopting prescription (21).

We shall now show how the gauge coupling constant of the fundamental particles can be determined from the masslessness of the neutrino and their mass can be estimated from the experimental limit on the life-time of the electron. The ground state energy of the composite neutrino, as given by Dirac theory, is

$$\mathcal{E} = m\sqrt{1 - \alpha_g^2},\tag{31}$$

Setting this to zero gives

$$\alpha_g = 1 \tag{32}$$

In our model, the composite electron can make a transition to the neutrino state by emitting a M1 photon :

$$e \longrightarrow \nu + \gamma$$
 (33)

Since electric charge is not a fundamental quantum number in our model, there is no problem with non-conservation of charge in this decay. The life-time of this transition is given by

$$\frac{1}{\tau} = \frac{4w^3}{3} < \nu \mid M \mid e > \mid^2 \tag{34}$$

where M is the transition magnetic dipole operator and w is the frequency of the emitted photon which, in this case equals electron mass. From dimensional considerations

$$< \nu \mid M \mid e > \sim \alpha_g/m$$

$$\tag{35}$$

Substituting this in eqn(34), we get

$$m^2 = \alpha_g^2 m_e^3 \tau \tag{36}$$

Using the experimental limit

$$\tau > 2.7 \times 10^{23} years \tag{37}$$

on the electron life-time, we get

$$m > 10^{22} GeV \tag{38}$$

This corresponds to distances less than $10^{-36}cm$ which is in the Planckian regime. This is an indication of the fact that the compositeness of the electron, photon and other particles discussed in the note will be revealed at Planckian energies.

In order to accommodate other leptons and quarks it would be necessary to extend the Lorentz group to $SU(5) \otimes SU(5)$. Details of this model will be presented in a separate communication,

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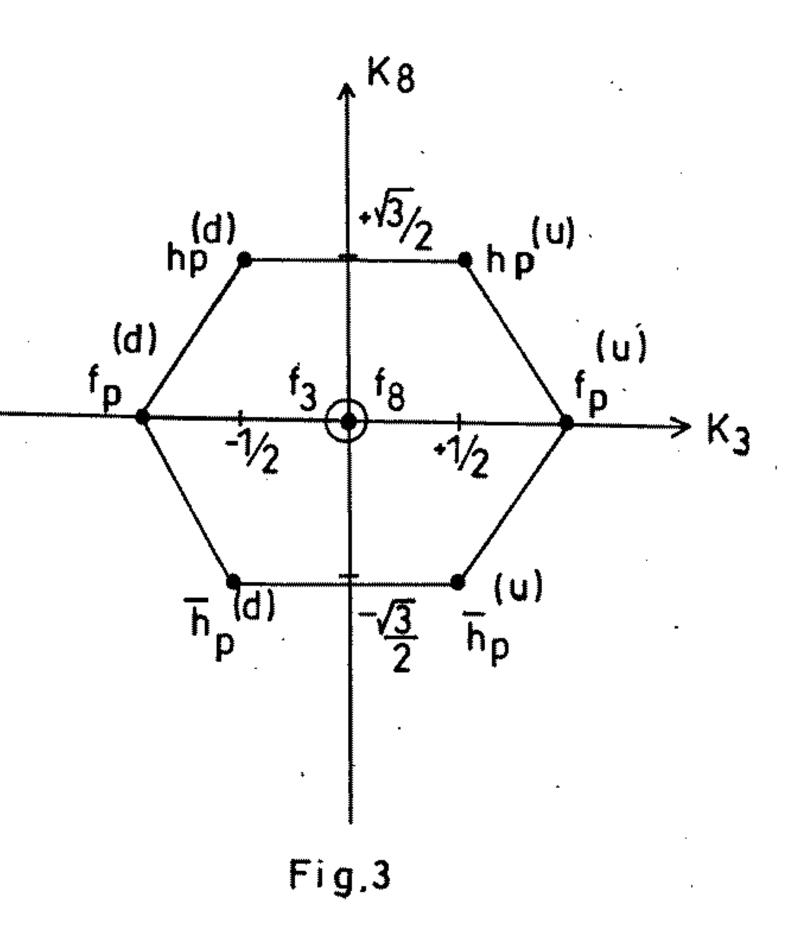
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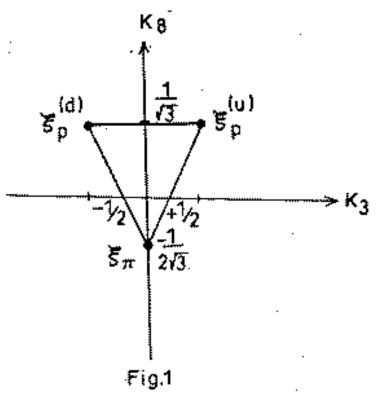
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Figure Caption

- Fig.1. Weight diagram for the fundamental particles.
- Fig.2. Weight diagram for electron and photon
- Fig.3. Weight diagram for gauge particles





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