Analysis of hadronic invariant mass spectrum in inclusive charmless semileptonic B decays

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Abstract

We make an analysis of the hadronic invariant mass spectrum in inclusive charmless semileptonic B meson decays in a QCD-based approach. The decay width is studied as a function of the invariant mass cut. We examine their sensitivities to the parameters of the theory. The theoretical uncertainties in the determination of $|V_{ub}|$ from the hadronic invariant mass spectrum are investigated. A strategy for improving the theoretical accuracy in the value of $|V_{ub}|$ is described.

I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is a fundemental parameter in the standard model of particle physics, which is not yet known with sufficient precision. A precise determination of it is very important for testing the standard model and for studying the origin of CP violation and quark masses.

Until recently, the sole evidence for charmless semileptonic B meson decays and for a non-zero value of V_{ub} , which occurs as a multiplicative factor in the decay amplitude, was reported from the inclusive charged lepton (electron or muon) energy spectra [1,2]. Since the rate of the charmless semileptonic B decay is very small, the main experimental challenge in these analyses is the suppression of backgrounds. The charged lepton from the inclusive charmless semileptonic decay $B \to X_u \ell \nu$ ($\ell = e \text{ or } \mu$) can be more energetic than that from the dominant semileptonic $b \to c$ decay; for a B meson at rest, the spectrum in the endpoint region $(M/2)(1 - M_D^2/M^2) < E_\ell \leq (M/2)(1 - M_\pi^2/M^2)$ (M being the mass of the B meson, M_D the mass of the D meson, and M_π the mass of the pion) results only from the $b \to u$ decay. This simple kinematic cut has been used to obtain the first sign of semileptonic $b \to u$ transitions¹ and determine $|V_{ub}|$ from the inclusive charged lepton energy spectra [4]. The theoretical description of the decays requires solving QCD in the nonperturbative regime. The accuracy of this inclusive determination of $|V_{ub}|$ is limited by model dependence, but recent work with a QCD-based approach shows promise of improving the situation [5].

Recently, the CLEO Collaboration has studied the exclusive charmless semileptonic B decays to $\pi \ell \nu$, $\rho \ell \nu$, and $\omega \ell \nu$ [6]. This is achieved through the reconstruction of semileptonic decays involving a $b \to u$ transition. The final states are reconstructed using an observed hadron, an e or μ , and a neutrino. Using the hermeticity of the CLEO detector, they reconstruct the neutrino by inferring its four-momentum from the missing energy and missing momentum in each event. With this neutrino reconstruction technique, they have measured the branching fractions for $B^0 \to \pi^- \ell^+ \nu$ and $B^0 \to \rho^- \ell^+ \nu$ and have extracted $|V_{ub}|$ from this study. Current determination of $|V_{ub}|$ by this exclusive method has large model dependence [6,7]. One should notice, however, recent significant theoretical progress in describing exclusive semileptonic $b \to u$ decays [8].

Various other methods to determine $|V_{ub}|$ have been proposed. These efforts are obviously necessary in order to find the most precise way to extract this important parameter. At least, different methods and analyses including two existing types of measurements lend confidence and indicate limitations. Among them it has been proposed [9] long time ago that the hadronic invariant mass spectrum in the inclusive semileptonic *B* decays may be useful for the determination of $|V_{ub}|$. For a *B* meson at rest, events with a kinematic cut on the hadronic invariant mass in the final states, $M_X < M_D$, come only from the $b \to u$ transition (the full $b \to u$ kinematic range is $M_{\pi} \leq M_X \leq M$) and are thus separated from those due to the $b \to c$ transition. Using this cut we have access to a much larger region of phase space than with the cut on E_{ℓ} . About 90% of the $b \to u$ events survive

¹A new inclusive analysis was presented by ALEPH which is the first evidence for semileptonic $b \rightarrow u$ transitions in *b*-hadrons produced at LEP [3].

this cut in contrast to the charged lepton energy cut, with which only about 10% of the $b \rightarrow u$ events survive. Hence the theoretical calculation of the decay rate is expected to be more reliable with the hadronic invariant mass cut. Recent experimental work on the neutrino reconstruction technique, which has been used by CLEO in the exclusive charmless semileptonic *B* decay measurement [6], shows promise of measuring the hadronic invariant mass spectrum. A number of recent papers [10] have renewed discussions on such possibility of determining $|V_{ub}|$. On the other hand, studies of the hadronic invariant mass spectra will also provide a testing ground for our understanding of the decay dynamics.

In this paper we will study the hadronic invariant mass spectrum and the decay width with a hadronic invariant mass cut in inclusive charmless semileptonic B meson decays in a QCD-based approach [11,12,5]. This approach describes inclusive semileptonic B meson decays by making use of the light-cone expansion and the heavy quark effective theory (HQET) and has been recently extended to inclusive semileptonic decays of an unpolarized b-flavored hadron [13]. The light-cone dominance attributes the nonperturbative QCD effects on the underlying weak decays to a single distribution function. The sum rules derived from operator product expansion and heavy quark effective theory constrain the distribution function considerably. The interplay between nonperturbative and perturbative QCD effects has been accounted for and a coherent treatment of both has been formulated. This approach has been tested on inclusive semileptonic decays of B mesons. The resulting lepton energy spectrum is in good agreement with the experimental data [5]. The calculation of the semileptonic decay width of the B meson has shown [12] that the kinematically enhanced nonperturbative QCD contributions play numerically an important role and $|V_{cb}|$ has then been determined, which is consistent with the exclusive measurements [14]. It has been shown [5] that an improved precision in $|V_{ub}|$ can be gained from the lepton energy spectra with a controlled theoretical uncertainty. In this work we turn our attention to investigate the theoretical accuracy of the alternative determination of $|V_{ub}|$, i.e., from the hadronic invariant mass spectrum.

Section II presents the theoretical methods to calculate the charmless hadronic invariant mass spectrum. We include both the bound-state effects and the QCD radiative corrections. In Sec. III we examine the sensitivities of the spectrum to the parameters of the theory. The effect of the QCD radiative corrections is also discussed. The theoretical uncertainties in the calculation of the semileptonic decay width to a given maximum hadronic invariant mass for $B \to X_u \ell \nu$ are studied in Sec. IV. We estimate the theoretical errors in the determination of $|V_{ub}|$ from the hadronic invariant mass spectrum. Finally, concluding remarks and a discussion are given in Sec. V.

II. THEORETICAL METHOD

The approach has been described in [11-13,5], which we refer to for a more complete exposition of the theory. We begin by briefly reviewing the methods relevant to this analysis. Because of the light-cone dominance, the nonperturbative QCD effects on the inclusive semileptonic *B* meson decays can be disentangled and encoded in a distribution function:

$$f(\xi) = \frac{1}{4\pi M^2} \int d(y \cdot P) e^{i\xi y \cdot P} \langle B \left| \bar{b}(0) P(1 - \gamma_5) b(y) \right| B \rangle |_{y^2 = 0},$$
(2.1)

where P denotes the four-momentum of the B meson. Several important properties of the distribution function were derived from QCD. Due to current conservation, it is exactly normalized to unity with a support $0 \le \xi \le 1$:

$$\int_{0}^{1} d\xi f(\xi) = 1.$$
 (2.2)

It obeys positivity. It contains the free quark decay as a limiting case with $f(\xi) = \delta(\xi - m_b/M)$, where m_b labels the *b* quark mass. In addition, two sum rules for the distribution function can be obtained by using the operator product expansion and the HQET method [15]. These two sum rules determine the mean value μ and the variance σ^2 of the distribution function, which characterize the position of the maximum and the width of it, respectively:

$$\mu \equiv \int_0^1 d\xi \xi f(\xi) = \frac{m_b}{M} (1 + E_b) , \qquad (2.3)$$

$$\sigma^2 \equiv \int_0^1 d\xi (\xi - \mu)^2 f(\xi) = \frac{m_b^2}{M^2} \left(\frac{2K_b}{3} - E_b^2\right) , \qquad (2.4)$$

where

$$G_b = -\frac{1}{2M} \left\langle B \left| \bar{h}_v \frac{g_s G_{\alpha\beta} \sigma^{\alpha\beta}}{4m_b^2} h_v \right| B \right\rangle , \qquad (2.5)$$

$$K_b = -\frac{1}{2M} \left\langle B \left| \bar{h}_v \frac{(iD)^2}{2m_b^2} h_v \right| B \right\rangle , \qquad (2.6)$$

with $E_b = G_b + K_b$. The first matrix element G_b measures the chromomagnetic energy due to the spin coupling between the *b* quark and the light constituents in the *B* meson and is determined by the mass splitting between B^* and *B* mesons. For the observed difference $M_{B^*} - M_B = 0.046$ GeV [16], one gets

$$m_b G_b = -\frac{3}{4}(M_{B^*} - M_B) = -0.034 \text{ GeV}.$$
 (2.7)

The second matrix element K_b measures the kinetic energy of the *b* quark in the B meson. A calculation using QCD sum rules yields [17]

$$2m_b^2 K_b = 0.5 \pm 0.2 \text{ GeV}^2.$$
(2.8)

Taking $m_b = 4.9 \pm 0.2$ GeV, the mean value and the variance of the distribution function are estimated to be

$$\mu = 0.93 \pm 0.04 \,, \tag{2.9}$$

$$\sigma^2 = 0.006 \pm 0.002 \,. \tag{2.10}$$

A non-zero value for σ^2 indicates the deviation from $f(\xi) = \delta(\xi - m_b/M)$, i.e., the free quark decay, and is a measure of the amount of the nonperturbative QCD contribution.

To obtain the hadronic invariant mass distribution $d\Gamma/dM_X$, it is technically convenient to start from the double differential decay rate $d^2\Gamma/dq^2dq^0$, since the momentum transfer to the lepton pair, q, is a quantity which has no distinction between the hadron level and the quark level. Following [5], one can account for the nonperturbative and perturbative QCD effects on $d^2\Gamma/dq^2dq^0$ in a coherent way through the convolution of the quark-level decay rate and the distribution function. This double differential decay rate is then straightforwardly converted to $d\Gamma/dM_X$. In the *B* meson rest frame, the hadronic invariant mass spectrum in the inclusive charmless semileptonic *B* decay is finally given by

$$\frac{d\Gamma}{dM_X} = \frac{M_X}{M} \int_0^{(M-M_X)^2} dq^2 \int_{\frac{q^0 + |\mathbf{q}|}{M}}^1 d\xi f(\xi) \left[\frac{d^2\Gamma_b}{dq^2 dq^0}\right]_{p_b = \xi P},\tag{2.11}$$

where p_b denotes the four-momentum of the *b* quark. The detailed expressions of the quarklevel double differential decay rate $d^2\Gamma_b/dq^2dq^0$ including the perturbative QCD corrections can be found in [18]. The masses of the leptons and the *u* quark have been neglected. To illustrate the effect of the radiative QCD corrections, it is instructive to retain only the terms which do not arise from such gluon radiation. Subtracting the radiative QCD corrections, Eq. (2.11) then reduces to

$$\frac{d\Gamma}{dM_X} = \frac{G_F^2 M_X |V_{ub}|^2}{24\pi^3} \int_0^{(M-M_X)^2} dq^2 |\mathbf{q}| \left[f(\xi_+) \left(\xi_+^2 + \frac{2q^2}{M^2} \right) + (\xi_+ \to \xi_-) \right], \quad (2.12)$$

where

$$\xi_{\pm} = \frac{q^0 \pm |\mathbf{q}|}{M}.\tag{2.13}$$

By integrating Eq. (2.11) over the hadronic invariant mass M_X , we obtain the decay width for the $B \to X_u \ell \nu$ decay to a given maximum hadronic invariant mass M_X^{cut} defined by

$$\Gamma(M_X^{\text{cut}}) = \int_0^{M_X^{\text{cut}}} dM_X \frac{d\Gamma}{dM_X}.$$
(2.14)

The known properties of the distribution function stated above improve the theoretical accuracy remarkably. However, since the distribution function has not yet been completely determined in QCD, for pratical calculations we shall adopt the following parametrization [12] of the distribution function which incorporates all known properties:

$$f(\xi) = N \frac{\xi(1-\xi)^{\alpha}}{[(\xi-a)^2 + b^2]^{\beta}} \theta(\xi) \theta(1-\xi) , \qquad (2.15)$$

where a, b, α , and β are four parameters, which are constrained by the sum rules (2.3) and (2.4), and N is the normalization constant. For $\alpha = \beta = 1$, Eq.(2.15) becomes the ansatz of [11]. In the following, $\alpha = \beta = 1$ is preset unless explicitly stated².

²In general, the parameters α and β need not be integer.



FIG. 1. The charmless hadronic invariant mass spectrum with (solid curve: $\alpha_s = 0.25$, dashed curve: $\alpha_s = 0.30$) and without (dotted curve) radiative QCD corrections. The mean value and the variance of the distribution function are kept fixed to be $\mu = 0.93$ and $\sigma^2 = 0.006$.

III. CHARMLESS HADRONIC INVARIANT MASS SPECTRUM

We proceed to examine the sensitivity of the charmless hadronic invariant mass spectrum to the parameters of the theory. We calculate the hadronic invariant mass spectrum in the inclusive charmless semileptonic B meson decay for a B meson at rest by use of Eqs. (2.11) and (2.15). The input parameters are the strong coupling constant α_s and the parameters which arise in the distribution function.

The effects of the radiative QCD corrections to the spectrum are demonstrated in Fig. 1. We calculate the spectra using two different values of the strong coupling constant: $\alpha_s = 0.25$ (solid curve) and $\alpha_s = 0.30$ (dashed curve). The spectrum appears to be insensitive to the value of α_s , varied within a reasonable range. For comparison, the spectrum without the radiative QCD corrections (dotted curve) is also shown, calculated by use of Eqs. (2.12) and (2.15). It is evident that the charmless hadronic invariant mass spectrum receives large radiative QCD corrections. This is easily understood because gluon bremsstrahlung strongly affects the hadronic final states.

We explore next the sensitivity of the spectrum to the form of the distribution function. The mean value and the variance of it vary in the ranges specified in Eqs. (2.9) and (2.10) inferred from the sum rules (2.3) and (2.4). We show in Fig. 2 the variation of the hadronic invariant mass spectrum due to the changes in the mean value μ and the variance σ^2 of the distribution function. The position of the maximum for the spectrum is a sensitive function of the mean value μ . The sensitivity of the spectrum to the variance σ^2 is seen to be relatively weak.

Furthermore, to get an idea of the sensitivity of the spectrum to the form of the distribu-



FIG. 2. Dependence of the charmless hadronic invariant mass spectrum on the mean value and the variance of the distribution function. The strong coupling constant is taken to be $\alpha_s = 0.25$.

tion function when keeping the mean value and the variance of it fixed, we change the shape of the distribution function by varying the values of the two additional parameters α and β in the parametrization (2.15) (for given values of α and β , the parameters a and b are fixed by the mean value and the variance). In Fig. 3 we plot the distribution functions for four different choices of the parameters α and β in (2.15) that give the same mean value $\mu = 0.93$ and variance $\sigma^2 = 0.006$. The spectra calculated using the two very different distribution functions with $\alpha = 1$ and $\beta = 1$ (solid curve), $\alpha = 2$ and $\beta = 2$ (dashed curve), respectively, at fixed mean value and variance are shown in Fig. 4. It is seen that the hadronic invariant mass spectrum relies strongly on the form of the distribution function, even when the mean value and the variance of it are kept fixed.

IV. PARTIAL (TOTAL) DECAY WIDTHS AND DETERMINATION OF $|V_{ub}|$

Using the charmless hadronic invariant mass spectrum, the semileptonic branching fraction $\Delta \mathcal{B}_u(M_X^{\text{cut}})$ for the charmless semileptonic *B* decay with the hadronic final states up to a given maximum invariant mass M_X^{cut} can be measured. Together with the measured lifetime of the *B* meson, τ_B , and the theoretically calculated decay width defined in Eq. (2.14) up to the CKM factor $|V_{ub}|^2$, this determines $|V_{ub}|$:

$$|V_{ub}|^2 = \frac{\Delta \mathcal{B}_u(M_X^{\text{cut}})}{\tau_B \gamma_u(M_X^{\text{cut}})}, \qquad (4.1)$$

where $\gamma_u(M_X^{\text{cut}})$ is defined by $\Gamma(M_X^{\text{cut}}) = |V_{ub}|^2 \gamma_u(M_X^{\text{cut}})$.



FIG. 3. The distribution functions from the parametrization (2.15) for four different pairs of the parameters α and β . All the distribution functions give $\mu = 0.93$ and $\sigma^2 = 0.006$.



FIG. 4. Comparison of the charmless hadronic invariant mass spectra in two distribution functions. The mean value and the variance of the distribution functions are fixed to be $\mu = 0.93$ and $\sigma^2 = 0.006$. We take $\alpha_s = 0.25$.

We investigate the theoretical uncertainties in this determination of $|V_{ub}|$. The semileptonic decay width $\Gamma(M_X^{\text{cut}})$ is displayed in Fig. 5 as a function of the hadronic invariant mass cutoff, M_X^{cut} . For $M_X^{\text{cut}} = M$, $\Gamma(M_X^{\text{cut}})$ is the total semileptonic decay width for $B \to X_u \ell \nu$. The solid and dotted curves correspond to two different values of α_s : $\alpha_s = 0.25$ (solid), $\alpha_s = 0.30$ (dotted), and the other parameters are identical. We find that a reasonable variation of the value of α_s leads to a small change in the value of the decay width $\Gamma(M_X^{\text{cut}})$.

In order to explore the impact of the form of the distribution function, as in the last section, we first study the sensitivities of $\Gamma(M_X^{\rm cut})$ to the mean value and the variance of the distribution function by using the parametrization (2.15) setting $\alpha = \beta = 1$. We vary the mean value and the variance in the ranges of Eqs. (2.9) and (2.10). The resulting decay widths are shown in Fig. 5 corresponding to $\mu = 0.93$ and $\sigma^2 = 0.006$ (solid curve), $\mu = 0.89$ and $\sigma^2 = 0.006$ (long-dashed curve), $\mu = 0.93$ and $\sigma^2 = 0.008$ (dot-dashed curve), with $\alpha_s = 0.25$ held fixed. Then we change the form of the distribution function by changing the values of the two additional parameters α and β in Eq. (2.15) in order to examine the further sensitivity of $\Gamma(M_X^{\text{cut}})$ to the distribution function when keeping the mean value and the variance of it fixed. The results are also shown in Fig. 5 for $\alpha = \beta = 1$ (solid curve), $\alpha = \beta = 2$ (short-dashed curve), with $\mu = 0.93, \sigma^2 = 0.006$, and $\alpha_s = 0.25$ held fixed for both curves. We find that for the cut M_X^{cut} less than about 1.2 GeV, the partial decay width $\Gamma(M_X^{\text{cut}})$ is rather sensitive to the form of the distribution function, whereas for $M_X^{\rm cut}$ greater than about 1.2 GeV, the decay width $\Gamma(M_X^{\rm cut})$ (including the total semileptonic decay width for $B \to X_u \ell \nu$ decays) is sensitive essentially only to the mean value of the distribution function, which is calculable from HQET with the results given in Eqs. (2.3) and (2.9). These imply that the theoretical uncertainties in the determination of $|V_{ub}|$ are under control for the value of the hadronic invariant mass cutoff greater than about 1.2 GeV, but very large for the value of the cutoff less than 1.2 GeV or so.

Adding the errors on the semileptonic decay width due to the value of α_s and the form of the distribution function linearly to be conservative, we estimate the theoretical errors in the determination of $|V_{ub}|$. Without any cut, the theoretical error in determining $|V_{ub}|$ from the total charmless semileptonic decay width is about 13%. Applying a cut on the hadronic invariant mass, the theoretical errors in determining $|V_{ub}|$ from the hadronic invariant mass spectrum are approximately 16% for $M_X^{\text{cut}} = 1.8$ GeV, 26% for $M_X^{\text{cut}} = 1.5$ GeV, and 44% for $M_X^{\text{cut}} = 1.2$ GeV, respectively.

V. CONCLUSION AND DISCUSSION

We have presented an analysis of the hadronic invariant mass spectrum and studied the decay width as a function of the invariant mass cut for inclusive charmless semileptonic B decays. An approach based on the light-cone expansion and the heavy quark effective theory has been exploited. Nonperturbative QCD effects in the processes are contained in a distribution function and several important properties of it are known from QCD. However, one needs a parametrization of the distribution function incorporating the known properties in order to make quantitative predictions, since it cannot yet been completely determined from QCD. For our analysis, we employ the distribution function parametrization given in Eq. (2.15). We have examined the sensitivities of the hadronic invariant mass spectrum and



FIG. 5. The semileptonic decay width defined in Eq. (2.14) for $B \to X_u \ell \nu$ as a function of the hadronic invariant mass cutoff. The dependence on the mean value and the variance of the distribution function is shown using $\alpha_s = 0.25$ and the parametrization (2.15) with $\alpha = \beta = 1$ and three pairs of the mean value and the variance: $\mu = 0.93$ and $\sigma^2 = 0.006$ (solid curve), $\mu = 0.89$ and $\sigma^2 = 0.006$ (long-dashed curve), $\mu = 0.93$ and $\sigma^2 = 0.008$ (dot-dashed curve). The dependence on the form of the distribution function at fixed mean value and variance is shown by the solid and short-dashed curves corresponding to the same values of α_s and of the mean value and the variance, but $\alpha = \beta = 2$ for the short-dashed curve. The dependence on the strong coupling constant is illustrated by the solid and dotted curves obtained from the same parameters except $\alpha_s = 0.30$ for the dotted curve.

the decay width with a hadronic invariant mass cut to the parameters of the theory, namely the strong coupling constant and the form of the distribution function.

It is shown that a reasonable variation of the value of α_s hardly affects the charmless hadronic invariant mass spectrum. The dependence of the decay width with a hadronic invariant mass cut on the value of α_s is small. The main theoretical uncertainties arise from the form of the distribution function.

We found that the charmless hadronic invariant mass spectrum exhibits a significant sensitivity to the form of the distribution function. Thus, if experimentally feasible, the measured hadronic invariant mass spectrum will impose strong constraints on the shape of the distribution function.

We have also studied the sensitivity of the semileptonic decay width to a given maximum hadronic invariant mass to the form of the distribution function. For the hadronic invariant mass cutoff greater than about 1.2 GeV, the value of the decay width is sensitive essentially only to the mean value of the distribution function whose value is known from HQET, and thereby the result is nearly model independent and the uncertainties in the calculation of the decay width are under control. However, the theoretical uncertainties in the partial decay width with a hadronic invariant mass cutoff less than 1.2 GeV are very large, obscuring its usefulness for determining $|V_{ub}|$, since it is rather sensitive to the form of the distribution function.

We have observed the feature that for $M_X^{\text{cut}} \ge M_D$ the decay width tends to show only little dependence on the invariant mass cut (see Fig. 5) and most of the $b \to u$ events lie below $M_X = M_D$ (see also Figs. 1, 2, 4).

We have estimated the theoretical errors in determining $|V_{ub}|$ in our approach. At the present time, the theoretical error on $|V_{ub}|$ from the total semileptonic decay width for the $B \to X_u \ell \nu$ decays is about 13%. Applying a hadronic invariant mass cutoff, the theoretical errors on $|V_{ub}|$ are approximately 16% for $M_X^{\text{cut}} = 1.8$ GeV, 26% for $M_X^{\text{cut}} = 1.5$ GeV, and 44% for $M_X^{\text{cut}} = 1.2$ GeV, respectively. The higher the hadronic invariant mass cutoff can be experimentally made to be, the smaller the theoretical error on $|V_{ub}|$; the smallest theoretical error on $|V_{ub}|$ would be achieved if the total charmless semileptoinc decay width can be measured³.

Besides the hadronic invariant mass cutoff, the mean value of the distribution function is another key quantity for precise determination of $|V_{ub}|$, which requires as precise knowledge of the mean value as possible. Based on this analysis and the previous one [5], a detailed fit to the measured charged-lepton energy spectrum and/or the hadronic invariant mass spectrum can be used to extract the parameters (especially, the mean value of the distribution function) experimentally. This procedure allows to reduce the uncertainties in the determination of $|V_{ub}|$. Future measurements of the distribution function and continuing efforts in calculations of hadronic matrix elements that govern B decays should enable to further reduce the uncertainties.

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³An attempt has been made [3] by the ALEPH Collaboration to measure the inclusive charmless semileptonic branching fraction of B hadrons.

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