

Determining Top Quark CP Violating Dipole Couplings from
 $e^+e^- \rightarrow t\bar{t}$

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Abstract

We show how to determine the electric and weak dipole moments of the top quark simultaneously and independently from $e^+e^- \rightarrow t\bar{t}$ at $\sqrt{s} = 500$ GeV NLC. To obtain the best accuracies with which the dipole moments can be measured, we apply the optimal observables to extract the CP violating effects and consider only purely hadronic, hadronic-leptonic final state events. Results with left- and right- handed longitudinal polarized as well as unpolarized electron beams are given. We find that with $50fb^{-1}$ integrated luminosity, the dipole moments can be measured to the accuracy of 10^{-18} *e cm*.

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I. INTRODUCTION

The top quark has its unique advantages in search for new physics beyond the Standard Model(SM) due to its large mass and decay properties. Many non-SM models predict several orders of magnitude large electric dipole moment(EDM) and weak dipole moment(WDM) of the top quark than the SM. These dipole moments give rise to CP violating effects in top quark pair productions. CP violating effects in top quark pair production at e^+e^- colliders have been widely studied [1]- [10]. Many of these studies concentrate on the observables which use purely leptonic or hadronic-leptonic (including $bl_1^+\nu_{l_1}\bar{b}q_2\bar{q}_2'$ and $b\bar{q}_1q_1'\bar{b}l_2^-\bar{\nu}_{l_2}$) final state events. Therefore, these studies can not give the best limits one can obtain from the experiments. To achieve the best goal, one needs to make use of the purely hadronic events which consist of 10/17 of the total events and to apply the optimal approach [3] [11]- [13]. The best limits on the top quark dipole moments given in Ref. [3] are $\sim 10^{-19} e cm$. Our work has some similarities with Ref. [3]. New features of our work are:(1) the W-boson polarization is not and indeed can not be determined completely by a single event, but statistically from its decay products; (2) we consider how to measure the electric and weak dipole moments simultaneously and independently ; (3) we give more realistic estimation on the limits by taking into account decay branching ratios, luminosity and detection efficiency. In Ref. [7], the optimal observables are also used, but without using the spin information of the hadronically decayed top quark. We consider $\gamma t\bar{t}$ and $Zt\bar{t}$ including EDM and WDM couplings, and for simplicity, assume the top quark decays via SM interactions.

II. CALCULATIONS AND OPTIMAL OBSERVABLES

We assume the couplings of electron with γ and Z bosons take the standard model values:

$$-ieg_e^V \gamma^\mu (1 + \alpha_e^V \gamma_5), \quad (1)$$

where $V = \gamma, Z$ and

$$g_e^\gamma = -1, \quad \alpha_e^\gamma = 0, \quad (2)$$

$$g_e^Z = \frac{4 \sin^2 \theta_W - 1}{4 \sin \theta_W \cos \theta_W}, \quad \alpha_e^Z = \frac{1}{4 \sin^2 \theta_W - 1}. \quad (3)$$

The couplings between the top quark and γ, Z bosons take the form:

$$-ie[g_t^V \gamma^\mu (1 + \alpha_t^V \gamma_5) + (p_t - p_{\bar{t}})^\mu (-id_t^V/e)\gamma_5], \quad (4)$$

where p_t , $p_{\bar{t}}$ are the momenta of the top quark and top antiquark. d_t^V is the dipole moment which we assume to have imaginary parts as well as real parts. We denote $\hat{d}_t^V = d_t^V/e$. The other couplings are:

$$g_t^\gamma = 2/3, \alpha_t^\gamma = 0, \quad (5)$$

$$g_t^Z = \frac{1 - \frac{8}{3} \sin^2 \theta_W}{4 \sin \theta_W \cos \theta_W}, \alpha_t^Z = -\frac{1}{1 - \frac{8}{3} \sin^2 \theta_W}. \quad (6)$$

The standard model amplitude of $e^+e^- \rightarrow t\bar{t}$ is

$$M_0 = ie^2 \sum_{V=\gamma,Z} g_e^V g_t^V \bar{v}(p_{e^+}) \gamma^\mu (1 + \alpha_e^V \gamma_5) u(p_{e^-}) \bar{u}(p_t) \gamma_\mu (1 + \alpha_t^V \gamma_5) v(p_{\bar{t}}) / (s - m_V^2), \quad (7)$$

and the correction from the dipole moments is

$$\delta M = ie^2 \sum_{V=\gamma,Z} g_e^V \bar{v}(p_{e^+}) \gamma^\mu (1 + \alpha_e^V \gamma_5) u(p_{e^-}) \bar{u}(p_t) [(p_t - p_{\bar{t}})_\mu (-i \hat{d}_t^V) \gamma_5] v(p_{\bar{t}}) / (s - m_V^2). \quad (8)$$

We shall assume the dipole moments are small enough that their quadratic contributions to the total cross section are negligible. Therefore the dipole moments contribute only to the CP violating effects through $2\text{Re}(M_0 \delta M^\dagger)$ which is linear in \hat{d}_t^V . To observe the CP violating effects, one needs to know the spins of the top quarks which can be determined statistically from their decay products. We assume the SM decay of the top quark and apply the narrow width approximations of the top quark and W-boson propagators:

$$\frac{1}{|q_X^2 - m_X^2 + im_X \Gamma_X|^2} \rightarrow \frac{\pi}{m_X \Gamma_X} \delta(q_X^2 - m_X^2), \quad (9)$$

where X stands for top quark and W-boson, Γ_X is the width of X .

The cross section for reaction $e^+e^- \rightarrow t\bar{t} \rightarrow bl_1^+ \nu_{l_1} \bar{b} l_2^- \bar{\nu}_{l_2}$ ($b\bar{q}_1 q_1' \bar{b} q_2 \bar{q}_2'$) can be written as

$$d\sigma = \frac{\beta}{(8\pi)^{10} s} \frac{\lambda_t |M_D|^2}{m_t^2 m_W^2 \Gamma_t^2 \Gamma_W^2} d\Omega_t d\Omega'_{W^+} d\Omega'_{W^-} d\Omega'_{l_1^+} d\Omega'_{l_2^-}, \quad (10)$$

where $\beta = \sqrt{1 - 4m_t^2/s}$ and

$$\lambda_t = \left(1 - \frac{(m_W + m_b)^2}{m_t^2}\right) \left(1 - \frac{(m_W - m_b)^2}{m_t^2}\right) \approx (m_t^2 - m_W^2)^2 / m_t^4, \quad (11)$$

$d\Omega'_{W^+}$ ($d\Omega'_{W^-}$) is the solid angle element of W^+ (W^-) in the rest frame of the (anti) top quark, $d\Omega'_{l_1^+}$ ($d\Omega'_{l_2^-}$) denotes the solid angle element of l_1^+ (l_2^-) in the rest frame

of $W^+(W^-)$, $|M_D|^2$ is the amplitude square excluding the top quark and W-boson propagators after the decays of the top quarks:

$$|M_D|^2 = |M_0|^2 + 2Re(M_0\delta M^\dagger) . \quad (12)$$

If the electron(positron) beam is not polarized, additional spin average factor and summation are needed. In our calculations, $|M_D|^2$ is easily obtained from the amplitude of $e^+e^- \rightarrow t\bar{t}$ by the following substitutions:

$$\begin{aligned} \bar{u}(p_t) &\rightarrow \frac{g^2}{8} \bar{u}_b \gamma_\mu (1 - \gamma_5) (\not{p}_t + m_t) \bar{u}_{\nu_1} \gamma^\mu (1 - \gamma_5) v_{l_1} , \\ v(p_{\bar{t}}) &\rightarrow \frac{g^2}{8} \bar{u}_{l_2} \gamma_\mu (1 - \gamma_5) v_{\nu_2} (\not{p}_{\bar{t}} - m_t) \gamma^\mu (1 - \gamma_5) v_{\bar{b}} , \end{aligned} \quad (13)$$

where g is the weak $SU(2)$ coupling constant. The above expressions are calculated numerically.

Denoting $g_1 = Re(\hat{d}_t^\gamma)$, $g_2 = Re(\hat{d}_t^Z)$, $g_3 = Im(\hat{d}_t^\gamma)$ and $g_4 = Im(\hat{d}_t^Z)$, we can write the amplitude square $|M_D|^2$ as

$$|M_D|^2 = \Sigma_0 + g_1 \Sigma_1 + g_2 \Sigma_2 + g_3 \Sigma_3 + g_4 \Sigma_4 , \quad (14)$$

where $\Sigma_0 = |M_0|^2$. For unpolarized beams, $\Sigma_{1,2,3,4}$ are independent. But for left- or right-handed polarized electron beam, there are only two independent terms:

$$\begin{aligned} |M_D|_L^2 &= \Sigma_{0L} + g_1^L \Sigma_1^L + g_2^L \Sigma_2^L , \\ |M_D|_R^2 &= \Sigma_{0R} + g_1^R \Sigma_1^R + g_2^R \Sigma_2^R , \end{aligned} \quad (15)$$

where L, R stand for left- or right-handed polarized electron beam, and

$$\begin{aligned} g_1^L &= g_1 + (\xi - \eta)g_2, & g_2^L &= g_3 + (\xi - \eta)g_4, \\ g_1^R &= g_1 + (\xi + \eta)g_2, & g_2^R &= g_3 + (\xi + \eta)g_4, \end{aligned} \quad (16)$$

where

$$\begin{aligned} \xi &= \frac{1 - 4 \sin^2 \theta_W}{4 \sin \theta_W \cos \theta_W} \frac{s}{s - m_Z^2}, \\ \eta &= -\frac{1}{4 \sin \theta_W \cos \theta_W} \frac{s}{s - m_Z^2}. \end{aligned} \quad (17)$$

In our calculation, we have set the electron masses to be zero and do not consider the radiative corrections to $e^+e^-\gamma(Z)$ couplings. Therefore, even with polarized electron beams, only the initial CP eigenstates couple to γ and Z .

To measure g_i , one needs to extract the CP violating effects which can be picked out by CP-odd observables. It has been shown in the literature [3] [11]- [13] that the optimal observables defined below have the smallest statistical errors. The optimized CP-odd observables in the full final state phase space with unpolarized beams are defined by

$$O_{1i} = \frac{\Sigma_i}{\Sigma_0} . \quad (18)$$

It is shown in Ref. [12] that a linear transformation of the above set of observables O_{1i} are still optimal.

When the top quark decays hadronically, we can not distinguish quark and anti-quark jet. For hadronic-leptonic events, the missing neutrino momenta can be fully reconstructed using energy momentum conservation equations, so that we are left with two fold ambiguity of the jet momenta. For purely hadronic events, we have four fold ambiguity. Considering this ambiguity, one can define alternatively the optimal observables:

$$O_{2i} = \frac{\sum_j \Sigma_i}{\sum_j \Sigma_0}, \quad O_{4i} = \frac{\sum_{j'} \Sigma_i}{\sum_{j'} \Sigma_0}, \quad (19)$$

where the sum j is over the two possible assignments of the jet momenta to the quark and antiquark in hadronic-leptonic events. j' is over the possible assignments of the jet momenta to the quark and antiquark in purely hadronic events.

All the above definitions can be applied to the polarized beam cases. We now consider separately the polarized and unpolarized beams.

We first look at the unpolarized beams. In this case, we can define four optimized observables for each of the two kinds of final state events mentioned above. They can be separated into two categories: $O_{n1}, O_{n2} (n = 2, 4)$ are \hat{T} -odd with \hat{T} being the transformation that inverse the particle spins and momenta but does not interchange initial and final states, O_{n3}, O_{n4} are \hat{T} -even.

The mean value of the observable O_{2i} is defined as

$$\langle O_{2i} \rangle = \frac{\int d\sigma^+ O_{2i}^+ + \int d\sigma^- O_{2i}^-}{\int d\sigma^+ + \int d\sigma^-}, \quad (20)$$

where the superscript $+, -$ mean that the integrations are over $bl_1^+ \nu_{l_1} \bar{b}q_2 \bar{q}'_2$ and $b\bar{q}'_1 q'_1 \bar{b}l_2^- \bar{\nu}_{l_2}$ final states, respectively. The mean value of the observable O_{4i} is simply

$$\langle O_{4i} \rangle = \frac{\int d\sigma O_{4i}}{\int d\sigma}. \quad (21)$$

We can express the mean value of an observable O_{ni} by

$$\langle O_{ni} \rangle = c_{ij} g_j, \quad (22)$$

where $c_{ij} = \langle O_{ni} O_{nj} \rangle$. The symmetric matrix c is just the covariance of the observables. Due to the different \hat{T} symmetry, the matrix c will be block diagonal between $i, j = 1, 2$ and $i, j = 3, 4$. Non-zero off-diagonal elements mean the statistical dependences of the observables and therefore the couplings. It will be difficult to estimate the statistical error of a particular coupling without assuming the other couplings are zero. The way to solve the problem is to find a linear combination of the couplings and the observables that are statistically independent [12]. To do so, one needs to diagonalize the matrix c by a matrix A . For simplicity, we choose A to be orthogonal:

$$c' = A^{-1} c A, \quad (23)$$

$$O'_{ni} = A_{ji} O_{nj}, \quad (24)$$

$$g'_i = A_{ji} g_j. \quad (25)$$

The 1σ statistical error of coupling g'_i is now given by

$$\Delta g'_i = \frac{1}{\sqrt{N c'_{ii}}}, \quad (26)$$

where N is the number of events. To reduce the statistical errors, one can combine the measurements of O'_{2i} and O'_{4i} to get a combined error $\Delta g'_{ci}$ [14]:

$$\frac{1}{(\Delta g'_{ci})^2} = \frac{1}{(\Delta g'_{2i})^2} + \frac{1}{(\Delta g'_{4i})^2}, \quad (27)$$

where $\Delta g'_{2i}$ and $\Delta g'_{4i}$ are the errors of the measurements using O'_{2i} and O'_{4i} , respectively.

From Eq.(15), we see that when the electron is left- or right- handed polarized, we can only define two independent observables O_{n1} , O_{n2} ($n = 2, 4$) which have different \hat{T} parity. They are sensitive to $g_1^{L,R}$, $g_2^{L,R}$, respectively. Therefore we have a diagonal matrix c . Only a linear combination of d_t^Y and d_t^Z can be measured with a particular polarization beam. The combination depends on the energy(cf. (16)(17)).

III. RESULTS AND CONCLUSIONS

We present our results of the matrix c at $\sqrt{s} = 500$ GeV unpolarized e^+e^- collider in Table I. It shows the relations between the mean values of the observables and the coupling constants. The corresponding orthogonal matrix A and the diagonal matrix c'

are given in Table II. Matrix A is useful for extracting g_i from g'_i and for the calculation of O'_{ni} . In Table III., we give the results of the matrix elements c_{ii} at the collider with polarized electron beams.

We assume: (1) the overall detection efficiency is $\epsilon = 0.1$; (2) the integrated luminosity is $\mathcal{L} = 50fb^{-1}$; (3) the branching ratio of hadronic-leptonic final state events is $B_{lj} = 0.29(l = e, \mu)$, the branching ratio of the purely hadronic events is $B_{jj} = 0.46$. The number of events is given by

$$N = \epsilon\mathcal{L}\sigma B, \quad (28)$$

where σ is the total $t\bar{t}$ production cross section, $B = B_{lj}$ or B_{jj} . With $\alpha_{em} = 1/128.8$ and $m_t = 176$ GeV, we get the following total cross sections for different electron beams:

$$\sigma(e^+e^-) = 563 \text{ fb}, \quad (29)$$

$$\sigma(e^+e^-_L) = 785 \text{ fb}, \quad (30)$$

$$\sigma(e^+e^-_R) = 341 \text{ fb}. \quad (31)$$

By using the results of c'_{ii} in Table II. and c_{ii} in Table III., we obtain the 1σ level statistical errors of g'_i and $g_i^{L,R}$ given in Table IV. From this table we see that the accuracies are about $10^{-18} e \text{ cm}$ for $d_t^{\gamma,Z}$. In the unpolarized case, the best limit is on $Im(d_t^{\gamma})$ which is the main component of g'_3 . Better limits can be obtained by using polarized electron beams with the same integrated luminosity. In this case, one can only measure the combination of d_t^{γ} and d_t^Z . One can combine the two modes of electron polarization to obtain d_t^{γ} and d_t^Z separately. But that needs two periods of running. Although with the right-handed electron beam, one gets a relative larger c_{ii} (cf. Table III.), the statistical errors are the same as that with the left-handed electron beam.

In conclusion, we have used the optimal observables to extract the CP-violating dipole couplings of the top quark at the NLC. The accuracies with which these couplings can be measured at $\sqrt{s} = 500$ GeV e^+e^- collider with an integrated luminosity of $50fb^{-1}$ are about $10^{-18} e \text{ cm}$.

TABLES

TABLE I.

Matrix elements c_{ij} of O_{2i} and O_{4i} at $\sqrt{s} = 500$ GeV unpolarized e^+e^- collider. Unit: 10^4 GeV².

	O_{21}	O_{22}	O_{23}	O_{24}	
	1.85	1.40	0	0	O_{21}
		4.11	0	0	O_{22}
O_{41}	0.817		7.59	1.40	O_{23}
O_{42}	0.81	2.39		1.05	O_{24}
O_{43}	0	0	4.27		
O_{44}	0	0	0.80	0.46	
	O_{41}	O_{42}	O_{43}	O_{44}	

TABLE II.

Matrix elements A_{ij} and c'_{ii} of O_{2i} and O_{4i} at $\sqrt{s} = 500$ GeV unpolarized e^+e^- collider.

	A_{ij}				$c'_{ii}(10^4\text{GeV}^2)$
O_{2i}	0.90	0.43	0	0	1.18
	-0.43	0.90	0	0	4.78
	0	0	0.98	-0.20	7.88
	0	0	0.20	0.98	0.763
O_{4i}	0.92	0.39	0	0	0.475
	-0.39	0.92	0	0	2.73
	0	0	0.98	-0.20	4.43
	0	0	0.20	0.98	0.299

TABLE III.

Matrix elements c_{ii} of O_{2i} and O_{4i} at $\sqrt{s} = 500$ GeV e^+e^- collider with left- and right-polarized electron beams. Unit: 10^4 GeV².

	O_{21}	O_{22}	O_{41}	O_{42}
$e^+e^-_L$	9.74	7.31	4.82	3.48
$e^+e^-_R$	22.2	17.4	10.9	8.23

TABLE IV.

1σ statistical errors of the coupling constants g'_i and $g_i^{L,R}$ at $\sqrt{s} = 500$ GeV collider with an integrated luminosity $50fb^{-1}$. Unit: 10^{-18} cm.

	g'_1	g'_2	g'_3	g'_4	g_1^L	g_2^L	g_1^R	g_2^R
O_{2i}	6.35	3.15	2.46	7.90	1.87	2.16	1.88	2.12
O_{4i}	7.94	3.31	2.60	10.0	2.11	2.48	2.13	2.45
combined	4.96	2.28	1.79	6.20	1.40	1.63	1.41	1.60

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REFERENCES

- [1] For a review see, for example, C.-P. Yuan, *Mod. Phys. Lett.* **A 10** (1995) 627; D.Atwood and A. Soni, hep-ph/9609418, Published in ICHEP 96:(1996) 1119.
- [2] G.Kane, G.A.Ladinsky and C.-P. Yuan, *Phys. Rev.* **D 45** (1992) 124.
- [3] D.Atwood and A.Soni, *Phys. Rev.* **D 45** (1992) 2405.
- [4] W. Bernreuther, O.Nachtmann, P.Overmann, T. Schröder, *Nucl. Phys.* **B 388** (1992) 53;(E) *ibid* **B406** (1993) 516.
- [5] B.Grzadkowski and W.Y.Keung, *Phys. Lett.* **B 319** (1993) 526.
- [6] W. Bernreuther and P. Overmann, *Z. Phys.* **C 61** (1994) 599.
- [7] W. Bernreuther and P. Overmann, *Z. Phys.* **C 72** (1996) 461; W. Bernreuther,A.Brandenburg and P. Overmann, *e⁺e⁻ Collisions at TeV Energies:The Physics Potential; Part D*, edited by P.Zerwas,DESY 96-123 D(1996) 49.
- [8] P.Poulose and S.D. Rindani, *Phys. Lett.* **B 349**,379 (1995); F.Guypers and S.D. Rindani, *Phys. Lett.* **B 343**,333 (1995); P.Poulose and S.D. Rindani, *Phys. Lett.* **B 383** (1996) 212; P.Poulose and S.D. Rindani, *Phys. Rev.* **D 54** (1996) 4326.
- [9] M.S. Baek, S.Y.Choi and C.S.Kim, *Phys. Rev.* **D 56** (1997) 6835.
- [10] B.Grzadkowski and Z. Hioki, hep-ph/9805318, and references therein.
- [11] M.Dovier, L.Duflot, F.Le Diberder, A.Rougé, *Phys. Lett.* **B 306** (1993) 411.
- [12] M.Diehl, O. Nachtmann, *Z.Phys.* **C 62** (1994) 397; hep-ph/9702208.
- [13] J.F.Gunion, B.Grzadkowski and X.-G.He, *Phys. Rev. Lett.* **77** (1996) 5172.
- [14] Review of Particle Properties, *Phys. Rev.* **D 54** (1996) 1.