

# Neutrino spin-flip effects in active galactic nuclei

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(October 19, 2018)

We study the effects of neutrino spin-flip in the magnetic field,  $B_{AGN}$ , of active galactic nuclei (AGN) for high-energy neutrinos ( $E \geq 10^6$  GeV) originating from AGN induced by an interplay of the violation of equivalence principle parameterized by  $\Delta f$  and the twist in  $B_{AGN}$ . We point out that a conversion effect may exist for  $\Delta f \sim 10^{-34}(\delta m^2/10^{-5}\text{eV}^2)$  independent of gravity mixing angle. Observational consequences for this conversion effect are discussed.

## I. INTRODUCTION

In this paper, we study the spin-flip effects for high-energy neutrinos ( $E \geq 10^6$  GeV) originating from active galactic nuclei (AGN) induced by the violation of equivalence principle (VEP) and/or the magnetic field twist as AGNs are presently considered to be a likely source of high-energy neutrinos [1]. The VEP arises as different flavors of neutrinos may couple differently to gravity [2–4]. This essentially results from the realization that flavor eigenstates of neutrinos may be the admixture of the gravity eigenstates of neutrinos with different gravitational couplings. A magnetic field twist occurs when the direction of the magnetic strength lines in the plane transverse to the neutrino momentum originating from AGN may not be fixed. Several general descriptions of the possible effects of magnetic field twist are available [5], as well as related to Sun [6], Supernovae [7] and the early Universe [8].

The present study is particularly welcome as the new under ice or water Čerenkov light detector arrays namely AMANDA, Baikal (as well as NESTOR and ANTARES), commonly

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known as high-energy neutrino telescopes, based on muon detection will have not only the energy, angle and flavor resolutions but also possibly the particle and antiparticle resolution in the electron neutrino channel near the Glashow resonance energy,  $E \sim 6.4 \cdot 10^6$  GeV [9–12]. These characteristics make these neutrino telescopes especially suitable for the study of high-energy neutrino conversions.

We study here the spin-flip effect for Majorana type neutrinos in the vicinity of the cores of active galaxies which we hereafter refer to as AGNs. Some AGNs give off a jet of matter that stream out from the core in a transverse plane and produces hot spots when the jet strikes the surrounding matter at its other ends. For a discussion of neutrino spin-flip in jets and hot spots, see [13]. Previously, the spin-flip effects for AGN neutrinos due to VEP are studied in [14,15]. The VEP is parameterized by a dimensionless parameter  $\Delta f$ . In [14], by demanding an adiabatic conversion to occur, a lower bound on neutrino magnetic moment  $\mu$  was obtained in terms of  $\Delta f$ , whereas in [15], the effect of possible random fluctuation in the magnetic field of AGN,  $B_{AGN}$  on neutrino spin precession is considered. In [16], neutrino spin-flip in AGN due to gravitational effects (not due to VEP) and due to the presence of a magnetic field is studied. Here we address two aspects of spin-flip for high-energy neutrinos originating from AGNs, viz, the spin (flavor)-precession with (or without) VEP and the twist in  $B_{AGN}$ ; and the adiabatic/nonadiabatic conversion due to an interplay of twist in  $B_{AGN}$  and the VEP. We point out that, for latter type of conversion effect, a  $\Delta f$  of the order of  $10^{-39} - 10^{-29}$  depending on  $\delta m^2$  gives reasonably large conversion probabilities. In particular, we point out that the neutrino spin-flip in AGN induced by an interplay of VEP and twist in  $B_{AGN}$  may give rise to *changes* in particle/antiparticle ratio as compared to no spin-flip situation in electron neutrino channel near the Glashow resonance energy.

The plan of the rest of the paper is as follows. In section II, we briefly discuss a matter density and a magnetic field profile in AGN. In section III, we discuss the spin (flavor)-precession due to VEP and determine the value of  $\Delta f$  needed to have the precession probability greater than 1/2. In the same section, we consider in some detail the adiabatic and non adiabatic conversions induced by an interplay of a conceivable twist in  $B_{AGN}$ , and the VEP and estimate the resulting neutrino spin (flavor)-conversion probabilities. In section IV, we discuss a possible observational consequence of neutrino spin-flip in AGN and contrast it with the pure vacuum flavor oscillations. Finally in section V, we summarize our results.

## II. THE MATTER DENSITY AND MAGNETIC FIELD IN AGN

Neutrino spin-precession in the context of the Sun was discussed in [17]. It was pointed out that the matter effects tend to suppress the neutrino spin-precession effect. As shown below, for AGN, matter effects arising due to coherent forward scattering of neutrinos off the matter particle background are negligible<sup>1</sup>. The essential conditions needed for appreciable spin-precession are: i)  $\mu B \Delta r \gtrsim 1$ , i.e.,  $B$  must be large enough in the region of width  $\Delta r$ ; ii) the smallness of the matter effects, so that neutrino spin-precession is not suppressed (see below); and iii) there should be no reverse spin-precession of neutrinos on their way to earth. As for the third essential condition the typical observed intergalactic magnetic field for the nearby galaxies is estimated to be  $\sim O(10^{-9})$  G at a scale of Mpc, where  $1 \text{ pc} \sim 3 \cdot 10^{18} \text{ cm}$  [19]. Taking a typical distance between the earth and the AGN as  $\sim O(10^2)$  Mpc, we note that the effect induced by intergalactic and galactic magnetic field is quite small as the galactic magnetic field is  $\sim O(10^{-6})$  G, thus causing negligible reverse neutrino spin-precession.

According to [20], the matter density in the vicinity of AGN has the following profile:  $\rho(x) = \rho_0 f(x)$  where  $\rho_0 \simeq 1.4 \cdot 10^{-13} \text{ g/cm}^3$  and  $f(x) \simeq x^{-2.5}(1 - 0.1x^{0.31})^{-1}$  as we take the AGN photon luminosity to be  $10^{46} \text{ erg/s}$  with  $x \equiv r/R_S$ ,  $R_S$  being the Schwarzschild radius of AGN:  $R_S \simeq 3 \cdot 10^{13} \left(\frac{M_{AGN}}{10^8 M_\odot}\right) \text{ cm}$ . We take the distance traversed by the neutrinos to be  $10 < x < 100$  in the vicinity of AGN. These imply that the width of the matter traversed by neutrinos in the vicinity of the AGN is  $l_{AGN} \sim (10^{-2} - 10^{-1}) \text{ g/cm}^2$ . In the presence of matter, the effective width of matter needed for appreciable neutrino spin-flip, on the other hand, is  $l_0 \equiv \sqrt{2\pi} m_N G_F^{-1} \sim 2 \cdot 10^9 \text{ g/cm}^2 \gg l_{AGN}$ . Hence, from now on, we ignore the matter effects.

We consider now the magnetic field in the vicinity of AGN with the following profile [20]

$$B_{AGN}(x) = B_0 g(x), \tag{1}$$

where  $B_0 \sim 5.5 \cdot 10^4 \text{ G}$  and  $g(x) = x^{-1.75}(1 - 0.1x^{0.31})^{-0.5}$  for  $10 < x < 100$ . We will use this  $B_{AGN}$  in our estimates as an example.

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<sup>1</sup>Similar estimate for other astrophysical systems like Sun and Supernovae shows that the matter effects are indeed non negligible in most part of these systems [18].

### III. NEUTRINO SPIN-FLIP DUE TO VEP AND TWIST IN $B_{AGN}$

The evolution equation for the two neutrino state for vanishing gravity and vacuum mixing angles may be written in a frame rotating with the magnetic field as [21]

$$i\dot{\psi} = H_{eff}\psi, \quad (2)$$

where  $\psi^T = (\nu_e, \bar{\nu}_\alpha)$  and  $H_{eff}$  is a  $2 \times 2$  matrix with  $H_{11} = 0$ ,  $H_{12} = H_{21} = \mu B$  and  $H_{22} = \delta - V_G + \dot{\phi}$ . Here  $\alpha = \mu$  or  $\tau$ , and  $\dot{\phi} \equiv d\phi/dr$  defines the direction of rotation of  $B_{AGN}(\equiv B)$  in the plane orthogonal to the neutrino momentum.  $\delta = \delta m^2/2E$ , where  $\delta m^2 = m_j^2 - m_1^2 > 0$  with  $E$  being the neutrino energy and  $j = 2$  or  $3$ . In Eq. (2), for latter convenience, we have subtracted from the lower diagonal element, the upper diagonal element of the effective Hamiltonian in order to make the upper diagonal element equal to zero. This is equivalent to the renormalization of the two neutrino wave functions by the same factor, which does not change the relevant precession (conversion)/survival probabilities [21].  $V_G$  is the effective potential felt by the neutrinos at a distance  $r$  from a gravitational source of mass  $M$  due to VEP and in its rather simpler form is given by [2]

$$V_G \equiv \Delta f \beta(r) E, \quad (3)$$

where  $\Delta f = (f_\alpha - f_1)(f_\alpha + f_1)^{-1}$  and  $\beta(r) = G_N M r^{-1}$  is the gravitational potential in the Keplerian approximation, with  $G_N$  being the gravitational constant. Here  $f_1 G_N$  and  $f_\alpha G_N$  are the gravitational couplings respectively for  $\nu_e$  and  $\bar{\nu}_\alpha$ , such that  $f_1 \neq f_\alpha$ . Let us note that in the vicinity of AGN, the  $V_G$  due to AGN dominates [22].

We consider mainly the following two neutrino flavors:  $\nu_e$  and  $\nu_\tau$  in the subsequent discussion in this section, motivated by the fact that the initial fluxes of these neutrino states are estimated to be maximally asymmetric, typically with  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_e + \bar{\nu}_e) \lesssim 10^{-5}$ , according to various models of AGN [23]. Presently, the high-energy neutrino flux from AGNs can dominate over the atmospheric neutrino background typically for  $E \geq 10^6$  GeV. The current empirical upper bounds on high-energy neutrino flux, for instance, from AMANDA (B10), is relevant typically for  $E \leq 10^6$  GeV [24]. Let us mention here that the upper bound discussed in [25] does not apply to (diffuse) high-energy neutrino flux originating from cores of AGNs because these sources do not contribute dominantly to the observed ultrahigh-energy cosmic ray flux.

In this section, we intend to discuss in some detail the possible effects arising due to interaction of neutrino magnetic moment,  $\mu$ , with  $B_{AGN}$ , to enhance this ratio, that is, to

obtain  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_e + \bar{\nu}_e) \gg 10^{-5}$ . In this context, we now propose to study the various main possibilities arising from the relative comparison between  $\delta$ ,  $V_G$  and  $\dot{\phi}$  in Eq. (2).

**Case 1.**  $V_G = \dot{\phi} = 0$ . For constant  $B$ , we obtain the following expression for spin-flavor precession probability  $P(\nu_e \rightarrow \bar{\nu}_\alpha)$  by solving Eq. (2):

$$P(\nu_e \rightarrow \bar{\nu}_\alpha) = \left[ \frac{(2\mu B)^2}{(2\mu B)^2 + X^2} \right] \sin^2 \left( \sqrt{(2\mu B)^2 + X^2} \cdot \frac{\Delta r}{2} \right), \quad (4)$$

with  $X = \delta$ . We now discuss the relative comparison between  $2\mu B$  and  $\delta$  and evaluate  $P$  for corresponding  $\delta m^2$  range.

a)  $\delta \ll 2\mu B$ . Using  $B$  given in Eq. (1) for  $\mu \sim 10^{-12} \mu_B$  [26], the condition  $\delta \ll 2\mu B$  implies  $\delta m^2 \ll 5 \cdot 10^{-4} \text{ eV}^2$  with  $E \sim 10^6 \text{ GeV}$ . We take here  $\delta m^2 \sim 5 \cdot 10^{-6} \text{ eV}^2$  as an example. The expression (4) for  $P$  then reduces to

$$P(\nu_e \rightarrow \bar{\nu}_\alpha) \simeq \sin^2(\mu B \Delta r). \quad (5)$$

The phase of  $P$  can be of the order of unity if  $\mu B \Delta r = \frac{\pi}{2}$  (or if  $\mu B \Delta r \gtrsim 1$ ) for a constant  $B$ . Evidently, this  $P$  is independent of  $E$ . According to Eq. (1), the  $B_{AGN}$  varies with distance so that to have maximal depth of spin-flavor precession, we need to integrate the strength of the magnetic field along the neutrino trajectory. Thus, for maximal depth of  $\nu_e \rightarrow \bar{\nu}_\alpha$  precession, we require in Eq. (5) that

$$\mu \int_0^{r'} dr' B(r') \gtrsim 1. \quad (6)$$

We note that Eq. (5) [along with Eq. (6)] give  $P(\nu_e \rightarrow \bar{\nu}_\alpha) > 1/2$  for the  $B_{AGN}$  profile given by Eq. (1) with  $\mu \sim 10^{-12} \mu_B$ . Thus, an energy independent permutation (exchange) between  $\nu_e$  and  $\bar{\nu}_\alpha$  may result with  $P > 1/2$ . This energy *independent* permutation of energy spectra of  $\nu_e$  and  $\bar{\nu}_\alpha$  for small  $\delta m^2$  follows from the fact that Eq. (5) also gives  $P(\bar{\nu}_\alpha \rightarrow \nu_e)$  since we are considering a two neutrino state system. For another magnetic field strength profile of AGN [27] we obtain  $P(\nu_e \rightarrow \bar{\nu}_\alpha) > 1/2$  for  $\mu \sim 2 \cdot 10^{-16} \mu_B$  [this profile suggests a constant magnetic field  $\sim O(10^4) \text{ G}$  for  $x \gtrsim 10$ ]. We thus obtain the same  $P$  value ( $P > 1/2$ ) with a 4 orders of magnitude small  $\mu$  for this  $B_{AGN}$  profile for same  $\delta m^2$ . Therefore, if  $\mu$  turns out to be  $\sim O(10^{-16}) \mu_B$  and if empirically it is found that, for instance,  $P(\nu_e \rightarrow \bar{\nu}_\tau) > 1/2$  for small  $\delta m^2$  then this situation may be an evidence for the latter  $B_{AGN}$  profile. Let us further note that this small value of  $\delta m^2$  ( $\delta m^2 \sim 5 \cdot 10^{-6} \text{ eV}^2$ ) is not only interesting in the context of Sun [6] but also Supernovae [7].

b)  $\delta \simeq 2\mu B$ . Here  $\delta m^2$  corresponds to  $5 \cdot 10^{-4} \text{ eV}^2$ . In this case expression (4) for  $P$  reduces to

$$P(\nu_e \rightarrow \bar{\nu}_\alpha) \simeq 1/2 \sin^2(\sqrt{2}\mu B \Delta r). \quad (7)$$

Thus, for  $\delta m^2 \simeq 5 \cdot 10^{-4} \text{ eV}^2$ , energy dependent distortions may result in *survived* and *precessed* neutrino energy spectra with  $P \lesssim 1/2$ .

c)  $\delta \gg 2\mu B$ , that is,  $\delta m^2 \gg 5 \cdot 10^{-4} \text{ eV}^2$ . Energy dependent distortions may result for relatively large  $\delta m^2$  with  $P < 1/2$ . For instance, consider  $\delta m^2 \sim 10^{-3} \text{ eV}^2$  relevant for atmospheric neutrino problem [28]. The  $(\nu_e + \bar{\nu}_e)/(\nu_\tau + \bar{\nu}_\tau)$  ratio as well as  $(\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu)$  will have energy dependence in this case. Among the  $\nu_\mu$  and  $\nu_\tau$  channels, the spin-flavor precessions lead to an energy dependent  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_\mu + \bar{\nu}_\mu)$ . This situation may be realized by replacing  $\nu_e$  by  $\nu_\mu$  and  $\alpha$  by  $\tau$  in Eq. (2) with the corresponding changes in  $\delta m^2$  and in  $V_G$ . For comparison, let us note that the pure vacuum flavor oscillations lead to an energy independent ratio equal to  $1/2$ , that is,  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_\mu + \bar{\nu}_\mu) \sim 1/2$  [29]. Therefore, an energy dependent ratio different from  $1/2$  may provide an evidence for high-energy neutrino spin-flip in AGN. It is relevant here to mention that the future/existing high-energy neutrino telescopes may attempt to measure the three ratios  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_\mu + \bar{\nu}_\mu)$ ,  $(\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu)$  as well as  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_e + \bar{\nu}_e)$  of the absolute fluxes of high-energy neutrinos and possibly the energy dependence in this ratio [see section IV for some further discussion].

Let us note that all these spin (flavor)-precession situations are realized without VEP and magnetic field twist in  $B_{AGN}$  as a spin (flavor)-precession for AGN neutrinos may develop even without VEP and gravitational neutrino flavor dependent effects. Thus, the cause of change (as compared to no precession situation) in the ratios of the  $(\nu_\tau + \bar{\nu}_\tau)$ ,  $(\nu_\mu + \bar{\nu}_\mu)$  and  $(\nu_e + \bar{\nu}_e)$  fluxes, as well as an energy dependence in these ratios, in future/existing high-energy neutrino telescopes may not only be attributed to VEP and/or gravitational effects depending on relevant  $\delta m^2$  range.

**Case 2.**  $V_G = 0$ ,  $\dot{\phi} \neq 0$ . For constant  $B$  and  $\dot{\phi}$ , we obtain the expression for precession probability (for small  $\delta$ ) by substituting  $\dot{\phi}$  for  $X$  in Eq. (4). We first take  $\dot{\phi} \sim 2\mu B$ , thus  $\delta \ll \dot{\phi}$  for  $\delta m^2 \sim 5 \cdot 10^{-6} \text{ eV}^2$  [as considered in case 1a)]. Note that in this expression for precession probability, the sign of  $\dot{\phi}$  is unimportant. It is natural to suggest that the total rotation angle of the AGN magnetic field is restricted by  $\Delta\phi \lesssim \pi$ . Thus, for instance, a twist appears, when high-energy neutrinos cross the toroidal magnetic field with magnetic strength lines winding around the spherically accreting matter disk in AGN. In this case the maximal

rotation angle is  $\pi$ , i.e., the above bound is satisfied. The field twist can be characterized by the scale of the twist,  $r_\phi$ , such that  $r_\phi \equiv \pi/\dot{\phi}$ , so that on the way, the total rotation angle (for uniform rotation), equals to  $\Delta\phi = \pi$ . Let us define the critical rotation scale as  $r_\phi^c \equiv \pi/2\mu B$  [30]. Note that this  $r_\phi^c$  coincides with the precession length  $l_p [\equiv (2\mu B)^{-1}]$  apart from a factor of  $\pi$  and on dimensional grounds is the simplest possibility. For appreciable magnetic field twist effects, evidently we require  $r_\phi \lesssim r_\phi^c$ . Comparing  $r_\phi^c$  with the distance from the center of AGN in units of  $x$  (or  $R_S$ ), we find that  $r \sim r_\phi^c$  for a  $B$  that is smaller than the available  $B_{AGN}$  given by Eq. (1). In this case  $P(\nu_e \rightarrow \bar{\nu}_\alpha)$  reduces to Eq. (7). Thus, for small  $\delta m^2$ , we obtain here  $P \lesssim 1/2$ . This case can therefore be differentiated from the previous one by concentrating on  $P$  value. For small  $\delta m^2$  [case 1a)] previously we have  $P > 1/2$ . The magnetic field twist effects here may give rise to energy independent spin (flavor)-precession between  $\nu_e$  and  $\bar{\nu}_\tau$ . However, here unlike previous case for small  $\delta m^2$ , the required  $B$  has an upper bound for a naturally scaled field twist. For  $\dot{\phi} \ll 2\mu B$ , we obtain case 1a) whereas for  $\dot{\phi} \gg 2\mu B$ , we obtain case 1c).

For  $\delta \sim -\dot{\phi}$ , the spin (flavor)-precession results from a cancellation between  $\delta$  and  $\dot{\phi}$  which for a naturally scaled  $\dot{\phi}$  corresponds to  $\delta m^2 \lesssim 5 \cdot 10^{-4} \text{ eV}^2$  with  $P > 1/2$ , whereas the opposite sign of  $\dot{\phi}$  results in suppression of  $P$ . Thus, for large  $\delta m^2$  (but comparable to  $\dot{\phi}$ ), energy dependent distortions may occur with  $P > 1/2$ . For  $\delta \gg \dot{\phi}$ , this case reduces to case 1c).

**Case 3.**  $V_G \neq 0$ ,  $\dot{\phi} = 0$  (with small  $\delta$ , that is,  $\delta \ll 2\mu B$ ). For constant  $V_G$  and  $B$ , we obtain from Eq. (4) the relevant precession probability expression by replacing  $X$  with  $V_G$ . If  $V_G \ll 2\mu B$  then using Eq. (1) and Eq. (3), we obtain  $\Delta f \ll 6 \cdot 10^{-32}$ . We take here  $|\Delta f| \lesssim 10^{-34}$  as our criteria and so consequently the corresponding  $P$  reduces to (5). This results in  $P > 1/2$  with no energy dependence. Thus this case coincides with case 1a) for small  $\Delta f$  ( $\lesssim 10^{-34}$ ) depending on the given  $B_{AGN}$  profile. Consequently, if there is a VEP at the level of  $10^{-34}$  or less, a spin (flavor)-precession for neutrinos may occur in the vicinity of AGN with small  $\delta m^2$ . Evidently, this value of  $\Delta f$  is independent of the gravity mixing angle [2]. Let us note in passing that this value of  $\Delta f$  is (much) lower than the one obtained in [4]. For  $\Delta f \gtrsim 10^{-34}$ , energy dependence in  $P$  results with  $P \lesssim 1/2$ . For large  $\delta$  ( $\delta \simeq V_G$ ) see case 5 and if  $\delta \gg V_G$  then this case reduces to 1c). The upper bound for  $\Delta f$  obtained in this case has only a linear energy dependence, whereas the other necessary requirement [Eq. (6)] does not depend on  $E$  for small  $\delta$ . This is in sharp contrast to the situation discussed in case 5, where both the level crossing as well as the adiabaticity conditions depend on  $E$ . Thus, to

summarize, we have pointed out in this case that for high-energy neutrinos originating from AGN, a spin (flavor)-precession may develop in the vicinity of AGN if  $\Delta f \lesssim 10^{-34}$  yielding, for instance,  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_e + \bar{\nu}_e) \gg 10^{-5}$ .

The observational consequences of the high-energy neutrino spin-flavor precessions discussed in the previous three cases are the energy dependence in the relevant ratio of the fluxes as well as a possible change in the ratios with respect to the pure vacuum flavor oscillations. With the improved information on the relevant neutrino mixing parameters, these cases may in principle be disentangled from each other.

**Case 4.**  $V_G = \dot{\phi}$  (for small  $\delta$ ). This results in conversion effect in contrast to the previously considered three cases [which are spin (flavor)-precession effects].

Two conditions are essential for an adiabatic conversion: i) level crossing and ii) adiabaticity. The level crossing is obtained by equating the diagonal element of the effective Hamiltonian in Eq. (2), i.e.,  $V_G = \dot{\phi}$  implying  $\Delta f \propto E^{-1}$  (or a *linear* dependence of  $\dot{\phi}$  on  $E$ ). For  $\bar{\nu}_e \rightarrow \nu_\alpha$  conversions, if  $\Delta f > 0$  (both for particles and antiparticles) then there is no level crossing as  $\dot{\phi}$  is negative for this channel. If  $\Delta f < 0$ , then the level crossing shifts to antiparticle channel ( $\bar{\nu}_e \rightarrow \nu_\alpha$ ). Thus, a simultaneous deficit/enhancement in both  $\nu_e$  and  $\bar{\nu}_e$  spectra (and in  $\nu_\tau$  and  $\bar{\nu}_\tau$  spectra) is not expected due to an interplay of VEP and the twist in  $B_{AGN}$  unless  $\Delta f$  has different sign for particles and antiparticles. The level crossing is induced by a naturally scaled field twist for  $\Delta f \lesssim 10^{-34}$ , that is, when  $r_\phi^c/r \gtrsim 1$  (see case 2 also). Let us note that this level crossing is induced by an interplay of magnetic field twist and VEP for neutrinos with small  $\delta m^2$  ( $\delta m^2 < 5 \cdot 10^{-6} \text{ eV}^2$ ). This is a characteristically distinct feature of a more realistic situation of having magnetic strength lines winding around the nearly spherical matter disk. However, level crossing alone is not a sufficient condition for a complete conversion. As stated earlier, adiabaticity is the other necessary condition that determines the extent of conversion. If there is only level crossing and no adiabaticity at the level crossing then there is no conversion of electron neutrinos into anti tau neutrinos. In the remaining part of this case, we discuss quantitatively the latter condition, that is, the adiabaticity.

The adiabaticity condition assumes the slowness of variation in  $V_G$  and is given by [31]:

$$\kappa_R = \frac{2(2\mu B)^2}{|\dot{V}_G|}. \quad (8)$$

This is the adiabaticity parameter in the resonance for uniform magnetic field twist ( $\ddot{\phi} = 0$ ). A conversion is adiabatic if  $\kappa_R \gtrsim 1$ . Notice that here  $\kappa_R \propto E^{-1}$ . Since  $\kappa_R$  depends



(quadratically) on  $B$ , thus adiabaticity of conversion is essentially determined and controlled by the given  $B$  profile. By requiring an adiabatic conversion to occur, we can obtain  $B_{\text{ad}}$  from Eq. (8). Using Eq. (1) and for  $\Delta f \sim 10^{-34}$  (a requirement of level crossing), we obtain  $B_{\text{ad}} < B_{\text{AGN}}$ . It is interesting to note that the  $B_{\text{ad}}$  does not depend on any  $B$  profile of AGN, it is determined rather by the gradient of  $V_G$ . Thus, an adiabatic conversion may occur for  $\Delta f \sim 10^{-34}$  or less depending on  $\delta m^2$  in a uniform magnetic field twist. Let us emphasize that this adiabatic level crossing is induced by the change in the gravitational potential rather than the change in effective matter density. A general expression for neutrino spin-flavor conversion probability including the effect of non adiabaticity ( $\kappa_R < 1$ ), using Eq. (2) is [32]

$$P(\bar{\nu}_e \rightarrow \nu_\tau) = \frac{1}{2} - \left\{ \frac{1}{2} - \exp\left(-\frac{\pi}{2}\kappa_R\right) \right\} \cos 2\theta_{B_i} \cos 2\theta_{B_f}, \quad (9)$$

where  $\tan 2\theta_{B_i} = (2\mu B)/(\delta - V_G)$  is evaluated at the high-energy neutrino production site in the vicinity of AGN and  $\tan 2\theta_{B_f} = (2\mu B)/(\delta - V_G)$  is evaluated at the exit. In Fig. 1, we display  $P$  using Eq. (9) for some representative values of  $\Delta f$  with  $\delta m^2 \sim 10^{-10}$  as a function of  $E$  for illustrative purpose only. From Fig. 1, we notice that for  $E \sim 6.4 \cdot 10^6$  GeV, the  $P$  is rather large ( $\sim 0.6 - 0.7$ ), thus leading to a suppression in the  $\bar{\nu}_e$  flux.

Nonuniform field twist ( $\ddot{\phi} \neq 0$ ) changes the adiabaticity condition (8). It now reads

$$\kappa_\phi = \frac{2(2\mu B)^2}{|\dot{V}_G - \ddot{\phi}|}. \quad (10)$$

Thus, for  $\ddot{\phi} \simeq \dot{V}_G$ , we may have a large enhancement in  $\kappa_\phi$ . For a naturally scaled field twist, the total rotation angle for a nonuniform magnetic field twist is given by [7]

$$\Delta\phi \sim \kappa_R^{-1}, \quad (11)$$

i.e., the total rotation angle is given by the inverse of the adiabaticity parameter for a uniform magnetic field twist. Clearly, only modest improvement in  $\kappa_R$  may be achieved for a naturally scaled magnetic field twist. The corresponding conversion probability  $P$  in this case is energy dependent. Thus, observationally, we may obtain here  $(\nu_\tau + \bar{\nu}_\tau) \sim (\nu_e + \bar{\nu}_e)$ , due to an adiabatic conversion induced by an interplay of  $\dot{\phi}$  and  $V_G$  in the vicinity of the AGN. For large  $\delta$ , comparable to  $V_G$  and  $\dot{\phi}$ , see case 6.

**Case 5.**  $V_G \simeq \delta$ ,  $\dot{\phi} = 0$ . This situation also results in conversion effects (as opposed to cases 1-3), however see case 3 also. The level crossing implies

$$\Delta f \simeq 2 \cdot 10^{-34} \left( \frac{\delta m^2}{10^{-5} e V^2} \right). \quad (12)$$

Note that relative sign between  $\delta$  and  $V_G$  is important for level crossing.

If  $\Delta f > 0$ , both for particles and antiparticles, then both  $\nu_e$  and  $\bar{\nu}_e$  will transform simultaneously, whereas if  $\Delta f < 0$ , both for particles and antiparticles, no level crossing takes place. On the other hand, if  $\Delta f$  changes sign for particles and antiparticles, level crossing between particles *or* antiparticles will take place. Thus, this case can be distinguished from the previous case.

It is important to note that from the level crossing it follows that  $\Delta f \propto E^{-2}$ , i.e., an inverse quadratic  $E$  dependence on  $\Delta f$ . Thus, the level crossing induced by the VEP alone has a *different* energy dependence on  $\Delta f$  as compared to the level crossing induced by an interplay of  $\dot{\phi}$  and the VEP (see previous case). The relevant adiabaticity condition may be written as

$$B_{\text{ad}} \gtrsim 3 \cdot 10^2 \text{ G} \left( \frac{10^{-12} \mu_B}{\mu} \right) \left( \frac{\Delta f}{10^{-29}} \right)^{\frac{1}{2}} \left( \frac{10 R_S}{r} \right). \quad (13)$$

We note that  $B_{\text{ad}} \lesssim B_{\text{AGN}}$  for  $10 < x < 100$ . The adiabaticity parameter here has the *same* energy dependence on  $E$  as in case 4. Thus, the adiabatic conversion may occur giving rise to energy dependent distortions with corresponding conversion probability greater than 1/2. For large  $\delta$  whereas a spin (flavor)-precession is suppressed [see case 1b) and 1c)], an adiabatic conversion may result with  $P > 1/2$  for large  $\Delta f$  thus resulting in correspondingly different observational consequences. For  $\delta \ll V_G$  this case reduces to case 3 whereas for  $\delta \gg V_G$ , we obtain case 1c).

It follows from the discussion in cases 4 and 5 that a *nonzero*  $\Delta f$  is needed to induce an adiabatic level crossing with  $P > 1/2$ . It is in contrast to cases 1, 2 and 3 where a spin-flip may occur through spin (flavor)-precession without  $\Delta f$  with  $P > 1/2$ .

**Case 6.** If  $\delta$ ,  $V_G$  and  $\dot{\phi}$  are of the same order of magnitude then we have two possibilities: the  $V_G$  and  $\dot{\phi}$  terms cancel each other. Then, effectively case 1 a) is recovered. On the other hand, if  $V_G$  and  $\dot{\phi}$  tend to add up, then effectively (apart from a factor of 2) we obtain either case 2 or case 5.

From the discussion in the previous cases, it follows that neutrino spin-flavor precessions/conversions may occur in several situations depending on the range of relevant neutrino mixing parameters.

## IV. POSSIBLE OBSERVATIONAL CONSEQUENCES OF NEUTRINO SPIN-FLIP IN AGN

In this section, we discuss in some detail the potential of the future high-energy neutrino telescopes to possibly determine some observational consequences of neutrino spin-flip in AGN through examples.

The planned high-energy neutrino telescopes may in principle differentiate between the three neutrino flavors ( $e$ ,  $\mu$  and  $\tau$ ) considered so far in this paper [10]. The particular relevance here is of the electron neutrino channel, in which the downward going  $\bar{\nu}_e$  interaction rate (integrated over all angles) is estimated to be an order of magnitude higher than that of  $(\nu_e + \bar{\nu}_e)$  per Megaton per year at  $E \sim 6.4 \cdot 10^6$  GeV [33]. This an order of magnitude difference in interaction rate of downward going  $\bar{\nu}_e$  relative to  $(\nu_e + \bar{\nu}_e)$  deep inelastic scatterings is due to Glashow resonance encountered by  $\bar{\nu}_e$  with  $E \gtrsim 10^6$  GeV when they interact with electrons near or inside the detector. The upward going  $\bar{\nu}_e$ , on the other hand, while passing through the earth, at these energies, are almost completely absorbed by the earth. Thus, for instance, if  $E \sim 6.4 \cdot 10^6$  GeV, an energy resolution  $\Delta E/E \sim 2\Gamma_W/M_W \sim 1/20$ , where  $\Gamma_W \sim 2$  GeV is the width of Glashow resonance and  $M_W \sim 80$  GeV, may be needed to empirically differentiate between  $\nu_e$  and  $(\nu_e + \bar{\nu}_e)$ . The  $\bar{\nu}_e$  and  $(\nu_e + \bar{\nu}_e)$  essentially produce a single shower event. Thus, the planned high-energy neutrino telescopes may in principle attempt to measure the  $\nu_e/\bar{\nu}_e$  ratio near the Glashow resonance energy in addition to identifying  $(\nu_\tau + \bar{\nu}_\tau)$ ,  $(\nu_\mu + \bar{\nu}_\mu)$  as well as  $(\nu_e + \bar{\nu}_e)$  events separately by measuring the ratio of these fluxes. This may allow us to corroborate the neutrino mixing effects somewhat meaningfully.

The near future large high-energy neutrino telescopes may attempt to utilize this enhancement in the  $\bar{\nu}_e$  cross-section over electrons to measure the high-energy (antielectron) neutrino flux. Therefore, it is useful to ask for what possible range of neutrino mixing parameters, the high-energy  $\bar{\nu}_e$  flux could be suppressed (or enhanced). In the remaining part of this section, we elaborate such a possibility. Let us remark here that at present the absolute normalization of the high-energy neutrino flux is basically unknown [34]. The suppression or enhancement for high-energy  $\bar{\nu}_e$  flux correlated to the direction of the source alongwith the corresponding changes in the remaining neutrino flavors as pointed out in this paper depends only on the neutrino mixing parameters (and on the source).

Let us comment on the implications of current atmospheric and solar neutrino results on our analysis presented in section III. A recent global three neutrino oscillation study [35] of

neutrino data indicates that the best fitted  $\delta m^2$  and  $\sin^2 2\theta$  values to solve the atmospheric neutrino problem in terms of  $\nu_\mu \rightarrow \nu_\tau$  oscillations are typically  $\sim 10^{-3} \text{ eV}^2$  and  $\sim 1$ . On the other hand, presently there exists more than one solution to solve the solar neutrino problem in terms of  $\nu_e \rightarrow \nu_\alpha$  oscillations. For SMA (MSW) solution, the  $\delta m^2$  and  $\sin^2 2\theta$  values are  $\sim 10^{-5} \text{ eV}^2$  and  $\sim 10^{-2}$ , for LMA (MSW) solution, these are  $\sim 10^{-5} \text{ eV}^2$  and  $\sim 1$ , whereas for VAC solution, these are  $\sim 10^{-10} \text{ eV}^2$  and  $\sim 1$ , respectively. The LOW solution values are close to that of LMA (MSW) solution. Following [36], and using these values of  $\delta m^2$  and  $\sin^2 2\theta$ , we note that *energy independent* pure vacuum neutrino flavor oscillations occur between the AGN and the earth irrespective of the specific oscillation solution for solar neutrino problem.

In order to further contrast the spin-flip effects studied in this paper with the pure vacuum flavor oscillations for (downward going) high-energy neutrinos originating from AGNs let us emphasize that vacuum flavor oscillations lead to an energy independent same ratio for the three flavors, i.e.,  $F_e : F_\mu : F_\tau = 1 : 1 : 1$ , where  $e \equiv (\nu_e + \bar{\nu}_e)$ , etc., at the level of intrinsic electron neutrino flux  $F_e^0$ . It is so because firstly the matter effects are basically negligible in the vicinity of the yet known sources of high-energy neutrinos as well as between the source and the earth and secondly the sources are considered to be cosmologically distant and that the intrinsic ratio of the high-energy neutrinos is  $1 : 2 : 0$ . Therefore, a deviation from  $1 : 1 : 1$  for the final ratios correlated to the direction of the source as well as an energy dependence may provide an example of neutrino spin-flip effect in AGN. A simple relevant remark is in order here. The pure vacuum flavor oscillation length is given by,  $l_f \sim 4E/\delta m^2$ , whereas the spin-flavor precession length is (defined earlier as  $l_p$ ),  $l_{sf} \sim 1/2\mu B$ . For  $\delta m^2$  range under discussion, i.e.,  $10^{-10} \leq \delta m^2/\text{eV}^2 \leq 10^{-3}$ , and for the typical  $E$  value range, i.e.,  $10^6 \leq E/\text{GeV} \leq 10^7$ , with  $\mu \sim 10^{-12}\mu_B$  and  $B \equiv B_{AGN}$  given by Eq. (1), we note that  $l_{sf} < l_f$ . Therefore, spin-flip effects may dominate in the vicinity of the AGN. The pure vacuum flavor oscillations that may dominate between the AGN and the earth are essentially an energy independent effect. Thus the energy dependence due to neutrino spin-flip in AGN in for instance  $(\nu_e + \bar{\nu}_e)/(\nu_\tau + \bar{\nu}_\tau)$  will survive providing a signature of neutrino spin-flip in AGNs (see Fig. 1).

To disentangle the neutrino spin-flip effects from pure vacuum flavor oscillation effects, a suitable energy range  $\sim 4M_W\Gamma_W/m_e$  can be centered at  $E \sim 6.4 \cdot 10^6 \text{ GeV}$ . The vacuum neutrino flavor mixing parameters (namely  $\delta m^2$  and  $\sin^2 2\theta$ ) will presumably be get measured in various terrestrial experiments and so the corresponding effects for high-energy neutrinos

can reliably be isolated from the spin-flip effects discussed here.

In case of spin-flavor precessions between  $(\nu_\mu + \bar{\nu}_\mu)$  and  $(\nu_\tau + \bar{\nu}_\tau)$ , which may happen for the range of  $\delta m^2$  values given in case 1c) of previous section, the observational consequence is a change in the value of  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_\mu + \bar{\nu}_\mu)$  ratio as compared to that of pure vacuum flavor oscillations along with possible energy dependence. The empirical distinction between  $\nu_\mu$  and  $\bar{\nu}_\mu$  as well as  $\nu_\tau$  and  $\bar{\nu}_\tau$  is currently not envisaged for the typical high-energy neutrino telescopes. The spin-flavor precession effects discussed in cases 1-3 leads to precessions of the type  $\nu_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\tau$  and  $\bar{\nu}_e \rightarrow \nu_\mu, \nu_\tau$  simultaneously. Thus, in this case the ratio  $\nu_e/\bar{\nu}_e$  is the same, however, energy dependence in the ratio  $\nu_e/\bar{\nu}_e$  and a change in the non electron neutrino flux ratios here remain a distinctive feature of spin-flavor precessions depending on  $\delta m^2$  values. The energy dependence in  $\nu_e/\bar{\nu}_e$  ratio due to production should be essentially absent in case  $\nu_e$  and  $\bar{\nu}_e$  come from the same parent particle, for instance, from  $\mu$ .

There are several situations (case 4-5) as discussed in the previous section in which  $\nu_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\tau$  spin-flavor conversions may occur. As pointed out earlier in this section, distinction between  $\nu_e$  and  $\bar{\nu}_e$  may become possible near the Glashow resonance energy so this possibly gives a better chance to identify an observational consequence of neutrino spin-flip through spin-flavor conversions. For  $\Delta f \sim 10^{-34}$ , if VEP is different for neutrinos and antineutrinos then the energy dependent spin-flavor conversions as discussed in case 5 may give rise to change in  $\nu_e/\bar{\nu}_e$  ratio, in addition to change in  $(\nu_\tau + \bar{\nu}_\tau)$  or  $(\nu_\mu + \bar{\nu}_\mu)$ , which ever the case may be. However, if VEP is the same for neutrinos and antineutrinos, then this situation coincides with the previous situation of spin-flavor precession, i.e., no change in the  $\nu_e/\bar{\nu}_e$  ratio. Thus, for instance, absence (or enhancement, depending on sign of  $\Delta f$ ) of  $\bar{\nu}_e$  events near Glashow energy and energy dependence and enhancement in ratios of other neutrino flavors from an AGN may provide an observational consequence for neutrino spin-flip in AGN.

An interesting situation may arise after the incorporation of magnetic field twist effects, as discussed in case 4 which also give rise to change in  $\nu_e/\bar{\nu}_e$  ratio but for different (small)  $\delta m^2$  values, irrespective of nature of VEP. The pure  $\nu_e \rightarrow \bar{\nu}_e$  or  $\bar{\nu}_e \rightarrow \nu_e$  (though suppressed [37]) may also take place giving rise to changes in  $\nu_e/\bar{\nu}_e$  ratio (for instance, different from unity) possibly with no energy dependence or change in  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_\mu + \bar{\nu}_\mu)$  ratio. This can be realized from the discussion in case 4 of section III where it is pointed out that an interplay between VEP and a naturally scaled field twist leads to conversions in either  $\nu_e$  or  $\bar{\nu}_e$  channel but not in both channels simultaneously. Note that in this case nonzero  $\Delta f$  and

a nonzero  $\dot{\phi}$  is needed.

A relevant remark is that “matter like” effects induced by the presence of nonzero  $\Delta f$  (along with nonzero  $\dot{\phi}$ ) differentiates between particles and antiparticles. Thus, if the measurement of  $\nu_e/\bar{\nu}_e$  ratio for high-energy neutrinos originating from AGNs were to become feasible, it may at least in principle constrain  $\Delta f$  up to (much) smaller values than which can currently be achieved by neutrinos from other astrophysical sources [4]. Let us note that the change in  $\nu_e/\bar{\nu}_e$  ratio is not expected from pure vacuum flavor oscillations. This can be a characteristic observational consequence of incorporating the effect of possible (uniform) rotation of magnetic strength lines along the high-energy neutrino trajectories originating from AGNs.

The expected event rates for different neutrino flavors in  $\text{km}^3$  volume high-energy neutrino telescopes using the rather optimistic diffuse upper flux limits, as an example, given in Ref. [20] typically range, for  $E \sim 10^6$  GeV, as follows: the downward going  $(\nu_e + \bar{\nu}_e)$  event rate is typically  $\sim O(10^{1.5})$ , the downward going  $(\nu_\mu + \bar{\nu}_\mu)$  event rate is typically  $\sim O(10^2)$ , whereas the downward going  $(\nu_\tau + \bar{\nu}_\tau)$  event rate is typically  $\sim O(10^1)$ , all in units of per year per steradian, the downward going  $\bar{\nu}_e$  event rate for  $E \sim 6.4 \cdot 10^6$  GeV being approximately half an order of magnitude higher than the  $(\nu_e + \bar{\nu}_e)$  event rate in the high-energy neutrino telescopes [38]. The three flavors are expected to have different event topologies [39], thus providing some prospects to search for the observational consequences pointed out in this section.

Summarizing, a possible observational consequence of neutrino spin-flip in the high-energy neutrino telescopes include a *change* in the expected  $\nu_e/\bar{\nu}_e$  ratio correlated to the direction of source with an energy resolution  $\Delta E/E \lesssim 1/20$  near the Glashow resonance energy as well as a possible energy dependence in the ratio of the three flavors. Some of the other situations in neutrino spin-flip discussed here tend to overlap with the pure vacuum flavor oscillations scenario.

## V. RESULTS AND DISCUSSION

The intrinsic fluxes of the high-energy neutrinos ( $E \geq 10^6$  GeV) originating from AGN are estimated to have the following ratios:  $(\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu) \simeq 1/2$ ,  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_{e,\mu} + \bar{\nu}_{e,\mu}) \lesssim 10^{-5}$ . Thus, if an enhanced energy dependent  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_e + \bar{\nu}_e)$  ratio (as compared to no precession/conversion situation) is observed correlated to the direction of source for

high-energy neutrinos, then it may be either an evidence for a spin-flip through spin (flavor)-precession alone or through a resonant conversion in the vicinity of AGN due to an interplay of VEP and/or a conceivable magnetic field twist in  $B_{AGN}$  depending on the finer details of the relevant high-energy AGN neutrino spectra. The spin (flavor)-precession and/or conversion effects discussed in this paper may be distinguished from the pure vacuum flavor oscillations by observing the *energy dependence* of the high-energy neutrino flux profiles. A mutual comparison of the relevant [that is, for instance,  $(\nu_e + \bar{\nu}_e)$  and  $(\nu_\tau + \bar{\nu}_\tau)$ ] high-energy neutrino spectra may in principle isolate the mechanism of neutrino conversions in the vicinity of AGN.

The incorporation of a possible magnetic field twist induces a level crossing in the vicinity of AGN due to VEP. This conversion can be made adiabatically resonant for a naturally scaled magnetic field twist with  $\Delta f \lesssim 10^{-34}$ . A resonant character in the oscillations of high-energy neutrinos originating from AGN for vanishing gravity and vacuum mixings may not be induced otherwise. Thus, a breakdown in the universality of gravitational coupling of neutrinos at the level of  $10^{-34}$  or less depending on relevant  $\delta m^2$  may provide a possible cause for observing energy dependence and change in the three neutrino flavors w.r.t. pure vacuum flavor oscillations, assuming that there is no appreciable reverse neutrino spin-flip between AGN and the earth.

For small  $\delta m^2$  ( $\delta m^2 < 5 \cdot 10^{-6} \text{ eV}^2$ ) a spin (flavor)-precession may result in an energy independent permutation of the relevant neutrino spectra with the corresponding spin (flavor)-precession probability greater than 1/2. This spin (flavor)-precession may occur for small  $\Delta f$  ( $\Delta f \lesssim 10^{-34}$ ). The spin-flip may occur through resonant conversions induced by the VEP and/or field twist in  $B_{AGN}$  as well. Assuming that the information on  $\Delta f$  may be obtained from various terrestrial/extraterrestrial experiments, a mutual comparison between the survived and transformed high-energy AGN neutrinos may enable one to distinguish the mechanism of conversion. If for small  $\delta m^2$  ( $\delta m^2 < 5 \cdot 10^{-6} \text{ eV}^2$ ), an energy dependent permutation are obtained empirically with corresponding  $P > 1/2$  then this situation may be an evidence for a conversion effect due to an interplay of VEP and twist in  $B_{AGN}$ .

For large  $\delta m^2$  ( $\delta m^2 > 5 \cdot 10^{-6} \text{ eV}^2$ ), if energy dependent distortions and for instance a change in  $(\nu_\tau + \bar{\nu}_\tau)/(\nu_e + \bar{\nu}_e)$  is observed with the corresponding conversion probability greater than 1/2 then the cause may be a relatively large  $\Delta f$  ( $\Delta f > 10^{-34}$ ) and/or a naturally scaled magnetic field twist. The level crossing induced by VEP and/or field twist has a *different E* dependence thus in principle with the improved information on either  $\Delta f$  or the

scale of magnetic field twist, the cause of the conversion effect may be isolated. Further, as the energy span in the relevant high-energy AGN neutrino spectra is expected to be several orders of magnitude, therefore, energy dependent spin (flavor)-precession/conversion probabilities may result in distortions in some part(s) of the spectra for relevant neutrino species and may thus be identifiable in future high-energy neutrino telescopes.

A possible observational consequence of neutrino spin-flip in AGN in electron neutrino channel only can be an observed *change* in  $\nu_e/\bar{\nu}_e$  ratio (as compared to no spin-flip situation) near the Glashow resonance energy which may be a result of an interplay of VEP and the magnetic field twist. This feature is absent in pure vacuum flavor oscillations.

An additional feature of the present study is that it may provide useful information on the strength/profile of  $B_{AGN}$  if the cause of  $\nu_e \leftrightarrow \bar{\nu}_{\mu,\tau}$  conversion/precession can be established due to VEP and/or magnetic field twist for high-energy AGN neutrinos.

*Acknowledgments.* The authors thank World Laboratory (WL) for financial support within project E-13. HA also thanks Physics Division of National Center for Theoretical Sciences for financial support.

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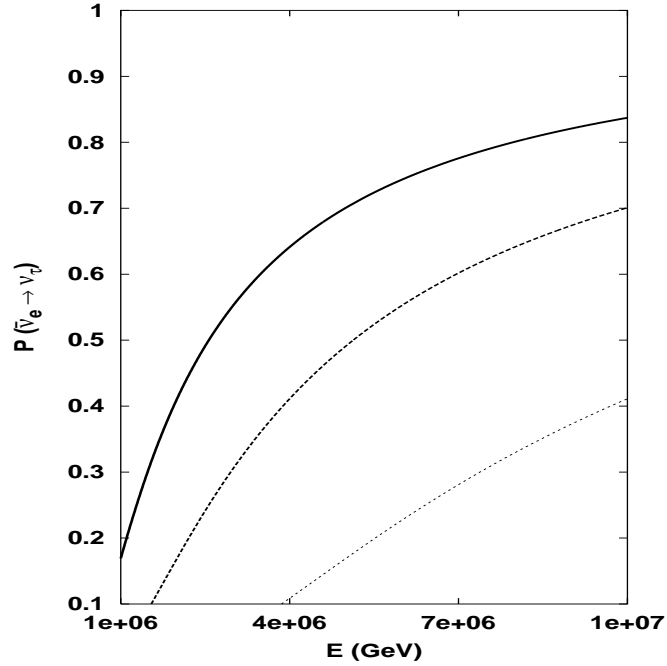


FIG. 1.  $P(\bar{\nu}_e \rightarrow \nu_\tau)$  as a function of  $E$  (GeV) for some representative values of  $\Delta f$  with  $\delta m^2 \sim 10^{-10} \text{ eV}^2$  and  $\mu \sim 10^{-12} \mu_B$  using Eq. (9) for illustrative purpose. Upper curve,  $\Delta f \sim 10^{-29}$ , middle curve,  $\Delta f \sim 5 \cdot 10^{-30}$ , lower curve,  $\Delta f \sim 2 \cdot 10^{-30}$ .